Ast 860 Midterm Exam 14 February 2012

Name ____________________________

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In the space outside a spherical mass M at the origin, the metric is

$$ds = -(1-2M/r)\,dt^2 + (1-2M/r)^{-1}\,dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right).$$

The Robertson-Walker metric is

$$ds^2 = -dt^2 + a(t)^2\left\{\left[1-(r/r_0)^2\right]^{-1/2} dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)\right\}.$$

Kepler's 3rd law for circular orbits: $\Omega^2 = M\big/r^3$, where $\Omega = \frac{d\phi}{dt}$.

Object	mass	radius [km]	semimajor axis [km]	period [yr]	ϵ
Earth	4.43 mm	6380	1.5×10^8	1	0.016
Mercury	0.245 mm	2425	5.8×10^7	0.24	0.21
Sun	1.48 km	6.96×10^5	□	□	□

Problem 1

The perihelion shift of a planet in orbit around a star of mass M is

$$6\pi M\big/\left[a\left(1-\epsilon^2\right)\right]$$

radians per orbit, where a is the semimajor axis, and ϵ is the eccentricity.

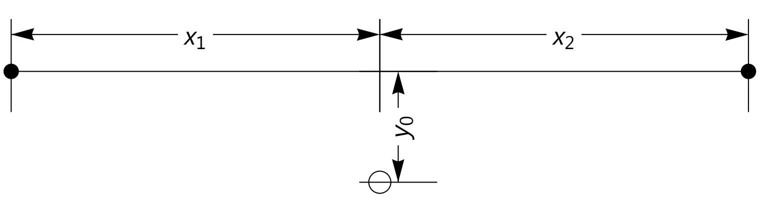
(a) (4 pts.)

Why is the perihelion shift in angle/century so much smaller for Earth than it is for Mercury? What kind of system would have a much larger perihelion shift in angle/century?

Problem 2

The coordinate time for light to travel from x_1 to x_2 while passing y_0 from the sun is

$$t = x_1 + x_2 + 4M\log\left[(4x_1x_2)^{1/2}\big/y_0\right].$$



(a) (2 pts.)

How much does the presence of the sun increase the coordinate time for light to go from earth at x_1 to a point opposite earth at $x_2 = x_1$? Assume the path grazes the sun. Express you answer in time units.

- **(b) (3 pts.)**

In what way is the Shapiro effect related to the deflection of light by the sun?

Problem 3

- **(a) (2 pts.)**

Consider two events $(t, x, y, z) = (0, 0, 0, 0)$ and $(2, 1, 0, 0)$ in Minkowski space. A frame of reference exists in which these two events occur at the same x, y, and z or at the same time. Which is it? Find the time separation or the spatial separation in that frame.

- **(b) (2 pts.)**

For the Schwartzschild metric given above, write all 16 terms of $g_{\mu\nu}$.

- **(c) (3 pts.)**

A particle of mass m at radial coordimate r near a star of mass m is moving in the radial direction. $p^0 = E$. Find p^r.

Problem 4

An Earthling at $r_E = 1$ AU $= 1.5\times10^8$ km sends signals every 1s to a Plutonian at $r_P = 40$ AU. You may ignore the mass of Earth and Pluto. The sun is at $r = 0$.

- **(a) (4 pts.)**

For this part, assume that the Earthling and Plutonian are stationary. Compute the rate at which the Plutonian receives signals.

- **(b) (4 pts.)**

For this part, the Earthling and Plutonian are in orbit around the sun, and they and the sun are in a line. Compute the rate at which the Plutonian receives signals. Why do the results of parts (a) and (b) differ?

- **(c) (4 pts.)**

For this part, the Earthling and Plutonian are in orbit around the sun, and their relative positions change. Explain why the rate at which the Plutonian receives signals depends on the location of Pluto relative to the Sun and Earth. At what two points is the rate shifted by the largest amount?