Physics 842 – Fall 2012
Classical Electrodynamics II

Problem Set #8 – due Tuesday November 13

1. The magnetic moment of an atom is proportional to the total angular momentum of the electrons (spin and orbital). $\vec{m} = -\gamma\vec{J}$. If we apply a magnetic field $\vec{H}$, there will be a torque on the magnetic moment. Averaging over all the individual moments in a ferromagnetic material to get the macroscopic magnetization, we obtain the Landau-Lifshitz equation:

$$\frac{d\vec{M}}{dt} = -\gamma\vec{M} \times \vec{H}$$

   Find the time evolution of $\vec{M}$ in a ferromagnet placed in a constant field $\vec{H}_0$. Hint: You should find oscillations at the Larmor frequency, $\omega_L = \gamma H_0$.

2. Consider an infinite ferromagnet in a uniform constant field $\vec{H}_0$ along the z-axis, and an additional weak field $\vec{h}\cos(\omega t)$ along the x-axis. Calculate the steady-state small oscillations of the magnetization $\vec{M}$ at frequency ω using the Landau-Lifshitz equation, to first order in $\vec{h}$. (Do <u>not</u> include the solution to the homogeneous differential equation – oscillations at the Larmor frequency -- which you found in problem 1.) Hints: Write the oscillating field as $\vec{h}\left(e^{i\omega t} + e^{-i\omega t}\right)/2$ and just treat the first term. Then write the total magnetization as $\vec{M} = \vec{M}_0 + \delta\vec{M}e^{i\omega t}$, plug in and solve for $\delta\vec{M}$. Notice that your solution diverges on resonance; that is because we have not included the damping terms in the Landau-Lifshitz equation.

3. Continue our class discussion of magnetization reversal for a ferromagnet with uniaxial anisotropy.

   a) Analyze the case where the external field is applied along a hard axis rather than the easy axis. Describe the evolution of the energy density *u* as a function of the angle θ between the magnetization $\vec{M}$ and the easy axis $\hat{z}$ as the magnetic field varies from large positive H_x to large negative H_x, with $H_z = 0$. Make plots of *u* vs θ for several values of H_x; then make a plot of M_x vs. H_x. Your last plot should not show any hysteresis. How do you reconcile that fact with the asteroid diagram shown in Figure 21 in Section 41 of Landau and Lifshitz? (By the way, L & L discuss this physics using the thermodynamic potential $\tilde{\Phi}$ rather than the energy density; don't let that bother you.)

   b) Now consider the case where the external field is applied along an arbitrary direction in the x-z plane, but not along the z-axis or x-axis. Guess what the M vs. H curve will look like. (On next week's homework you'll analyze this is more depth, in a slightly different context.)

Quiz #8

The quiz on Thursday, November 15, will consist of one of the following problems:

- Problems 1 - 3 on Problem Set #8
- Problems 4, 5, and 7 at the end of Section 34. (Don't worry about the integrals in problem 4 – they are not easy!)