Physics 842 – Fall 2012
Classical Electrodynamics II

Problem Set #7 – due Tuesday November 6

1. A right circular cylinder of radius R and length L is uniformly-magnetized along its axis.

   a) Last week you calculated $\vec{B}$ along the axis of the cylinder (both inside and outside) using the surface current density J_s. This week I want you to calculate $\vec{B}$ along the axis using the magnetic scalar potential and the fictitious magnetic charge density $\rho_m = -\vec{\nabla} \bullet \vec{M}$. Solve the Poisson equation $\nabla^2 \phi_m = -4\pi\rho_m$ the same way you would do if this were an electrostatics problem:

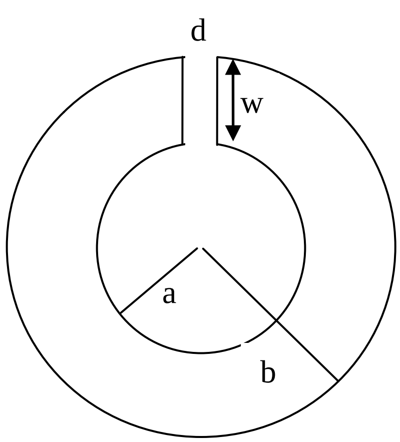
   $$\phi_m(\vec{r}) = \int \frac{\rho_m(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} d^3r'$$

   where the magnetic charge is actually a surface charge density on the two ends of the cylinder: $\sigma_m = \vec{M} \bullet \hat{n}$. Compare your answer with what you obtained last week using the surface current density. Hint: Be careful with absolute values when you take square roots.

2. Find $\vec{B}$ everywhere in space for a uniformly magnetized sphere. Use ϕ_m, but solve the problem using Legendre polynomials and the correct boundary conditions for $\vec{B}$ and $\vec{H}$ at the surface of the sphere.

3. A toroidal electromagnet consists of a soft iron core of permeability $\mu \gg 1$ with N turns of copper wire wound around it, carrying current I. The gap between the pole pieces is d, the width of the pole pieces is w, and the average radius of the toroid is $R=(a+b)/2$.

   a) Assuming that $d \ll w \ll R$, calculate the magnetic field B in the gap. Use $\vec{\nabla} \times \vec{H} = \frac{4\pi}{c}\vec{j}$ and the boundary condition on $B_\perp$ at the pole pieces.



   b) Simplify your expression in the limit $\mu \gg R/d$.

   Note that this problem can be solved in two lines once you've figured out how to set it up.

(over)

4. Consider a spherical shell of material with magnetic permeability μ, with inner and outer radii a and b, respectively, placed in a uniform external magnetic field $\vec{B}_0$.

   a) Calculate $\vec{B}$ everywhere in space. (This shouldn't be too difficult, because you already did the dielectric equivalent of this problem on Problem Set #3.)

   b) Simplify your expression for the field inside the cavity in the limit $\mu >> 1$. You should find that the field is proportional to $1/\mu$. This is how magnetic shielding works. The field lines are concentrated inside the magnetic material, so very little field leaks into the cavity.

5. In class I showed that the magnetostatic energy increases when a paramagnetic body is brought into a region with a pre-existing magnetic field. This result appears to contradict the known fact that paramagnetic materials are attracted to regions of high field. The resolution of this conundrum is that the current sources do work to maintain the field; the situation is analogous to electrostatic problems with fixed potentials rather than fixed charges. To show how this works in a specific problem, consider a long solenoid of area A with n turns per unit length, carrying current I. A long paramagnetic cylinder of permeability μ just fits inside the solenoid. Assume that the cylinder enters the solenoid moving at a speed v. (You may assume that the solenoid is infinitely long to calculate the magnetic field in this problem.)

   a) What is the rate of increase of magnetostatic energy of the system?

   b) By how much has the magnetostatic energy increased when the cylinder has moved in by a distance L? Express your answer in terms of the cylinder's susceptibility χ_m, the field B_0 in the solenoid before the cylinder entered, and the cylinder volume $V = AL$.

   c) Calculate the emf induced in the coil by the moving cylinder, using Faraday's Law. Assuming that the external current sources provide an emf that exactly cancels the induced emf, calculate the rate of work done by those sources.

   Your answer to part (c) should be exactly twice your answer to part (a). This shows that the total energy of the "magnetostatic system + current sources" actually decreases as the cylinder enters the solenoid, which is consistent with the cylinder being pulled in. (The cylinder would gain kinetic energy if we didn't hold onto it.)

Quiz #7

The quiz on Thursday, November 8, will consist of one of the following problems:

- Problems 1 - 5 on Problem Set #7

- Problems 1 - 4 at the end of Section 30 (problem 2 won't be on the quiz, but I want you to look at it.)