Physics 842 – Fall 2012

Classical Electrodynamics II

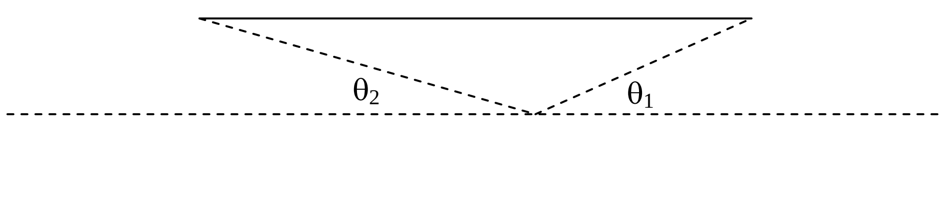
Problem Set #6 – due Tuesday October 30

1. Consider a classical model of a paramagnetic material, consisting of a gas of atoms with intrinsic magnetic dipole moments *m*, which can point in any direction. The gas has number density *n*. The gas is subject to a uniform applied magnetic field *B*. Using classical statistical mechanics, calculate the average magnetic moment per unit volume *M* at temperature *T*. Make a plot of *M* vs. *B*, for fixed *T*. (You can do this "by eye" or using Mathematica, whichever you prefer.) Calculate the magnetic susceptibility χ_m, in the limit $B \to 0$, and make a plot of χ_m vs. *T*. The temperature dependence you find is called the "Curie Law."

2. A solenoid of finite length has *n* turns of wire per unit length, carrying current *I*. In the limit of large *n*, show that the magnetic field *B* on the axis of the solenoid can be written in the following form:

$$B = \frac{2\pi nI}{c}\left(\cos(\theta_1) + \cos(\theta_2)\right)$$

where θ_1 and θ_2 are the angles between the z-axis and the ends of the coils, as seen from the observation point on the axis (see picture).



3. A right circular cylinder of radius *R* and length *L* is a uniformly-magnetized along its axis.

   a) Using the equivalence discussed in class between uniform magnetization and a surface current density, and the result from the previous problem, calculate the magnetic field *B* everywhere along the axis of symmetry – both inside and outside the cylinder. Put the origin $z = 0$ in the middle of the cylinder.

   b) Calculate *B* in the very center of the cylinder and at the end of the cylinder for the following limiting cases: $L >> R$, $L << R$, and $L = 2R$.

   c) Make a plot of *B* and *H* vs. *z* (where *z* is the coordinate along the symmetry axis) for the case of $L = 2R$. Put both *B* and *H* on the same plot, and label them clearly.

4. Using the same equivalence between uniform magnetization and a surface current density, calculate the magnetic fields B and H everywhere along the axis of symmetry for a uniformly-magnetized sphere, both inside and outside the sphere. You may use Mathematica to help with the integral, but be careful about square-roots and absolute values! Make a plot of B and H vs. z as you did in the previous problem.

5. A sphere of radius a is uniformly charged, with total charge Q. The sphere is spinning about its axis with angular velocity ω. Find the magnetic field inside and outside the sphere. Express it in terms of the magnetic moment m of the sphere.

   Hint: Read Sections 29 and 30 in Chapter 4 of Landau & Lifshitz before you do this problem. In particular, see the discussion before Eqn. (30.11). The difference here is that you will use spherical rather than cylindrical coordinates. Nevertheless, the current and vector potential have only one nonzero component in these coordinates. Use the "Coulomb gauge" in which the vector potential can be found from the equation:

$$\nabla^2 \vec{A} = -\frac{4\pi}{c}\vec{j}$$

Quiz #6

The quiz on Thursday, November 1, will be taken from the 5 problems on this problem set.