1. Make certain you are able to do problem 5.1 in the textbook. The answers are at the end.

2. (5 pts.) Do problem 5.2 in the textbook.

3. (5 pts.) Derive the addition law of velocities by observing the 4-momentum of a particle moving in the x-direction with speed v_1 in a frame moving at speed v_2 in the negative x-direction.

4. The coordinate time for a light pulse to go from $(-x_1, 0)$ to $(x_2, 0)$ is

$$t = (x_1^2 + \delta^2)^{1/2} + (x_2^2 + \delta^2)^{1/2} + 2M \log \frac{4x_1 x_2}{(y_0 + \delta)^2}. \tag{1}$$

The sun is at $(x, y) = (0, -y_0)$, and the light ray passes nearest the sun at $(0, \delta)$. In class we found δ_0 for which the time is an extremum.

$$\delta_0/x_1 + \delta_0/x_2 = 4M/(y_0 + \delta_0),$$

and we considered only the case $y_0 \gg \delta_0$.

(a) (2 pts.) Find the difference in time between the actual path and the path for which $\delta = 0$. Let $y_0 = 700\,\mathrm{Mm}$, which means the undeflected ray grazes the sun's surface. Let $x_1 = x_2 = 1\,\mathrm{AU} = 1.5 \times 10^8\,\mathrm{km}$.

(b) (5 pts.) Now pretend that the sun is a point mass. Find the second path for which the time is minimal. Sketch the path. Let $x_1 = x_2 = 1\,\mathrm{AU}$. Let $y_0 = 700\,\mathrm{Mm}$.

(c) (5 pts.) Although the situation in (b) does not occur for the sun, the result that you have derived applies to galaxies. A special case is that of an Einstein ring, where two galaxies and Earth are in a direct line. Let the distance $x_1 = x_2 = 1000$ Mpc $= 3 \times 10^{24}$ m and $M = 10^15$ m. Find the radius of the Einstein ring from equation (1).

Answers

(a) 5.1

- 4-vector a is time-like; b is null.
- $a - 5b = (-27, 0, -15, -19)$.
- $a \cdot b = 14$.