9 Feb 2012

16 Feb 2012

21 Feb 2012

23 Feb 2012

28 Feb 2012

Sound waves before recombination

- Outline
  - Is sound carried by radiation or by matter?
  - Sound speed
  - Calculation of the angular scale for a universe dominated by radiation
  - Precise calculation of the angular scale

Physical conditions at recombination

At recombination, which has the greater mass density, pressureless matter or radiation?

$\Omega_{m0} = 0.26$ pressureless matter, mostly dark matter, matter that does not interact with light
$\Omega_{b0} = 0.043$ baryons, ordinary matter
$\Omega_{r0} = 1.2\times 10^{-5}$ radiation

$\rho_b = \rho_{b0}\, a^{-3}$
$\rho_r = \rho_{r0}\, a^{-4}$

$\rho_m / \rho_r = \rho_{m0} / \rho_{r0}\, a$
At $a_{eq} = 0.00028$ ($z = 3600$), the mass-energy density of baryonic matter and radiation are equal.
Q: At recombination (a=0.0009), which has greater mass density, pressureless matter or radiation?

We will discuss sound waves, which has to do with radiation and matter.
Q: Does dark matter participate in the sound waves?

Q: At recombination, are electrons pressureless? The energy of a CBR photon is 2.3×10^{-4} eV.

Which has greater number density, baryonic matter or radiation?

At the present time, the mass of baryonic matter is 938MeV.
The mass of a photon is

$$2.73\, K/(11\,600\, K/\text{eV}) = 2.3\times 10^{-4}\ \text{eV}.$$

The number density

$$n_r / n_b = 0.00028 \times 938\ \text{MeV} / 2.3\times 10^{-4}\ \text{eV} = 1.1\times 10^{9}.$$

More precisely, because photons have different energies, I need to integrate the Planck number spectrum.

$$n_{\mathrm{r}} = 0.41\times 10^{9} \text{ photon } m^{-3}(T/2.725\,\mathrm{K})^{3}$$
$$n_{\mathrm{b}} = 0.25 \text{ nucleon } \mathrm{m}^{-3}(\Omega_{\mathrm{b}0}/.043)\,(H_{0}/72\,\mathrm{km/s/Mpc})^{2}$$
$$n_{\mathrm{r}}/n_{\mathrm{b}} = 1.64\times 10^{9}.$$

The number of photons and baryons do not change. As the universe expands, the number of baryons in a coexpanding box does not change. The number of baryons entering must equal the number exiting, because of homogeneity. Same argument is true for photons.

How can you say $n_{\mathrm{r}}/n_{\mathrm{b}} = 1.64\times 10^{9}$ in a dramatic way?

■ Is sound carried by photons or by matter?

The composition of the gas is ordinary matter and photons.

■ Does matter or radiation provide more pressure?

Pressure $P = n\, p_x\, v_x$, where n is number density, p and v are momentum and speed.
For matter,

$$P = n_{m}\, m\, v_x^2 = \frac{1}{3}\, n_m\, m\, v^2.$$

For radiation,

$$P = \frac{1}{3}\, n_r\, E \text{ (from earlier class).}$$

Equipartition: In thermal equilibrium, the average energy of each particle is the same. (More precisely, the energy of each degree of freedom is $\frac{1}{2}\, k\, T$. In QM, degrees of freedom may be frozen with 0 energy.)

For matter

$$P_m = \frac{2}{3}\, n_m\, \frac{3}{2}\, k\, T = n_m\, k\, T.$$

For radiation: $P_r \approx \frac{1}{3}\, n_r\, \frac{1}{2}\, k\, T$. More accurately,

$$P_r = 0.90\, n_r\, k\, T.$$

There are 10^9 photons for every baryon. Therefore the <u>pressure of radiation is 10^9times the pressure of matter.</u>

■

Sound speed is $\left(\frac{\partial P}{\partial \rho}\right)^{1/2}$

The temperature at recombination is 3000K
There are many photons for every baryon or electron.

$$n_{\mathrm{r}}/n_{\mathrm{b}} = 1.64\times 10^{9}$$

At $a_{\mathrm{eq}} = 0.00028$ ($z = 3600$), the mass-energy density of baryonic matter and radiation are equal.

The speed of sound

$$v_s = \left(\frac{\partial P}{\partial \rho}\right)^{1/2}$$

where the derivative is for adiabatic changes.

Proof: Newton's 2nd law $F = m\,a$ determines the movement of a sound wave, The force is due to an excess pressure. The mass is due to an excess density. Consider a slab of gas between x and $x + dx$. Because of the presence of the disturbance, x moves to $x + \chi(t,\ x)$. The $m\,a$ term becomes

$$(\rho_0\,dx)\,\frac{\partial^2\chi}{\partial t^2}.$$

The force comes from the difference in pressure. The force term is

$$-\frac{\partial P}{\partial x}\,dx.$$

I need to relate pressure to mass density:

$$P = -\frac{\partial P}{\partial \rho}\,\rho_0\,\frac{\partial \chi}{\partial x}$$

Collect all; cancel ρ_0 and dx:

$$\frac{\partial^2\chi}{\partial t^2} = \frac{\partial P}{\partial \rho}\,\frac{\partial^2\chi}{\partial x^2}$$

The speed of sound is

$$v_s = \left(\frac{\partial P}{\partial \rho}\right)^{1/2}$$

The derivative is taken with no heat flow, if the wavelength is large compared to the mean-free path.

- **Sound speed for a perfect gas**

For adiabatic changes

$$P = \text{const}\,\rho^\gamma.$$

For a monotonic gas, $\gamma = \frac{5}{3}$.

Take derivative to get

$$v_s^2 = \gamma\,P/\rho$$
$$= \gamma\,P\,V/(\rho\,V)$$
$$= \gamma\,k\,T/m$$

Equipartition $\Rightarrow \frac{3}{2}\,k\,T = \frac{1}{2}\,m\,v_{\text{avg}}^2$ Therefore

$$v_s = \left(\frac{\gamma}{3}\right)^{1/2} v_{\text{avg}}$$

The sound speed is approximately the average speed of the gas particles.

- **Values**

Calculating the sound speed

Consider a box of gas with a fixed number of particles. The box expands or shrinks because of the sound wave.

1. $d\,U = d\,Q - P\,d\,V = -P\,d\,V$

Because there is no heat flow, $d\,Q = 0$.

$$d\,U = -P\,d\,V.$$

Recall $u = a_B\,T^4$ and $P = \frac{1}{3}\,a_B\,T^4$

$$d\,(u\,V) = V\,d\,u + u\,d\,V = -P\,d\,V$$
$$d\,u = -(u + P)\,d\,V/V$$
$$4\,T^3\,d\,T = -\left(T^4 + \frac{1}{3}\,T^4\right)\text{dV}/V$$
$$3\,d\,T/T = -d\,V/V$$

2. $d\,P = \frac{4}{3}\, a_B\, T^3\, d\,T$.
3. $d\,\rho = d\,\rho_b + d\,\rho_r$

   $d\,\rho_b = -\rho_b\, d\,V / V$, since mass of the baryons in the box ($\rho_b\, V$) is unchanged.

   $d\,\rho_b = 3\,\rho_b\, d\,T / T$

   $d\,\rho_r = 4\,a_B\, T^3\, d\,T$
4. Gather all:

   $v_s^2 = \frac{dP}{d\rho} = \left(\frac{4}{3}\, a_B\, T^3\, d\,T\right)\left(3\,\rho_b\, d\,T / T + 4\,a_B\, T^3\, d\,T\right)^{-1}$

   $= \left(3 + \frac{9}{4}\, \frac{\rho_b}{\rho_r}\right)^{-1}$

   $\boxed{v_s = [3\,(1 + R)]^{-1/2}}$,

where

$R = \frac{3}{4}\, \frac{\rho_b}{\rho_r}$

Q: If $R \ll 1$, how fast do sound waves travel?

Q: Why do baryons slow the speed of sound? Recall $v_s = \left(\frac{d\,P}{d\,\rho}\right)^{1/2}$.

Number density of photons:

```
Integrate[x^2 / (ⅇ^x - 1), {x, 0, ∞}]
```

```
2 Zeta[3]
```

Energy density:

```
Integrate[x^3 / (ⅇ^x - 1), {x, 0, ∞}]
```

$$\frac{\pi^4}{15}$$

Average energy:

```
% / %% // N
```

```
2.70118
```

$\frac{1}{3}\,\langle E \rangle$ is

```
% / 3
```

```
0.900393
```

Calculating the horizon

How far does a sound wave travels from the big bang (t=0) to the time of recombination?
Let

a_L	be the expansion parameter at last scattering (recombination)
a_E	be the expansion parameter at epoch when $\rho_r = \rho_b$.

The distance of the horizon is

$$d = \int v_s \, d\, t.$$

$v_s\, d\, t$ is how far the sound wave moves. As the wave is moving, the ending point is moving too. Calculation is wrong.

Better posed: What is the comoving coordinate r of a sound wave that travels from the big bang (t=0) to the epoch of recombination?

$$v_s\, d\, t = a\, d\, r$$

$$r = \int_0^{t_L} v_s\, a^{-1}\, d\, t.$$

Given r, how do you get the distance of the horizon? $d = a_L\, r$

$$d = a_L \int_0^{t_L} v_s\, a^{-1}\, d\, t.$$

The sound speed $v_s = [3\,(1+R)]^{-1/2}$ depends on $R = \frac{3}{4}\,\frac{\rho_b}{\rho_r} = \frac{3}{4}\left(\frac{a}{a_E}\right)$.

- **To gain understanding, consider this simplified case where matter is negligible: $R \ll 1$.**

Then

$$d = a_L\, v_s \int_0^{t_L} a^{-1}\, d\, t$$

Use Friedman's equation

$$\left(\frac{da}{H_0\, dt}\right)^2 = \left(\Omega_{k0} + \Omega_{m0}\, a^{-1} + \Omega_{r0}\, a^{-2} + \Omega_{v0}\, a^2\right) \to \Omega_{r0}\, a^{-2}$$

$$d = a_L\, v_s \int_0^{t_L} a^{-1}\, d\, t$$

$$= H_0^{-1}\, a_L\, v_s\, \Omega_{r0}^{-1/2} \int_0^{a_L} d\, a$$

$$= H_0^{-1}\, a_L^2\, v_s\, \Omega_{r0}^{-1/2}$$

Apply F's eqn at a_L

$$H(a_L) = H_0\, \Omega_{r0}^{1/2}\, a_L^{-2}$$

to get the transparent result

$$d = H^{-1}(a_L)\, v_s$$

Q: Interpret the formula for d.

A dense region produces a sound wave that goes in all directions to cover a length $2\,d$.
The angle subtended is

$$\theta = \frac{2\,d}{r\, a_L}$$

$$= 2\, H^{-1}(a_L)\, v_s / [a_L\, r(a_L)]$$

Q: Interpret the formula for θ.

- **Results for best cosmological values**

$$d = a_L \int_0^{t_L} a^{-1}\, v_s(a)\, d\, t$$

Change $d t = H^{-1}\, a^{-1}\, d a$, and integrate to get (Weinberg 2008, Cosmology, p. 145)

$$d = 2\, H_0^{-1}\, a_L^{-3/2} (3\, R_L\, \Omega_{m0})^{-1/2} \ln\left\{\left[(1+R_L)^{1/2} + (R_E + R_L)^{1/2}\right] \big/ (1 + \sqrt{R_E})\right\}$$

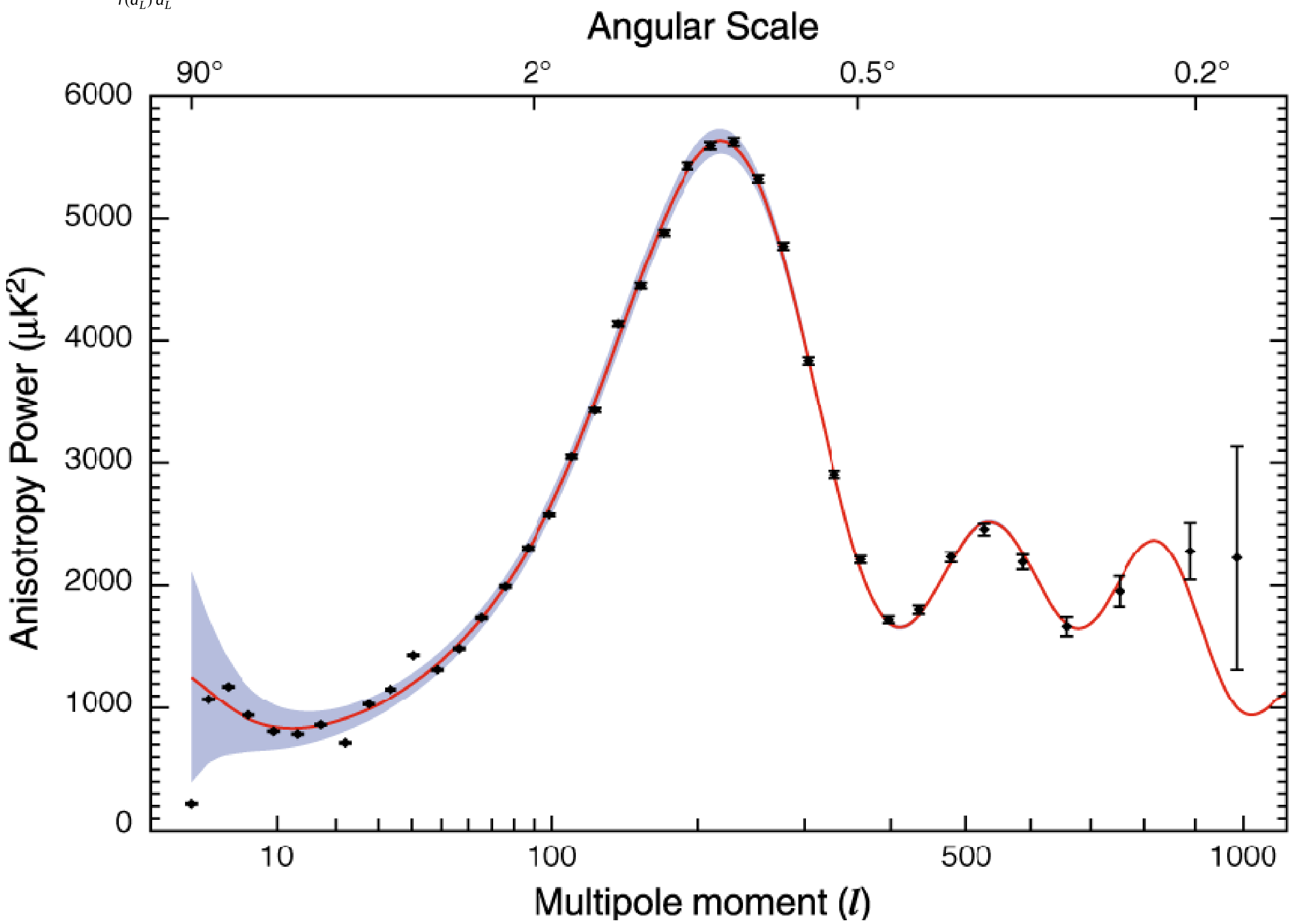
For $\Omega_{m0} = 0.26$, $\Omega_{v0} = 0.74$, $\Omega_{b0} = 0.043$,

$$R_L = 0.62$$

$$R_E = 0.21$$

$$d = 1.16\, H_0^{-1}\, a_L^{3/2}$$

$$\theta = \frac{2\,d}{r(a_L)\,a_L} = 1/48 = 1.2\,°$$



■ **Plot**