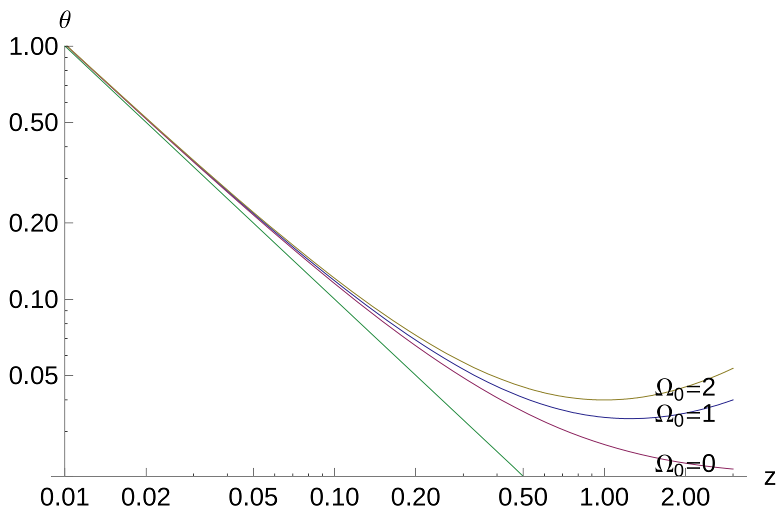
Evidence that vacuum energy dominates other forms of matter—9 Feb

- Announcements
  - Midterm is Tues, the 14th. Midterm from 2010 is on angel. (Link is on the syllabus.) Covers material through last week.
  - One cheat sheet, 8.5×11” front and back.
- Outline
  - Heuristic interpretation of the angle subtended by a ruler.
  - Flux of a star.
  - Measurement of the flux of supernovae. Big surprise. $\Omega_{vac} \neq 0$.
  - Questions found when studying for the midterm.

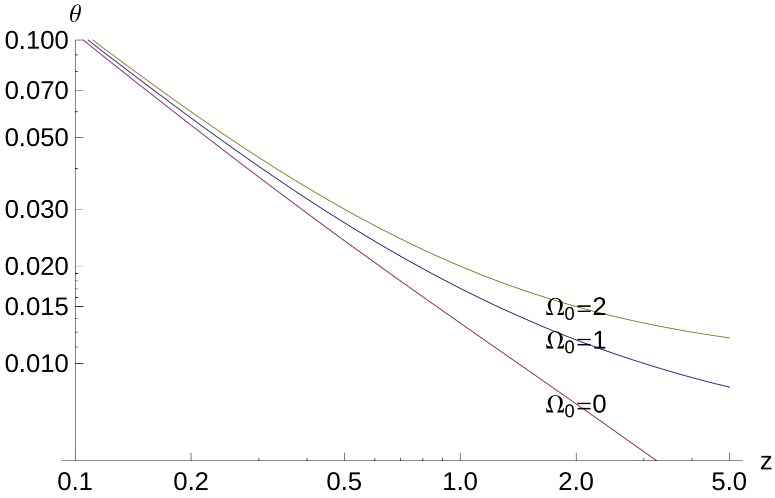
Angle subtended by a ruler

We observe a ruler at a great distance. What is the angle between the ends of the ruler? (This observation has been made by WMAP: The ruler was c×(the age of the universe at recombination).)

$$\theta = \frac{L}{a\, r(a)}$$



Caption: Angle subtended by a ruler of length 0.01 H_0^{-1} The green line is $\theta = L/\left(z\, H_0^{-1}\right)$.



Caption: Angle subtended by two galaxies separated by 0.01 $H_0^{-1}\,a$.

We want to understand heuristically why the angle is smaller for smaller density.

- **Plots**

How the comoving coordinate $r(a)$ depends on the density parameter Ω_0

Compare the comoving coordinate for $\Omega_0 = 0$, 1, and 2.
On Thurs, we integrated Friedman's equation for pressureless matter to find $r(a)$ for three cases.
For $\Omega_0 = 0$,

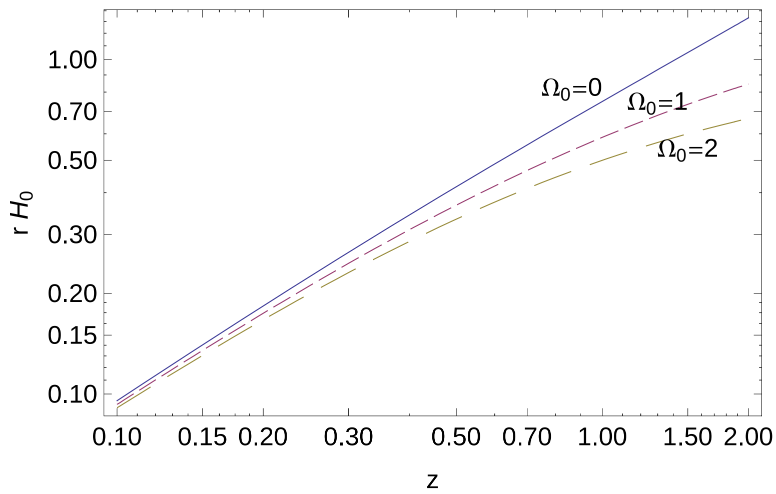
$$r(a) = H_0^{-1}\,\frac{1}{2}\left(a^{-1} - a\right)$$

For $\Omega_0 = 1$,

$$r\,(a) = 2\,H_0^{-1}\left(1 - a^{1/2}\right).$$

For $\Omega_0 = 2$,

$$r(a) = H_0^{-1}(1 - a)$$



For a given redshift, the comoving coordinate is greater for a lesser density.

For $z = 2$, $a = (1+z)^{-1} = (1+2)^{-1} = \frac{1}{3}$. The universe was 1/3 the present size. The universe expanded by a factor of 3 from when the light was emitted at $z = 2$ to the present.

Q: Simplicio: "It is impossible to know how big the universe is or was. What does it mean to say the universe was 1/3 the present size?"

- **Heuristic way of accounting for the fact that for a given redshift, the comoving coordinate is greater for a lesser density.**

Consider galaxies that are not far away. Then $a(t) \approx 1$ and $r \ll r_0$. The time for the light to get to us (and for the universe to expand) is found by integrating the light ray

$$dt = a(t)\, dr \left[1 - (r/r_0)^2\right]^{-1/2}$$

Becomes

$$dt = dr$$

The comoving coordinate is equal to the time for the universe to expand.

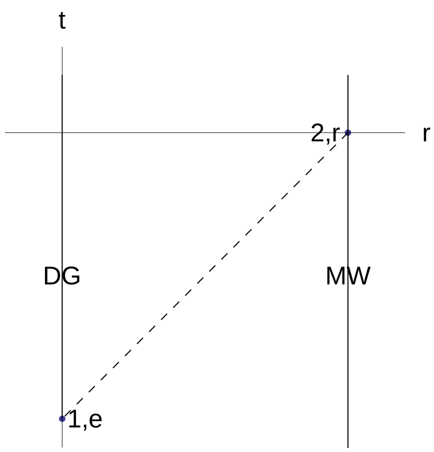
"For a given redshift, the comoving coordinate is greater for a lesser density" becomes "It takes a greater time for the universe to expand by a given factor for a lesser density."

Q: Write the equivalent statement for dropping a pencil from a height of 1 meter.

- **Plots**

Flux of a standard candle

A supernova emits luminosity $L(\nu_e)$ at frequency ν_e. The units are J/s/Hz. What is the flux $F(\nu_r)$ $\left[\text{J/s}/\text{m}^2/\text{Hz}\right]$ that we receive on earth?



Caption: A distant supernova emits a burst of photons over time $d\,t_e$ in a frequency band $d\,\nu_e$.

The total number of photons in the burst is

$$N = \frac{1}{h\,\nu_e}\,L(\nu_e)\,d\,t_e\,d\,\nu_e \quad .$$

We receive some number of photons, the ones that go into our detector of area A. The area of our detector is (from the metric)

$$A = (a\,r\,d\,\theta)\,(a\,r\,d\,\phi).$$

What do I use for a? a for supernova or a of astronomer?

Our detector subtends a solid angle of $\frac{A}{r^2}$, we catch

$$N\,\frac{1}{4\,\pi}\,\frac{A}{r^2}$$

photons.

Q: These photons have energy $h\,\nu$. What is $\frac{\nu}{\nu_e}$?

Q: These photons are spread out in time $d\,t$ and in frequency $d\,\nu$. What is $d\,t$ compared with $d\,t_e$? What is $d\,\nu_e$ compared with $d\,\nu$?

These photons have energy $h\,\nu$. What is $\frac{\nu}{\nu_e}$? These photons are spread out in time $d\,t$ and in frequency $d\,\nu$. What is $d\,t$ compared with $d\,t_e$? What is $d\,\nu$ compared with $d\,\nu$?

We already figured out this question. The wavelength of light expands by the same factor as the universe. $d\,t$ behaves the same as wavelength, since $d\,t$ behaves as the time between emission of wave peaks. Therefore $\frac{d\,t_e}{d\,t} = a_e.\ \frac{\nu}{\nu_e} = a_e.\ \frac{d\,\nu_e}{d\,\nu} = a_e^{-1}$.

The flux is related to the number of photons that we caught by

$$F\,d\,t\,d\,\nu\,A = N\,h\,\nu\,\frac{1}{4\,\pi}\,\frac{A}{r^2}$$

$$F(\nu) = L(\nu_e)\,\frac{\nu}{\nu_e}\,\frac{1}{4\,\pi\,r^2}\,\frac{d\,t_e}{d\,t}\,\frac{d\,\nu_e}{d\,\nu}$$

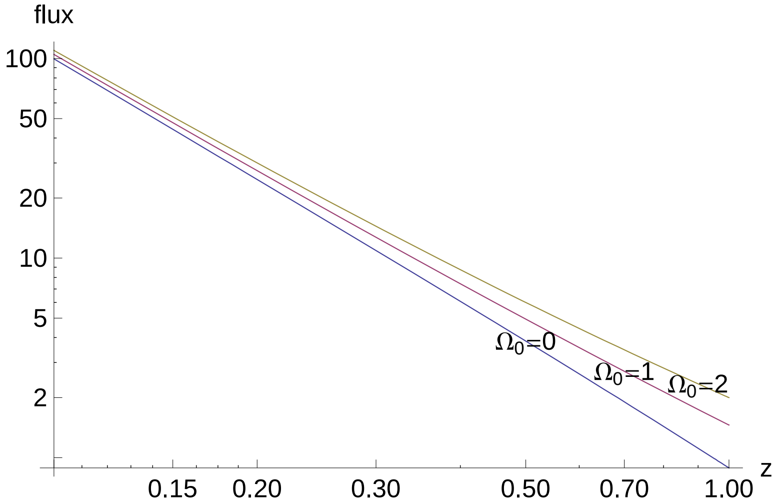
The frequency of the photons that we catch are shifted from the frequency that was emitted.

$$\boxed{F(\nu) = L\left(\nu\,a^{-1}\right)\frac{a}{4\,\pi\,r^2}}$$

We drop the subscript on a_e since a is clearly the expansion parameter when the light was emitted. It is handier to think that r is the comoving coordinate of the source.

- **Plots**

How does flux depend on the density parameter?



Caption: Flux vs redshift for three values of the density parameter.

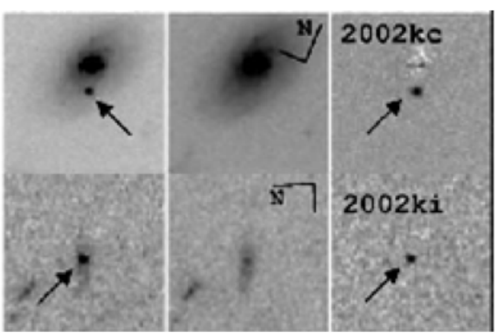
The flux

$$F(\nu) = L(\nu\, a^{-1}) \frac{a}{4\,\pi\, r(a)^2}$$

We have computed $r(a)$ for three cases with pressureless matter. By the same procedure, we can find $r(a)$ for radiation where $\rho \sim a^{-4}$ or the vacuum where ρ is a constant or for a mixture or all three.

Q: Why are the supernovae brighter for a universe with a higher mass density?

Measuring the flux of type I supernovae

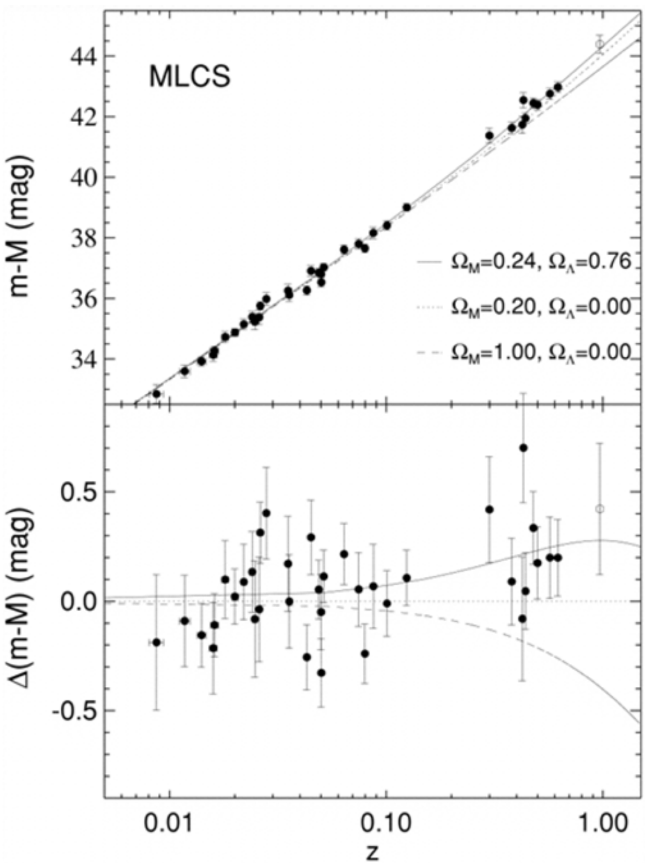


Riess et al, 2004, ApJ 607, 665.

- Type I supernovae do not have hydrogen in the spectrum. Type II supernovae do have hydrogen.
- A Type II supernova is a massive star that explodes when it runs out of fuel and pressure is insufficient to counter gravity.
- A Type I supernova is a white dwarf that explodes.
  A WD and giant orbit each other.
  Mass moves from the giant to the WD.
  WD explodes when it gets so much mass from the giant that degeneracy pressure can no longer oppose gravity.
- Type I supernovae are approximately “standard candles.” They have the same luminosity.

- How to find supernovae
  Look at many galaxies.
  Look again later. Find objects that were not there earlier.
- Measuring the flux
  Measure the flux over time. Identify the peak flux and the rate of flux decrease.
  Report the peak flux corrected by an empirical correlation between peak flux and the rate of decrease.

Observations



Upper plot: magnitude of the flux compared with the flux at a distance of 10 pc.

$$m - M = -2.5 \log_{10} F / F(10\,\text{pc})$$

Lower plot: difference between $m - M$ and a model with $\Omega_0 = 0.2$ for pressureless matter.
Other models: $\Omega_{0,\text{matter}} = 1$, $\Omega_{0,\Lambda} = 0$ (Λ stands for cosmological constant or vacuum)
$\Omega_{0,\text{matter}} = 0.24$, $\Omega_{0,\Lambda} = 0.76$

Supernovae are fainter than that for a universe with 0.2 of the critical density. Recall: to put supernovae at a given redshift farther, remove mass from the universe to get universe to expand slower in the past. Reiss et al say supernovae are too faint even if there is no mass. Need the negative pressure of the vacuum to accelerate the expansion.
Conclusion:

The dominant component of the universe is the vacuum energy. Its density parameter is 3 times that of pressureless matter.

Initialization