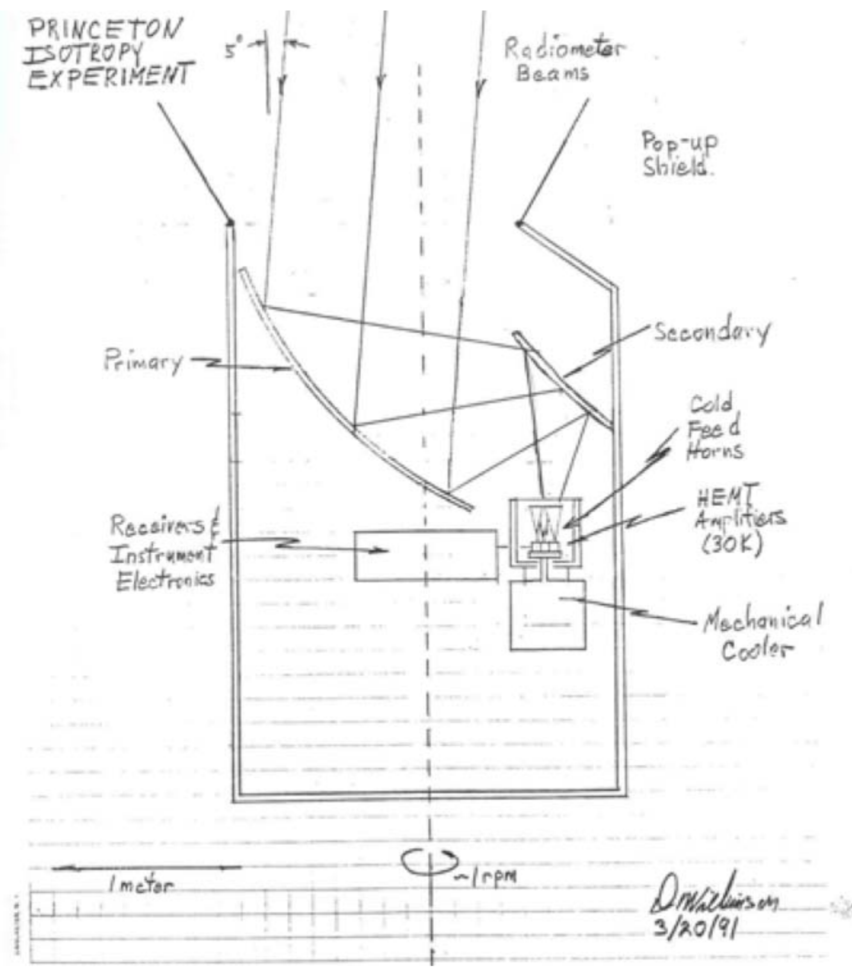
16 Feb 2012

21 Feb 2012

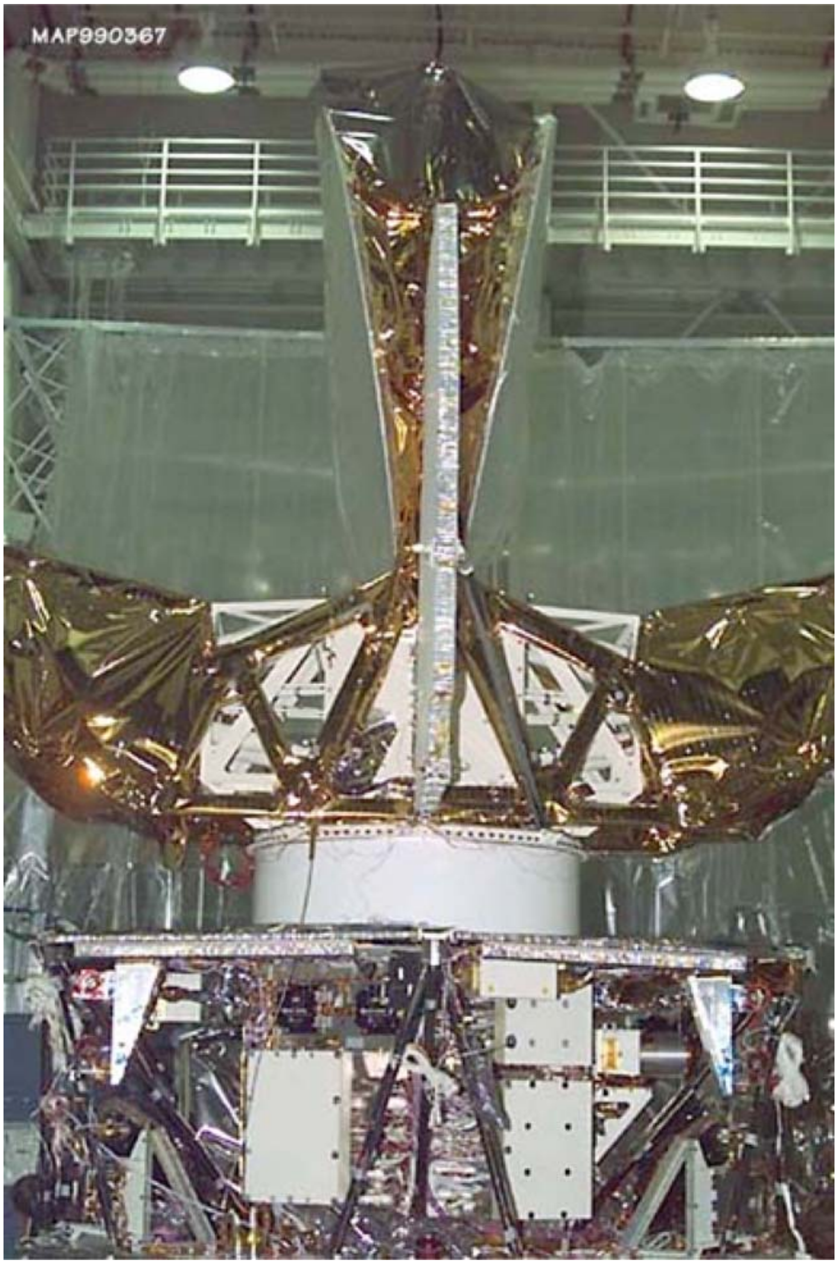
23 Feb 2012

- Announcements
  - Problem 5 (Hartle 18.3). Assume V_* is nonrelativistic. The relativistic case requires more complicated functions.
- Outline
  - WMAP satellite
  - Dipole anisotropy
  - Small-scale anisotropy
  - Rough calculation of the angular scale
  - Precise calculation of the angular scale
  - Sound waves

WMAP satellite



Dave's notebook, (Greg Tucker)

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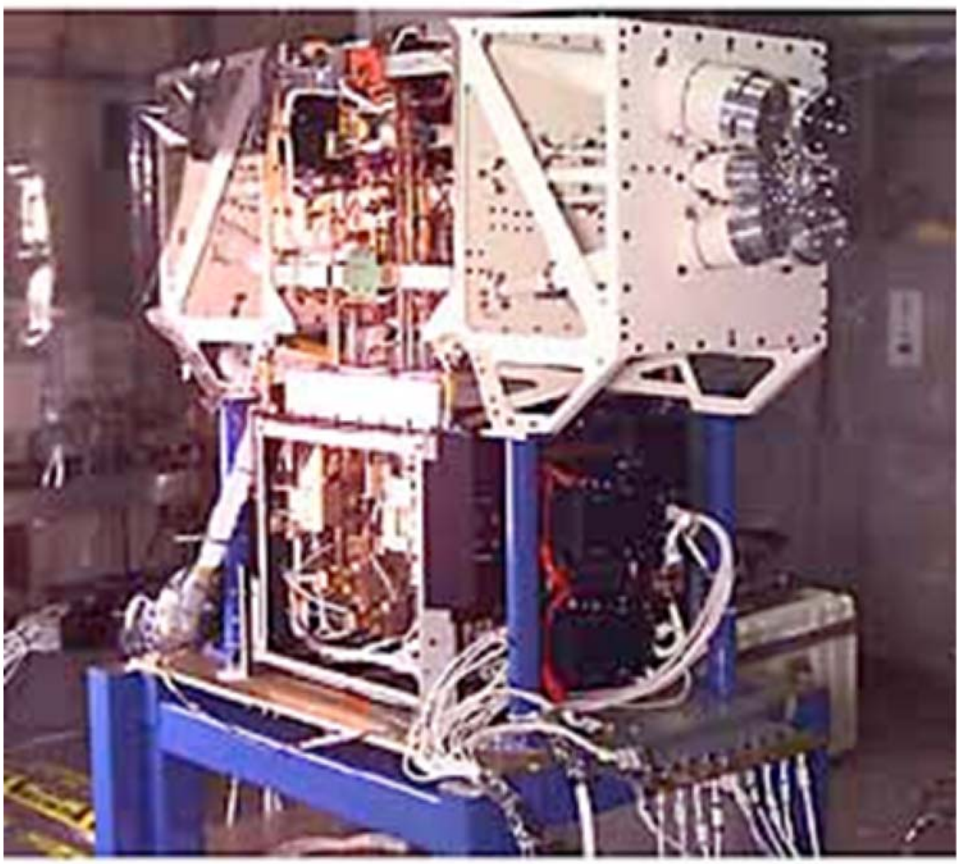
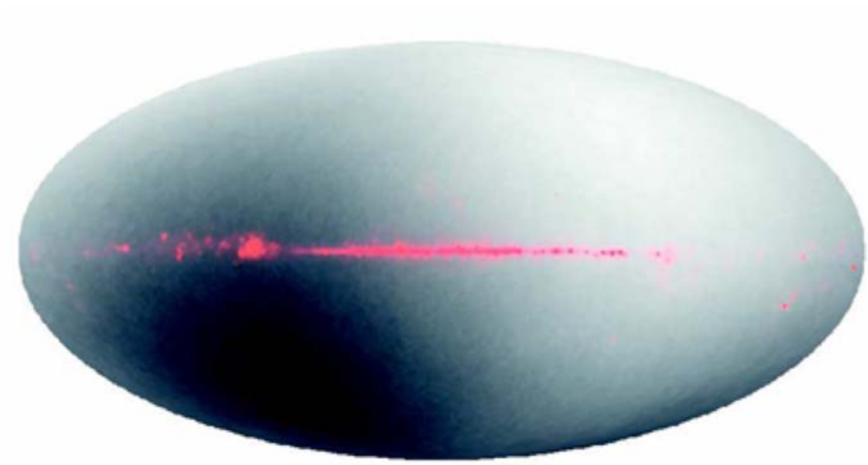


Five wavelength bands.
Each assembly compares two regions of the sky separated by 140°.
Q: Why does each assembly compare two regions of the sky?
One assembly operates at 22 GHz, one at 30 GHz, two at 40 GHz, two at 60 GHz, and four at 90 GHz.
Q: Why is there only one assembly at 22GHz yet there are four at 90GHz?

Dipole anisotropy



Temperature of the entire sky. Hottest spot is 3mK hotter than the average.
Pink is radiation of the Milky Way Galaxy, which has a different spectrum.

There is a special frame in which the universe is at rest.

We are moving with respect to this frame.
On 21 June, we are moving toward Pisces at 30 km/s = 0.0001.

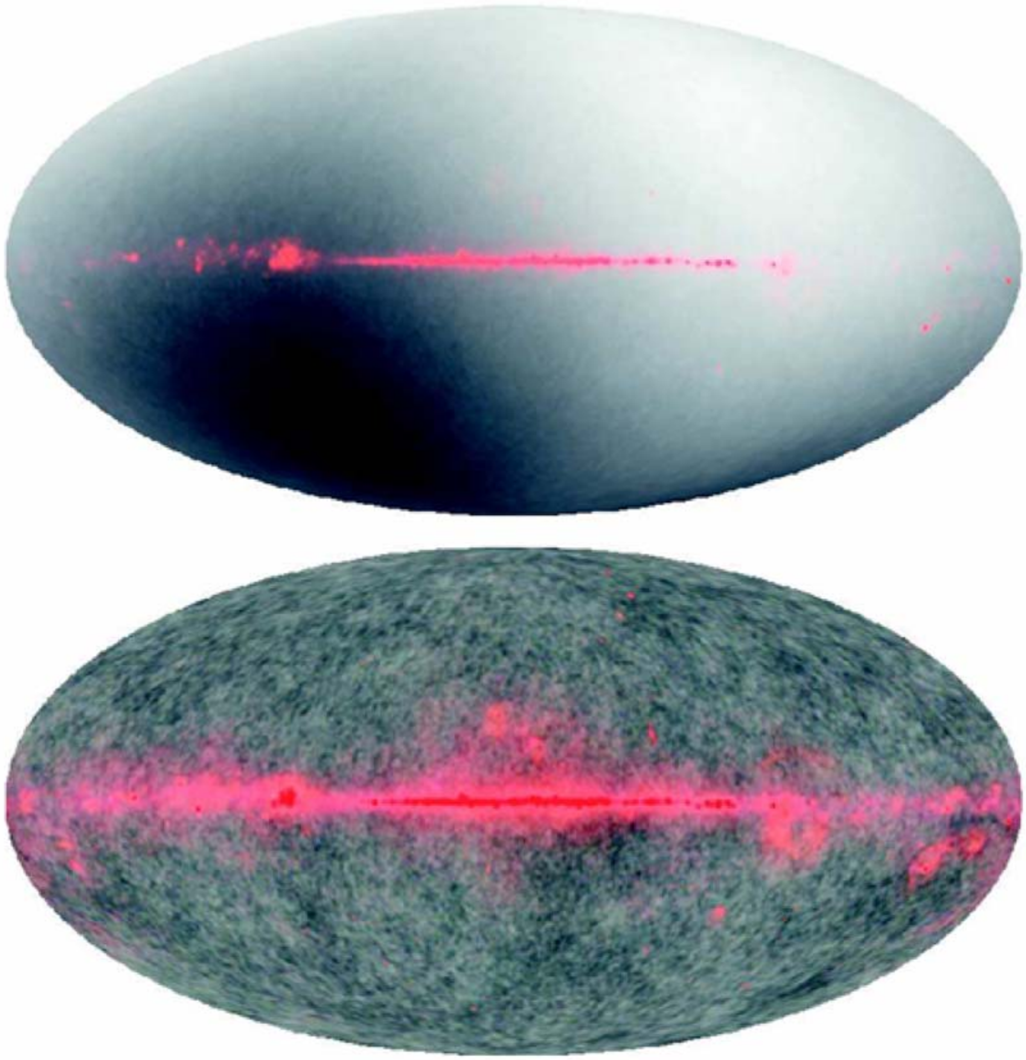
Q: On 21 June, what is the Doppler shift of the 2.7-K photons looking toward Pisces?
90° from Pisces? 180° from Pisces?

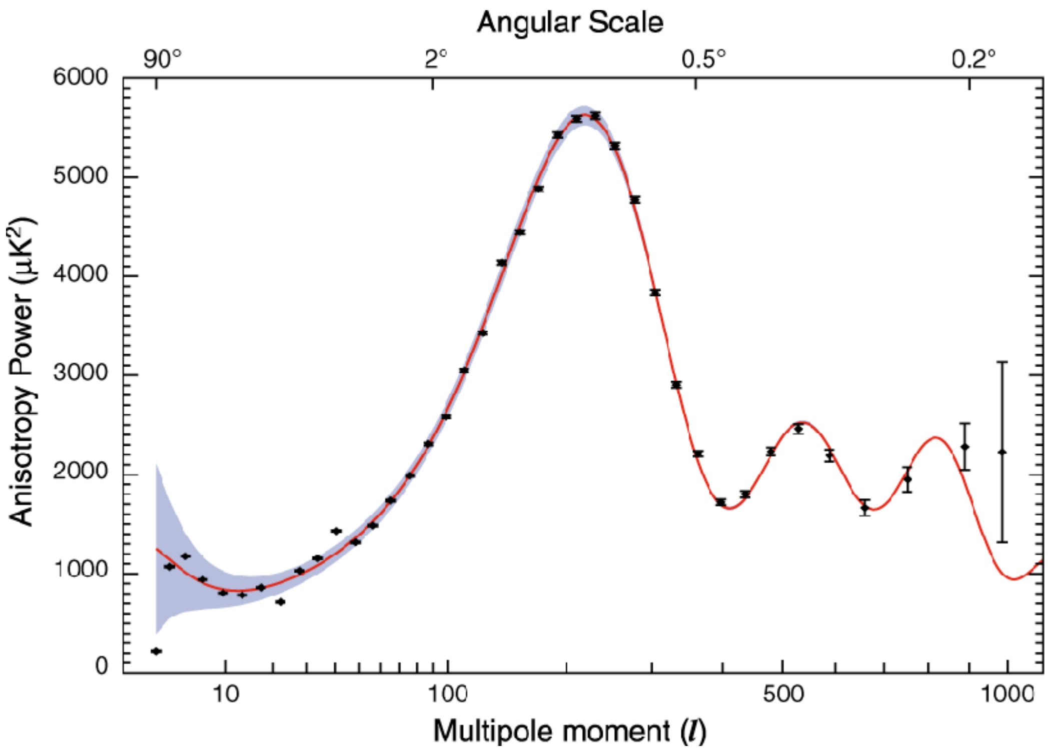
Q: On 21 June, what is the temperature shift of the CBR toward Pisces?
90° from Pisces? 180° from Pisces?

The peculiar velocities of galaxies are about 300 km/s = 0.001. A not unreasonable value of the dipole anisotropy is 2.7 mK.

The dipole anisotropy is 3.346 ± 0.017 mK toward (l, b) = (263.85 ± 0.1 °, 48.25 ± 0.04 °) (Bennett et al., 2003, ApJS, 148, 1). Motion of the sun in the Milky Way is 215 km/s. Milky Way moves at 200 km/s towards Andromeda. Net velocity of the local group of galaxies is 627 ± 22 km/s toward (l, b) = (276 ± 3 °, 30 ± 3 °) (Smoot et al, 1991, ApJ 371, L1,Kogut et al, 1993, ApJ 419, 1)

WMAP angular size of fluctuations





WMAP: http://map.gsfc.nasa.gov/media/060911/PowerSpectrum150.png

Rough calculation of the physical size of the fluctuations

Plan: Why is the largest anisotropy at an angular scale of 1°? Fluctuations can grow for the age of the universe. The size is about $c\,t$. We will calculate the age of the universe (for a given $\Omega_{0\,r}$). Then calculate the size L precisely. We already know that the angle subtended by a ruler at expansion paremeter a and comoving distance r is $\theta = L/(a\,r)$.

Age of the universe

Using middle-school physics

A car is moving at 50mph. It is 100mi from us. How long has the car been traveling since it left?
A galaxy is moving from us at speed v. Its distance is $v = D\,H_0$. Assume it has always moved at this speed. What is the time since the Big Bang?

$$\begin{aligned} t &= D/v \\ &= D/(D\,H_0) \\ &= H_0^{-1} \end{aligned}$$

Neglected effect: the galaxy can be slowing down or speeding up.

Proper calculation of expansion vs time

Friedman's equation relates the expansion parameter $a(t)$, matter density $\rho(t)$, and the radius of curvature r_0.

We want to integrate Friedman's equation to find $t(a)$.

$$\left(\frac{d\,a}{H_0\,d\,t}\right)^2 = \left(\Omega_{k0} + \Omega_{m0}\,a^{-1} + \Omega_{r0}\,a^{-2} + \Omega_{v0}\,a^2\right)$$

$$H_0\,d\,t = \left(\Omega_{k0} + \Omega_{m0}\,a^{-1} + \Omega_{r0}\,a^{-2} + \Omega_{v0}\,a^2\right)^{-1/2} d\,a$$

$$H_0\,t(a) = \int_0^a \left(\Omega_{k0} + \Omega_{m0}\,x^{-1} + \Omega_{r0}\,x^{-2} + \Omega_{v0}\,x^2\right)^{-1/2} d\,x$$

Important & instructive cases:
Matter:

$$H_0\,t(a) = \int_0^a \left(1 - \Omega_{m0} + \Omega_{m0}\,x^{-1}\right)^{-1/2} d\,x$$

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Integrate[1 / Sqrt[(1 - o) + o x^-1], {x, 0, a}, Assumptions → {1 > o > 0, 1 > a > 0}]
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$$\left(-\sqrt{a\,o\,(a + o - a\,o)} + o\sqrt{-\frac{o}{-1+o}}\ \text{ArcSinh}\left[\sqrt{a\left(-1+\frac{1}{o}\right)}\right]\right) \Big/ \left((-1+o)\sqrt{o}\right)$$

The age of the universe at expansion parameter *a*

$$t(a) = H_0^{-1}\left\{-\left[a\,\Omega_{m0}(a + \Omega_{m0} - a\,\Omega_{m0})^{1/2} + \Omega_{m0}^{3/2}(1-\Omega_{m0})^{1/2}\,\text{arcsinh}\left[\left(1 - \Omega_{m0}^{-1})\right]^{1/2}\right\} \Big/ \left[(-1+\Omega_{m0})\,\Omega_{m0}^{1/2}\right]$$

More specifically, for $\boldsymbol{\Omega_{m0} = 0}$,

$$H_0\,t = a$$

For $\boldsymbol{\Omega_{m0} = 1}$,

$$H_0\,t = \frac{2}{3}\,a^{3/2}$$

For $\boldsymbol{\Omega_{m0} = 2}$,

$$H_0\,t = \int_0^a \left[-1 + 2\,x^{-1}\right]^{-1/2} d\,x$$

$$= \int_0^a \frac{x}{\left[2\,x - x^2\right]^{1/2}}\,d\,x = \int_0^a \frac{1-(1-x)}{\left[1-(1-x)^2\right]^{1/2}}\,d\,x$$

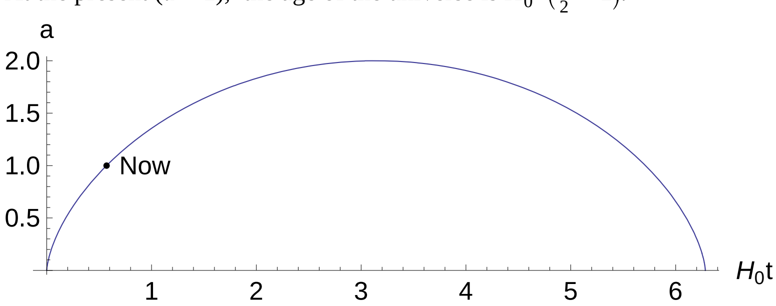
$$= \arccos(1-x)\,\Big|_0^a + \left[1-(1-x)^2\right]^{1/2}\,\Big|_0^a$$

Or.

$$a = 1 - \cos\eta$$
$$t = H_0^{-1}(\eta - \sin\eta)$$

What is the maximum value for the expansion parameter?

At the present ($a = 1$), the age of the universe is $H_0^{-1}\left(\frac{\pi}{2} - 1\right)$.



Radiation with $\boldsymbol{\Omega_{r0} = 1}$:

$$H_0\, t(a) = \int_0^a \left(x^{-2}\right)^{-1/2} d\,x = \int_0^a x\, d\,x$$

$$H_0\, t = \frac{1}{2}\, a^2$$

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Integrate[x / Sqrt[x^2 (1 - o) + o], {x, 0, a}, Assumptions → {o > 1, 1 > a > 0}]
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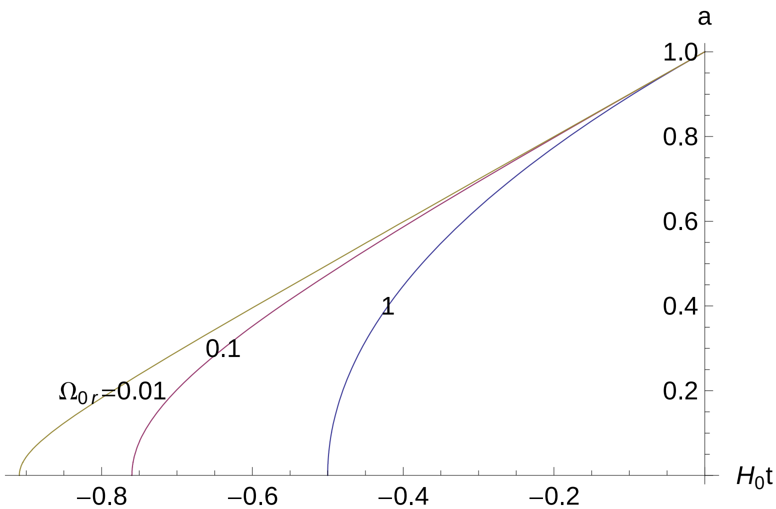
$$\frac{\sqrt{o} - \sqrt{-a^2\,(-1+o)+o}}{-1+o}$$

$$H_0\, t = \left[\Omega_{0\,r}{}^{1/2} - \left(\Omega_{0\,r} - a^2(\Omega_{0\,r}-1)\right)^{1/2}\right] \Big/ (\Omega_{0\,r}-1)$$

For $\Omega_0 = 1$,

$$H_0\, t = \frac{1}{2}\, a^2$$



Caption: time vs expansion parameter for a universe with radiation only.

Q: All of the models have the same slope at the present time. Why?

Q: Why does it take less time for the universe to expand with a larger density of radiation?

- **Plots**

Estimate of the angular size of the fluctuations for $\Omega_{\text{matter}\,0} = 1$.

$$\theta = L/(a\, r)$$

Assume the universe is matter dominated. a_r is expansion parameter at recombination. Assume $\Omega_0 = 1$.

$$L = \text{age of universe}$$
$$= H_0^{-1}\, \frac{2}{3}\, a_r^{3/2}$$

$$r\,(a) = 2\, H_0^{-1}\left(1 - a^{1/2}\right)$$

$$\theta = \frac{1}{3}\, a_r^{1/2} \Big/ \left(1 - a_r^{1/2}\right)$$

In[646]:= $\frac{1}{3}$ .001^1/2 / (1 - .001^1/2) / ° "°"

Out[646]= 0.623673 °

Now we need to do a better job.
1. Radiation dominates in the early universe.
2. How fast do sound waves go?
3. Part of the speed of sound is erased by the expansion of the universe. (Hartle 18.3).

Sound waves before recombination

Physical conditions at recombination

At recombination, which has the greater mass density, pressureless matter or radiation?

$\Omega_{m0} = 0.26$ pressureless matter, mostly dark matter, matter that does not interact with light
$\Omega_{b0} = 0.043$ baryons, ordinary matter
$\Omega_{r0} = 1.2\times 10^{-5}$ radiation

$\rho_b = \rho_{b0}\, a^{-3}$
$\rho_r = \rho_{r0}\, a^{-4}$

$\rho_m / \rho_r = \rho_{m0} / \rho_{r0}\, a$
At $a_{eq} = 0.00028$ ($z = 3600$), the mass-energy density of baryonic matter and radiation are equal.
Q: At recombination (a=0.0009), which has greater mass density, pressureless matter or radiation?

We will discuss sound waves, which has to do with radiation and matter.
Q: Does dark matter participate in the sound waves?

Q: At recombination, are electrons pressureless? The energy of a CBR photon is 2.3×10^{-4} eV.

At recombination, which has greater number density, baryonic matter or radiation?

At the present time, the mass of baryonic matter is 938MeV.
The mass of a photon is

$$2.73\, K / (11\,600\, K/\text{eV}) = 2.3\times 10^{-4}\ \text{eV}.$$

The number density

$$n_r / n_b = 0.00028 \times 938\ \text{MeV} / 2.3\times 10^{-4}\ \text{eV} = 1.1\times 10^{9}.$$

More precisely, because photons have different energies, I need to integrate the Planck number spectrum.

$$n_r = 0.41\times 10^{9}\ \text{photon}\ m^{-3} (T / 2.725\ \text{K})^3$$
$$n_b = 0.25\ \text{nucleon}\ \text{m}^{-3} (\Omega_{b0} / .043)\, (H_0 / 72\ \text{km/s/Mpc})^2$$
$$n_r / n_b = 1.64\times 10^{9}.$$

The number of photons and baryons do not change. As the universe expands, the number of baryons in a coexpanding box does not change. The number of baryons entering must equal the number exiting, because of homogeneity. Same argument is true for photons.