Initialization

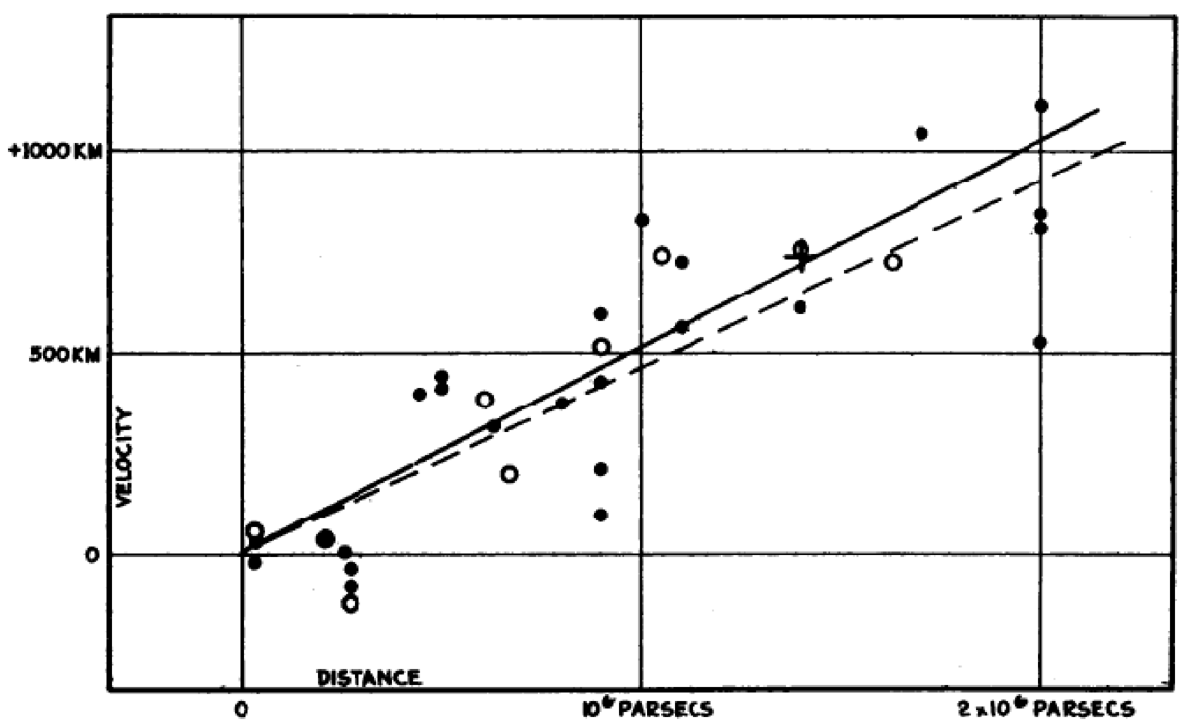
Cosmology—31 Jan

- Announcements
  - Homework 3 is due on 2/6. The link is on the syllabus on angel.
- Outline
  - Homework 1.
  - Hubble's Law
  - Expansion according to Hubble's Law
  - Robertson-Walker metric
  - Derivation of Hubble's Law and redshift.

Hubble's Law

In the 1920's, V. M. Slipher measured velocities of nearby galaxies. Hubble estimated their distances. Hubble (Hubble, E., 1929, Proc. Nat. Acad. Sci. 15, 168, "A relation between distance and radial velocity among extra-galactic nebulae.") found velocities v are proportional to distances D. This is called Hubble's Law.

$v = H\, D$



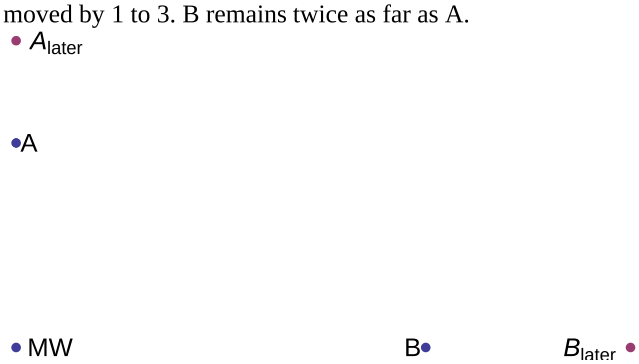
Velocity-Distance Relation among Extra-Galactic Nebulae.

Radial velocities, corrected for solar motion, are plotted against distances estimated from involved stars and mean luminosities of nebulae in a cluster. The black discs and full line represent the solution for solar motion using the nebulae individually; the circles and broken line represent the solution combining the nebulae into groups; the cross represents the mean velocity corresponding to the mean distance of 22 nebulae whose distances could not be estimated individually.

Hubble expansion is special

Expansion by Hubble's Law is very special. Consider Milky Way, galaxy A at distance 1, and galaxy B at distance 2.

1. Expansion is by a scale factor. $v_A = 1$, and $v_B = 2$. Suppose some time passed, and A has moved by 0.5 to 1.5. Then B has moved by 1 to 3. B remains twice as far as A.



2. Centerless expansion.
A is $5^{1/2}$ from B. MW is 2 from B.
After time passed, A is $\left(3^2 + 1.5^2\right)^{1/2} = \frac{3}{2}\, 5^{1/2}$ from B, and MW is $\frac{3}{2}\, 2$ from B.
Galaxy B is the center of the expansion too.

3. There exists a beginning, when the scale factor is 0. In this example, let time be -1.

4. Hubble did not find special directions. The universe is isotropic.

- **Plot**

Isotropic & homogeneous spaces

The universe is the same everywhere (homogeneous) and the same in all directions (isotropic).

Q: Simplicio says, “We see light from distant galaxies that were forming stars for the first time. There is not the same as here. Therefore the universe is not homogeneous.“ Explain in what way Simplicio’s conclusion is mistaken.

Q: Give an example of a 2-d homogeneous & isotropic space that is infinite. Give an example of a 2-d homogeneous & isotropic space that is finite.

Friedman-Robertson-Walker spaces

A 3-d space that is homogeneous and isotropic has a special choice of time. A choice of coordinates is

$$(t,\, r,\, \theta,\, \phi)$$

and the metric is

$$ds^2 = -dt^2 + a(t)^2\left[dr^2 \big/ \left(1 - (r/r_0)^2\right) + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right].$$

(r, θ, ϕ) is called the comoving coordinate. A galaxy stays at the same position; time changes.

r_0^2 can have any value, positive or negative.

$a(t)$ is called the expansion parameter.

Q: Descibe the effect of the expansion parameter.

What is r_0^2?

The FRW metric is

$$ds^2 = -dt^2 + a(t)^2\left[dr^2 \big/ \left(1 - (r/r_0)^2\right) + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

Galaxies are at fixed (r, θ, ϕ).

Let time be fixed. Consider the spatial part of the metric.

$$dr^2 \big/ \left(1 - (r/r_0)^2\right) + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

Suppose $r_0^2 \to \infty$. What is the space?

Suppose $r_0^2 > 0$. What is the space?

The radial distance between $r = 0$ and r is

$$\int_0^r dx \Big/ \left[1 - (x/r_0)^2\right]^{1/2} = r_0 \int_0^{r/r_0} \frac{dy}{\left(1-y^2\right)^{1/2}} = r_0 \sin^{-1}(r/r_0).$$

The circumferential distance is

$$r\,(\theta_2 - \theta_1)$$

This has a simple interpretation:

Map 3-d space into 2-d by supressing ϕ. The space is the 2-d surface of a sphere. This means the surface is the entire space. Above and below the surface is not part of the space.

I draw a slice through the sphere.

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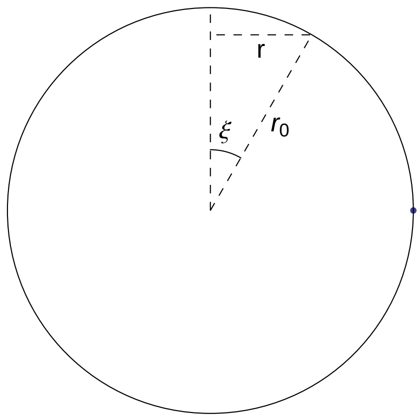
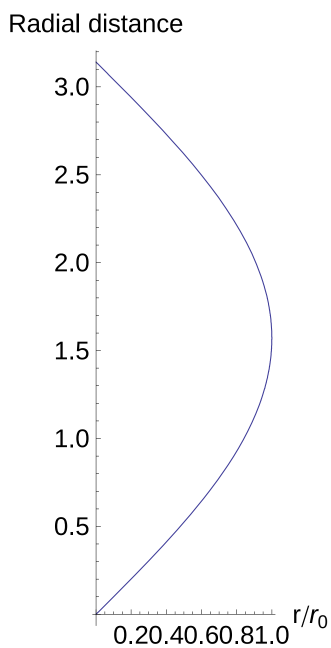


Define $\sin\xi = r/r_0$. $r_0\, d\,\xi = d\,r/\cos\xi = d\,r\Big/\left[1-(r/r_0)^2\right]^{1/2}$. Therefore

$$ds^2 = r_0^2\, d\text{ latitude}^2 + r^2\, d\text{ longitude}^2.$$

Lesson: If $r_0^2 > 0$, then r_0 is the radius of curvature of the 3-d space.

As r increases, the radial distance increases. Beyond $r/r_0 = 1$, r decreases, but the radial distance keeps increasing.



If $r_0{}^2 < 0$, the space is like a saddle, and the space is infinite.

- **Plots**

Hubble's Law derived from the R-W metric

Consider the case $r^2 \ll \left|r_0{}^2\right|$. The radial distance of a galaxy at r is

$$D(t) = a(t)\, r.$$

The galaxy moves away at speed

$$v = \frac{d}{dt}\ D(t) = \frac{1}{a}\ \frac{da(t)}{dt}\ a\,r = \frac{1}{a}\ \frac{da(t)}{dt}\ D.$$

Define Hubble's constant $\boxed{H = \frac{1}{a}\ \frac{da(t)}{dt}}$. Then

$$v = H\,D$$

This is Hubble's Law.

Redshift derived from the metric

A distant galaxy emits some light with wavelength λ_1 when the expansion parameter was $a(t_1)$. We see the light after some time. What is the wavelength of the light that we see?

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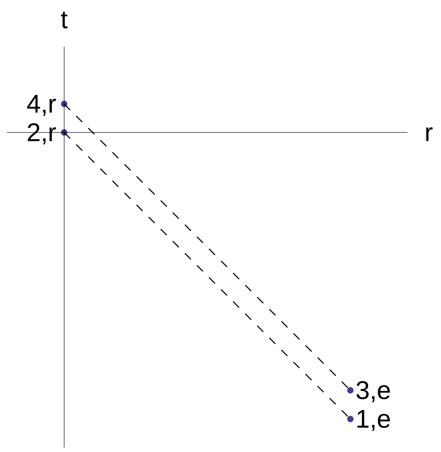


This is not our usual space-time diagram, since galaxies are at fixed spatial points, even though they are moving.

Events:
1. Distant galaxy emits a crest.
3. Distant galaxy emits a second crest.
2. Astronomer receives first crest.
4. Astronomer receives second crest.

The metric is

$$ds^2 = -dt^2 + a(t)^2\left[\,dr^2\big/\left(1-(r/r_0)^2\right) + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right].$$

Integrate between events 1 and 2. Because $ds = 0$, $dt = a\,(t)\,dr\left[1-(r/r_0)^2\right]^{-1/2}$, and

$$\int_1^2 \frac{d\,t}{a(t)} = \int_1^2 \left[1-(r/r_0)^2\right]^{-1/2} d\,r$$

Similarly for the second crest,

$$\int_3^4 \frac{d\,t}{a(t)} = \int_3^4 \left[1-(r/r_0)^2\right]^{-1/2} d\,r.$$

Since $r_1 = r_3$ and $r_2 = r_4$,

$$\int_1^2 \frac{d\,t}{a(t)} = \int_3^4 \frac{d\,t}{a(t)}.$$

$$\int_1^3 \frac{dt}{a(t)} = \int_1^2 + \int_2^3$$
$$= \int_3^4 + \int_2^3$$
$$= \int_2^4 \frac{dt}{a(t)}$$

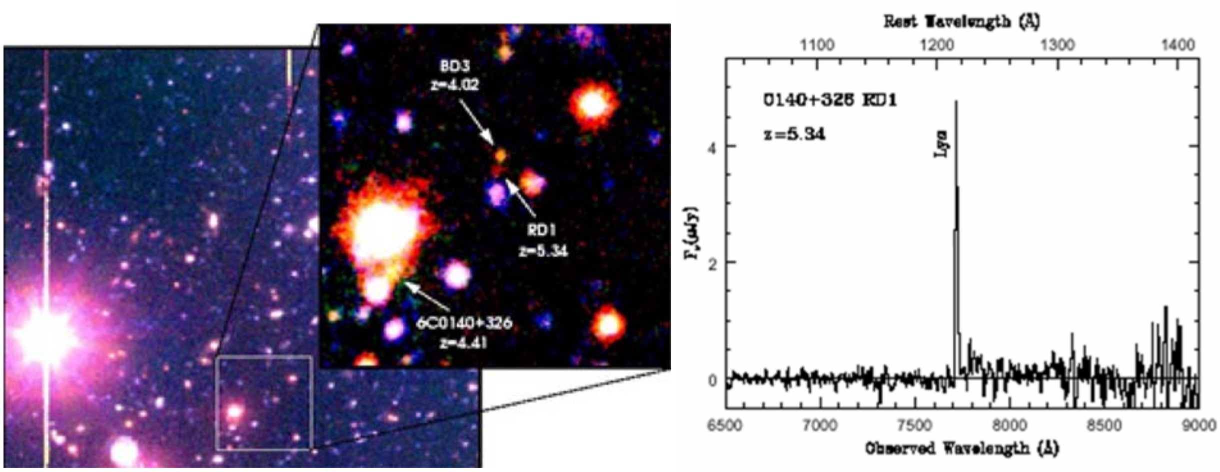
Events 1 and 3 are separated by the time to emit two wave crests, λ/c. $\int_1^3 \frac{dt}{a(t)} = \lambda / c / a(t_1)$.

$$\frac{\lambda_1}{a_1} = \frac{\lambda_2}{a_2}$$

Stated in words:

The wavelength of light expands by the same factor as the universe.

- **Plot**
- **Example**



When the light that we see left Galaxy 0140+326 RD1, its wavelength was 121.5 nm. We see its wavelength to be 771.0nm.

Q: By what factor has the universe gotten bigger?

Redshift derived from isotropy

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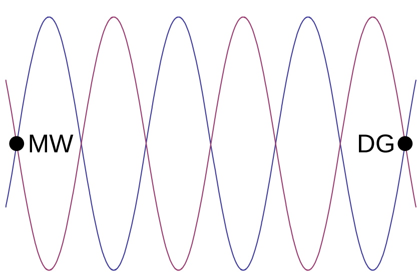

Space is permeated with standing waves of light. (To get a standing wave, waves goes in opposite directions.)

At some time 1, the Milky Way and a distant galaxy are N waves apart. (Here $N = 3$.) Later, MW and DG have moved apart by the factor a_2/a_1. Does the MW move to the right or the left of the node that it was on? Since there are no special directions, the MW has to stay on the node. Same for DG. Therefore

$$D_1 = N\,\lambda_1$$
$$D_2 = N\,\lambda_2$$
$$\frac{a_2}{a_1} = \frac{D_2}{D_1} = \frac{\lambda_2}{\lambda_1}$$

The wavelength of light expands by the same factor as does the universe.

More normal form: a_r at reception is 1. a_e at emission is written as a.

$$\lambda_r = \lambda_e/a.$$

Redshift is defined to be $z = (\lambda_r - \lambda_e)/\lambda_e$.

Then

$$z = a^{-1} - 1$$
$$a = (1+z)^{-1}$$

- **Plot**