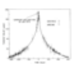
Deflection of light—17 Jan 2012

- Announcements
  - Homework 1 is due on Tues, the 24th.
  - Office hours: Tues, Thurs, 12:00–1:00? MF 12:00-1:00?
  - A few scribes have not sent answers to their questions.
- Outline
  - Results for Shapiro effect.
  - How does light figure out its path?
  - Bending of light by the sun

Shapiro effect. Results

Shapiro et al, 1971, PRL 26, 1132

Radar bounced off Venus.
Measured value is 1.02 that of general relativity.

In[1]:= **Show[Image[**  **], ImageSize → 400]**

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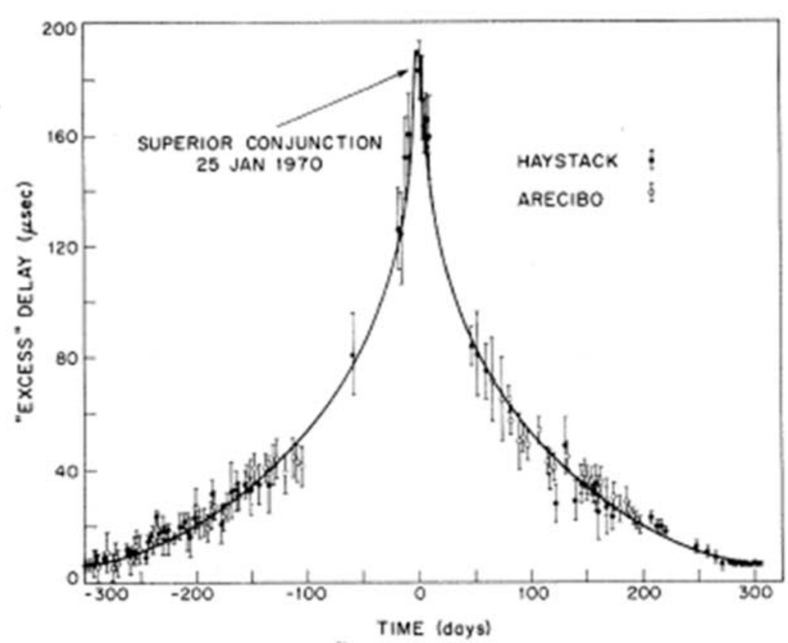
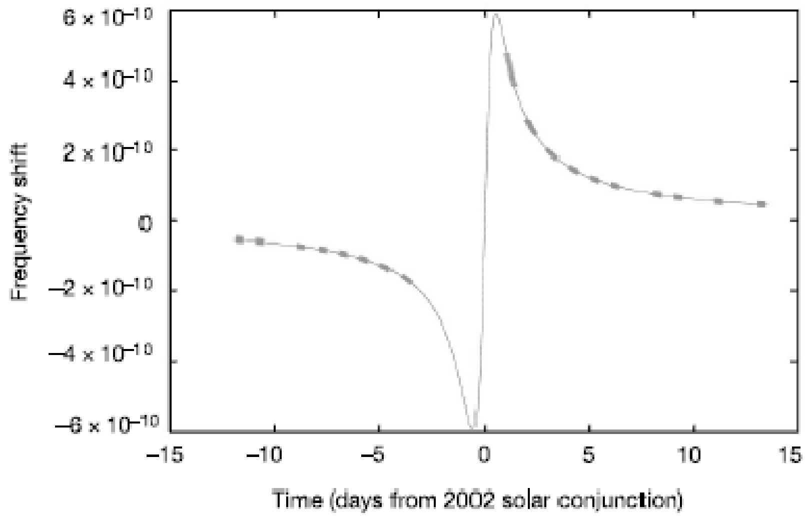


Bertotti, B., Iess, L, & Tortora, P, 2003, Nature, 425, 374, “A test of general relativity using radio links with the Cassini spacecraft”

Cassini was sent to Saturn.
Satellite is much smaller than a planet.

Two radio bands ⇒ measure and correct for the dispersion by electrons of the solar corona.
Closest light path was 1.6 $R_\odot$.



Result: Shapiro delay is $1 + (2.1 \pm 2.3) \times 10^{-5}$that of general relativity.

Historical importance of the bending of light by the sun

- **Einstein 1911 knew that energy depends on the gravitational potential. He did not know that space is curved. He calculates the bending of light.**

In terms of the metric, he knew

$$ds^2 = -(1+\Phi)^2\, dt^2 + dl^2,$$

where $\Phi = -M/r$ is the gravitational potential. The actual metric is

$$ds^2 = -(1+\Phi)^2\, dt^2 + (1-\Phi)^2\, dl^2.$$

He calculates the gravitational bending of light to be

$$\alpha = 2\,|\,\Phi\,|,$$

where Φ is at the closest distance the light ray is from the sun.

- **Erwin Freundlich attempted to measure the bending of light by the sun**

Failed because of the Crimean War.

- **18 Nov 1915. Einstein published a therory of gravity. The bending of light is double.**

Einstein publishes a theory of gravity that is almost correct. It is correct in empty space.

He calculates the gravitational bending of light to be

$$\alpha = 4\,|\,\Phi\,|,$$

where Φ is at the closest distance the light ray is from the sun.

The deflection angle is double. Doubling comes from the curvature of space.

- **25 Nov 1915. Einstein published the correct theory**

- **16 Jan 1916. Schwartzschild published his metric**

Einstein read the paper before the Prussian Acad. of Science.

- **11 May 1916. Schwartzschild died.**

- **1918. American attempt.**

No results.

- **Nov 1919 Eddington’s expedition**

deSitter in Holland told Eddington in Britain about Einstein’s prediction.
5/29/1919 Crommelin measures eclipse in Sobral, Brazil.
11/1919 Dyson, Eddington, & Davidson announce results of deflection at Royal Society.

Results announced. Einstein became famous.

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http://www.gettyimages.com/detail/90731681/SSPL
Photograph (bromide print) showing the instruments used by the British expedition sent to observe total solar eclipse on 29 May 1919 from Sobral in Brazil. Sir Arthur Eddington at Cambridge University organised the eclipse trip to try and test Einstein’s Theory of Relativity. During the event, two heliostats with moveable mirrors were used to direct images of the eclipsed Sun into a pair of horizontal telescopes. (Photo by SSPL/Getty Images)

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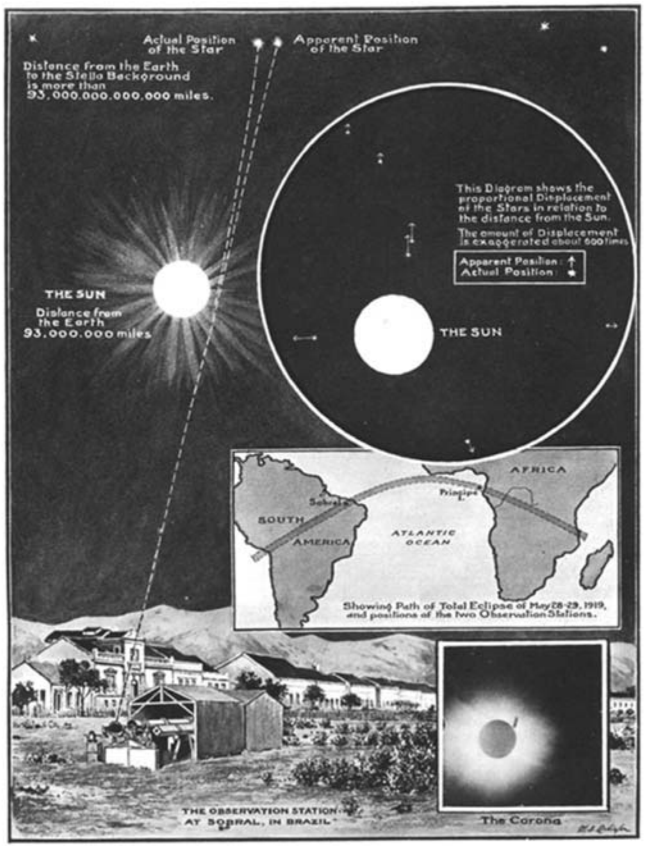

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LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.

}

11/22/1919 Illustrated London News (left) & NY Times (right)

Dyson, Eddington, & Davidson, 1920, A Determination of the Deflection of Light by the Sun's Gravitational Field, from Observations Made at the Total Eclipse of May 29, 1919, Philosophical Transactions of the Royal Society of London. Series A, 220, 291.

How does light figure out its path?

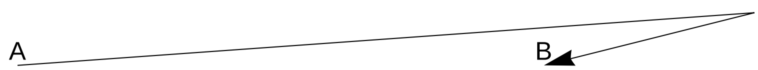
Fermat’s principle

Light goes from A to B. What path does light take?
Light tries all paths and picks out the path for which the time is an extremum.
(An extremum means that if the path is changed slightly, the change in time is 2nd order.)

Figures

Example

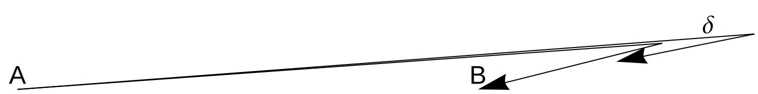
Why does light not take this path from A to B?



The time is not an extremum. Change the path slightly. The time

$$t_0 + 2\,\delta$$

is first order in δ.

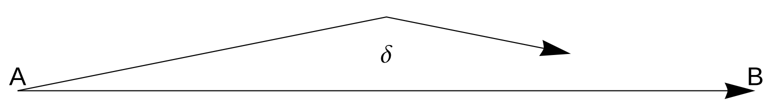


Why does like take the straight path from A to B?

The time is an extremum. Change the path slightly. The time

$$t_0 + O\,\delta^2$$

is second order in δ. (The hypotenuse of a triangle is $\left(1+\delta^2\right)^{1/2} \approx 1 + \frac{1}{2}\,\delta^2$.)



Example: Lens

Why does a lens focus light?
Light goes from the object to the image through a lens. Why can the light take any path through the lens?

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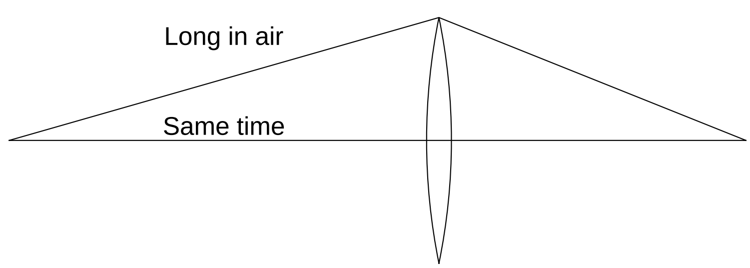
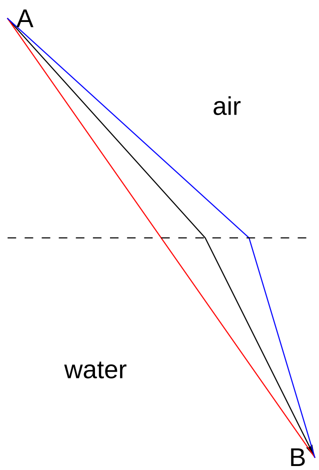


The time for light to go from the object to the images is the same for all straight paths through the lens. Through the edge of the lens, the path is long. Through the center, the time in air is shorter, but the time in glass is longer: The net is to make the time the same.

- **Snell's law**



Red path takes too much time because it goes through water too long. Blue path goes on a shorter path through water but on a much too long path through air.
Black path is an extremum.

Bending of light by the sun

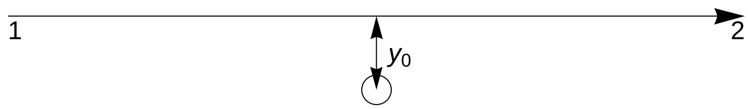
Recall Schwartzschild's metric in isotropic coordinates with $M/r \ll 1$

$$ds^2 = -\left(1 - \frac{M}{r}\right)^2 dt^2 + \left(1 + \frac{M}{r}\right)^2 dl^2.$$

On Thurs we found the time coordinate for light to go from point 1 to point 2 to be

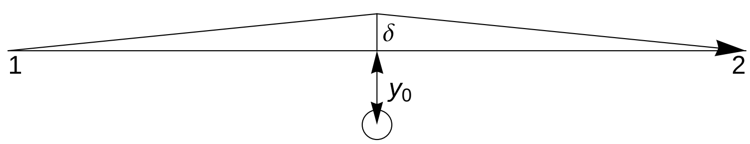
$$t = x_1 + x_2 + 2M \log \frac{4 x_1 x_2}{y_0^2} \qquad (1)$$

This was the Shapiro effect.



Q: The Shapiro effect for radar bounced off Venus is about 200μs. As measured on Earth, the effect is not t in equation 1. The measured time, after correcting for the position of Earth in eqn (1), is smaller in Jan and greater in July. Why?

The light ray says the term $2M \log \frac{4 x_1 x_2}{y_0^2}$ increases the time. Can light do better by choosing a path farther from the sun?



I move the path δ farther from the sun. The time is

$$t = \left(x_1^2 + \delta^2\right)^{1/2} + \left(x_2^2 + \delta^2\right)^{1/2} + 2M \log \frac{4 x_1 x_2}{(y_0 + \delta)^2}$$

Minimize the time:

$$\frac{dt}{d\delta} = \frac{\delta}{x_1} + \frac{\delta}{x_2} - 4\frac{M}{y_0} = 0$$

$$\frac{\delta}{x_1} + \frac{\delta}{x_2} = 4\frac{M}{y_0}$$

The deflection angle at earth is

$$\vartheta_2 \equiv \delta / x_2$$

For a star at ∞,

$$\vartheta_2 = 4\frac{M}{y_0}$$

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4 Convert[GravitationalConstant SolarMass / SpeedOfLight^2 / SolarRadius, 1]
  {1, 2600 / ° "arcsec"}
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{8.48939 × 10^-6, 1.26466 arcsec}

- **Measurements**

These are now called tests of post-Newtonian gravity. The parameter γ is 1 for general relativity.

Will, C.,
Living Rev.Relativity, 9, (2006), 3, http : // www.livingreviews.org/lrr - 2006 - 3, (Update of lrr - 2001 - 4)

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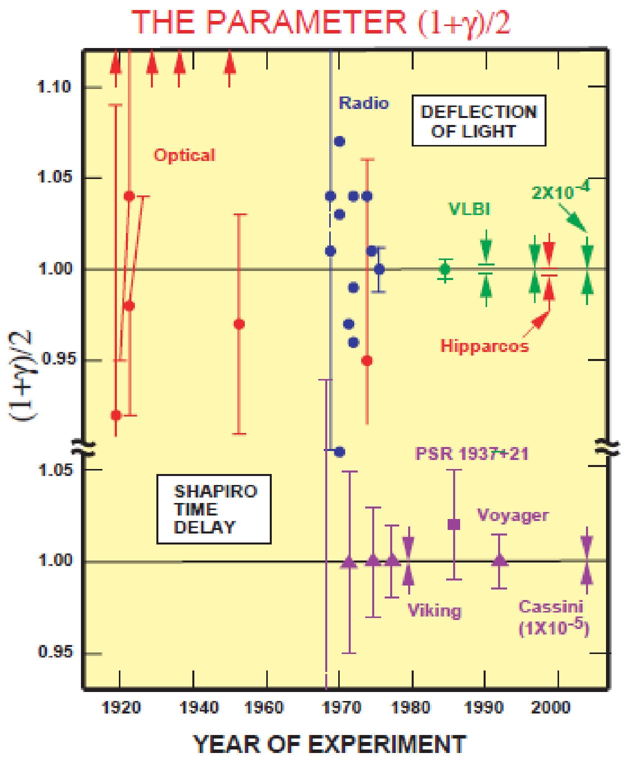

Example of VLBI measurements of quasars, Robertson, D., Carter, W., Dillinger, W., 1991, Nature, 349, 768.:

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