Physics 842 – Fall 2012
Classical Electrodynamics II

Problem Set #5 – due Tuesday October 23

1. Imagine that at time $t = 0$ we place a localized macroscopic charge distribution $\rho(\mathbf{r})$ inside a conducting medium of conductivity σ. Describe how the charge density evolves in time. (The time scale for the charge to disperse is called the Maxwell relaxation time.)

2. A capacitor of arbitrary shape is filled with a dielectric medium with dielectric constant ε. If that same capacitor were filled with a conducting medium of conductivity σ, it would have a resistance R. Find the capacitance C.

3. The magnetoresistivity tensor relates the electric field to the current density, $E_i = \rho_{ik} j_k$. For the Drude model of magnetoconductivity we discussed in class, calculate ρ_{ik} for a uniform conducting medium in the presence of a constant magnetic field B pointing in the z direction. (I essentially did this for 2D in class. Calculate the full 3D ρ_{ik} tensor.)

4. A vacuum diode consists of two parallel plates (cathode and anode) separated by a distance d, held at a constant potential difference ϕ_0, inside an evacuated chamber. Electrons are emitted from the cathode by heating it. (We will assume that the velocity of the emitted electrons is initially zero, which is a good approximation for large enough ϕ_0.)

   a) If the current density in the diode is j, calculate the electron velocity v and space charge density ρ as a function of x, where x=0 at the cathode and x=d at the anode. Assume that j is small enough that you can ignore the effects of ρ on the electric field.

   b) For large j we cannot neglect the space charge. Using Poisson's equation and current conservation, derive a differential equation for $\phi(x)$.

   c) Find a family of solutions to your equation that satisfy the boundary conditions $\phi(0)=0$ and $\phi(d)=\phi_0$. In the "space-charge limited" case, we have the additional condition that $d\phi/dx = 0$ at x=0. In this case, show that the current density j is proportional to $\phi_0^{3/2}$. This is a system that does not obey Ohm's Law!

Quiz #5

The quiz on Thursday, October 25, will consist of one of the following problems:

- Problems 1 - 4 on Problem Set #5
- Problems 1 - 3 at the end of Section 21