Physics 842 – Fall 2012
Classical Electrodynamics II

Problem Set #9 – due Tuesday November 27 (after Thanksgiving)

1. Critical dimension of single-domain magnetic particles. Calculate the magnetostatic energy of a uniformly-magnetized Ni sphere of radius R using the expression $U = -\frac{1}{2}\int \vec{M} \bullet \vec{H}\, dV$. (We are ignoring the constant term I discussed in class.) Now assume that this magnetostatic energy will be reduced by half if the particle forms two hemispherical domains. Calculate the energy of the domain wall, given that the surface tension of the wall is $\gamma = 0.7$ erg/cm^2 and the saturation magnetization of Ni is $M_0 = 510$ emu/cm^3. From these calculations, find the maximum size of a spherical Ni particle where the single-domain state is energetically favorable over the two-domain state.

2. Stoner-Wohlfarth model of magnetization switching in single-domain particles. (This problem is related to problem 3 on the previous Homework, but it goes further.)

   Consider a magnetic particle in the shape of a prolate ellipsoid with a = b < c, initially magnetized along its +c axis.) Assume that the particle is too small to form domains, and that there is no magnetocrystalline anisotropy. In that case, the only two terms in the energy are the magnetostatic energy (also known as shape anisotropy) and the dipole energy in the external magnetic field. You can express your answers in terms of the demagnetizing factors, n_c and $n_a = n_b$.

   a) The particle is aligned with its +c axis along the +z axis. We apply an external magnetic field first in the -z direction to switch the magnetization, then in the +z direction to switch it back. Make a plot of M_z vs. H. (We did this in class, and you did it last week, so this is just a warm-up exercise.)

   b) Now the particle is aligned with its c axis along the x axis, and we do the same experiment with H along the z axis. Make a plot of M_z vs. H in this case. (You also did this last week.)

   c) Finally, the particle is oriented in the x-z plane with its c-axis at an angle of $\alpha = 45°$ with respect to the z axis. We do the same experiment, always with H along the z axis. Make a plot of M_z vs. H in this case. (This is new. You will need to make several plots of magnetostatic energy vs. the angle θ between the magnetization and the c-axis of the particle. Find the values of H for which the energy minima are marginally stable.)

   If you had an ensemble of particles oriented in random directions, your overall plot of M vs. H would be a superposition of these plots for individual particles. This is one way to get the interesting shapes one finds in plots of M vs. H for real ferromagnetic systems.

3. In Section 43 of Landau & Lifshitz and in class, the energy due to non-uniform magnetization in a lattice with cubic symmetry is written in terms of the macroscopic magnetization in the form:

$$U_{non-uniform} = \frac{\alpha}{2}\left(\left|\nabla M_x\right|^2 + \left|\nabla M_y\right|^2 + \left|\nabla M_z\right|^2\right) \qquad (1)$$

In class, I motivated this expression by writing down a microscopic model for the exchange interaction between nearby atomic spins:

$$H_{exchange} = -\sum_{i \neq j} J_{ij} \vec{S}_i \bullet \vec{S}_j = -\sum_{i \neq j} J_{ij} S^2 \cos(\theta_{ij}) \qquad (2)$$

where S is a dimensionless spin variable. Assume that $S = 1$, that we have a simple cubic lattice and that J_{ij} is nonzero only for nearest-neighbor atoms. To show the equivalence of these two expressions, consider a Bloch domain wall, where the magnetization rotates by π in the plane perpendicular to the direction of magnetization change. In other words:

$$M_z(x) = M_0 \cos(qx) \qquad\qquad M_y(x) = M_0 \sin(qx)$$

with $qa << 1$, where a is the lattice constant. Calculate the energy per unit area of the domain wall using both models. By comparing your results, derive an expression for α in terms of J_{ij}, M_0, and a. Hint: Assume that the angle of rotation between adjacent planes of atoms in the domain wall, $\theta = qa$, is very small; expand the cosine in Eqn (2) for small θ. <u>Remember to consider only the difference in exchange energy between the situation in the domain wall and the situation with uniform magnetization.</u> At the end, check that your expression for α has the right units.

4. <u>Eddy Current Heating:</u> In class we discussed a semi-infinite conductor with conductivity σ filling the half-space z > 0. Outside the conductor, the magnetic field varies in time as $\vec{H}(t) = \mathrm{Re}\{\vec{H}_0 e^{-i\omega t}\}$. Inside the conductor, the field has the form $\vec{H}(z,t) = \mathrm{Re}\{\vec{H}_0 e^{-z/\delta} e^{iz/\delta} e^{-i\omega t}\}$, where $\delta = c/\sqrt{2\pi\sigma\omega}$ is the skin depth.

   a) Choose $\vec{H}_0 = H_0\hat{y}$ for convenience. Calculate $\vec{E}(z,t)$ inside the conductor using the Maxwell equation $\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$.

   b) Calculate the Joule heat dissipated inside the conductor, per unit area of the surface, from the equation: $Q = \int \vec{j} \bullet \vec{E} dV = \int \sigma E^2 dV$. Note that Landau & Lifshitz calculate Q a different way, so please do it this way to see if you get the same answer: equation (59.10).

Quiz #9

The quiz on Thursday, November 29, will consist of one of the above problems.