Diffraction and Interference of Plane Light Waves

Introduction

In this experiment you will become familiar with diffraction patterns created when a beam of light scatters from objects placed in its path. Such experiments were important historically as they were crucial in establishing the wave nature of light in face of competing theories that described light in terms of geometrical rays of discrete objects. Only a wave theory can give a quantitative explanation of the complex phenomena of diffraction and interference. A more complete description of diffraction and interference phenomena can be found in textbooks such as in chapters 21 and 22 of "Physics for Scientists and Engineers" by Knight, or Chapter 29 of Bauer & Westfall. A review lecture is also available on the 192 web site.

Theory Part I: Diffraction

When a plane light wave (in our case a laser beam) hits an obstacle it is diffracted. We can understand this phenomenon in terms of Huygens principle that states that every point in the wave front can be considered as a source of new wave fronts. This is illustrated in Figure 1 where a parallel beam of light strikes a barrier with a narrow slit and the diffracted wave can be considered to originate from a source at the slit.

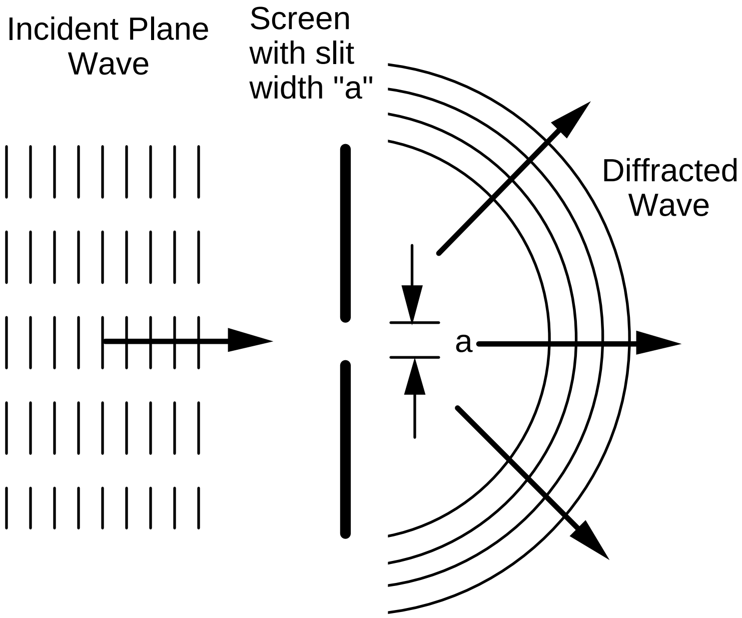

Fig. 1: Schematic representation of diffraction from a slit

After the slit, points of equal phase on the wave front are no longer in a plane but on concentric circles as indicated in the figure. This is exactly true in the limit where the slit width "a" goes to zero.

The result of this is that if we observe the image of the slit from a distance we will not only see a bright central image of the slit but, in addition, a pattern of light and dark areas around the central image. This pattern is called the diffraction pattern. Its origin is explained in Figure 2 where we consider two wave fronts leaving the slit at an angle Θ with respect to the normal to the slit.

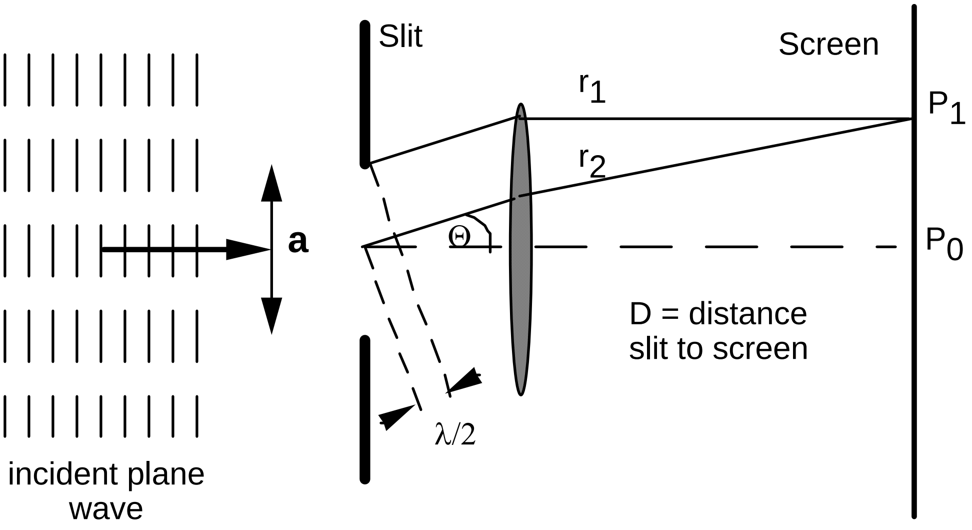

Fig. 2: Conditions for first minimum of diffraction from a slit
Ignore the lens: the equations are the same if D>>a.

These two wave fronts have the same phase at the slit, one starting at the edge, the other at the center. We will observe the resulting intensity pattern on a screen a distance D from the slit, where D is large compared to the slit width "a". This is sometimes called Fraunhofer diffraction. This can be achieved by using a focusing lens or by choosing D to be large (1 - 2 m) as is done in this lab. If the two light rays in Figure 2 are out of phase by 180° (at the point of observation P_1) the intensity at P_1 will be zero, i.e. it will be dark. This is the case when the paths of the two differ by n λ/2 where n = ± 1, ±2, ±3 etc. and λ is the wavelength of the light. So for the case shown, the condition for P_1 to be dark is:

$$\frac{a}{2}\sin\theta = n\,\lambda/2 \qquad (1)$$

Therefore, we will find diffraction minima at the following angles:

$$\sin\theta = n\,\lambda/a \qquad (2)$$

with n = ± 1, ±2, ±3 etc. For a single slit, the diffraction pattern intensity I, as a function of the angle θ is given by (Halliday and Resnick, Ch. 41):

$$\mathrm{I} \sim (\sin\alpha/\alpha)^2 \qquad (3a)$$

where

$$\alpha = (\pi\,\mathrm{a}/\lambda)\sin\theta \qquad (3b)$$

The observed intensity distribution, given by Eq. 3 is displayed in Figure 3 for different values of "a" in terms of the wavelength λ.

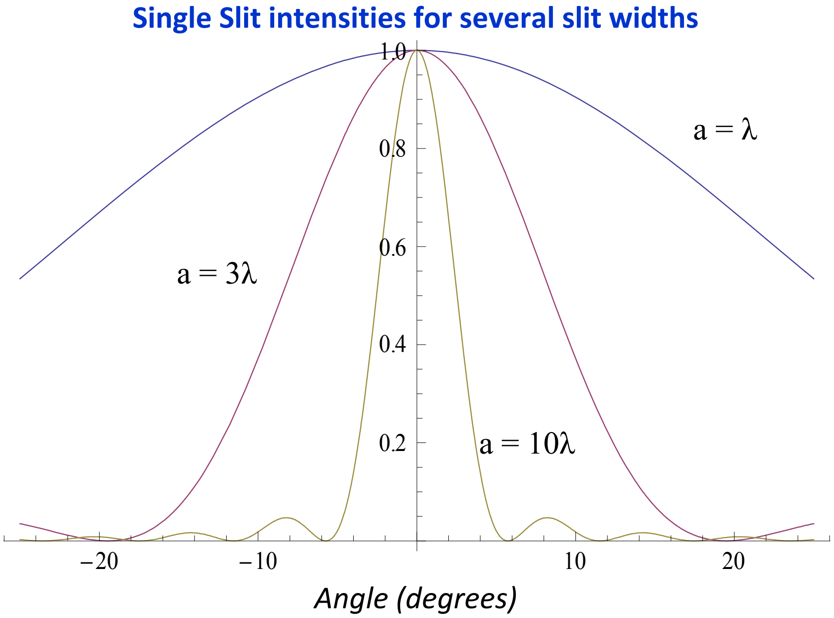

Fig. 3: Relative intensity vs. angle patterns for single slit diffraction.

The intensity in the higher order maxima is much less than the central peak. Another feature is that with increasing slit width, the central peak becomes narrower and the secondary maxima more pronounced.

Theory Part II: Diffraction and Interference

Double Slit Interference

Up to this point, only single slits have been discussed and now we want to turn to what happens in the case of multiple slits. For simplicity we will consider a plane wave falling onto a double slit consisting of slits S_1 and S_2 as indicated in Figure 4. Furthermore, we will assume that the width of these slits is negligible so that there is no diffraction from within each slit. Each of the slits will act as a source of light and waves originating from S_1 and S_2 will be coherent, i.e. in phase. The interaction of these two coherent waves is in general called interference and this interaction will give rise to an interference pattern (or fringes) on the screen, i.e. we will observe light and dark areas. Again, in Figure 4 a focusing lens has been drawn, whereas, in the lab the distance from slit to the screen, D, is large and thereby eliminates the need for such a lens.

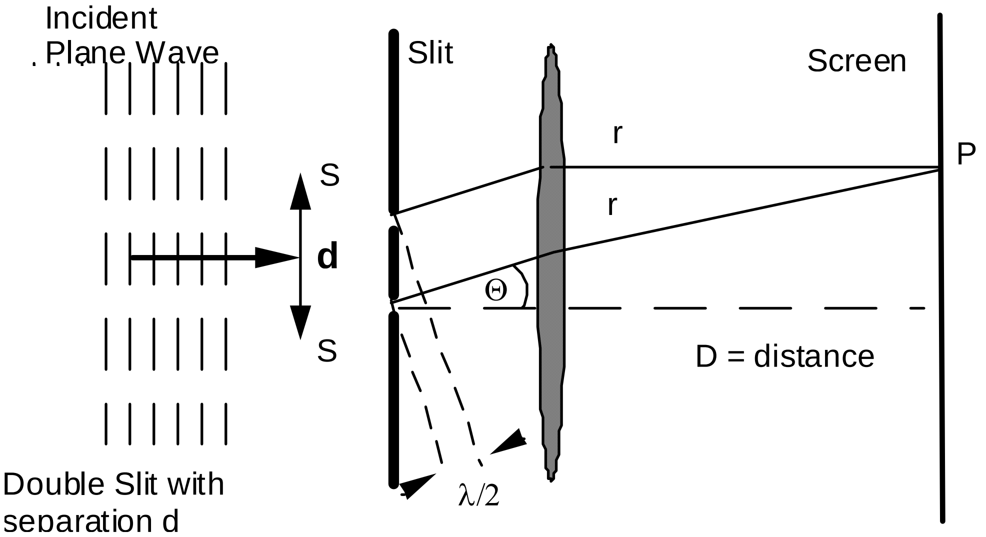

Fig. 4: Interference from a double slit.
Ignore the lens: the equations are the same if D>>d.

In a fashion similar to the first part of the lab, the conditions for destructive (intensity = zero) and constructive (intensity = maximum) interference are simple. Destructive interference results when the two waves are out of phase by 180° which is equivalent to saying that their path length (between slit and screen) differs by m λ/2. If the distance between the centers of the slits is "d" then the condition for interference minima is :

$$d \sin\theta = \left(m + \tfrac{1}{2}\right)\lambda \quad m = 0,1,2\cdots \qquad (4)$$

Similarly the condition for interference maxima is:

$$d \sin\theta = m\,\lambda \quad m = 0, 1, 2, \cdots \qquad (5)$$

The intensity pattern will have a central maximum (θ= 0) for m = 0 which is called the 0th order maximum. Other maxima occur at angles given by m = 1, 2, 3,.. etc. and "m" is called the order number.

The intensity distribution from the simplified double slit (assuming very small slit width) as a function of Θ is given by (see Halliday and Resnick Ch. 41-7):

$$\mathrm{I} \sim (cos\beta)^2 \quad \text{with} \quad \beta = (\pi\, d/\lambda)\sin\theta \qquad (6)$$

The distribution is shown in Figure 5a.

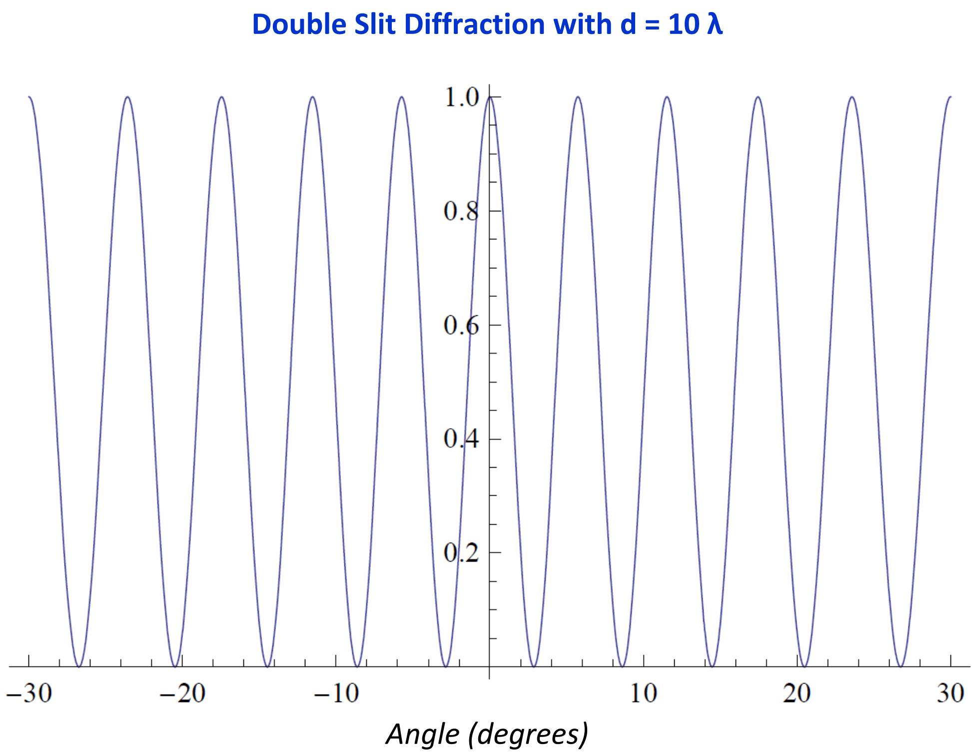

Fig. 5a: Diffraction from a double slit.

If we now allow the width of the two slits to be finite, diffraction will occur and the intensity distribution will be that given by Eq. 3 and shown graphically in Figure 5b.

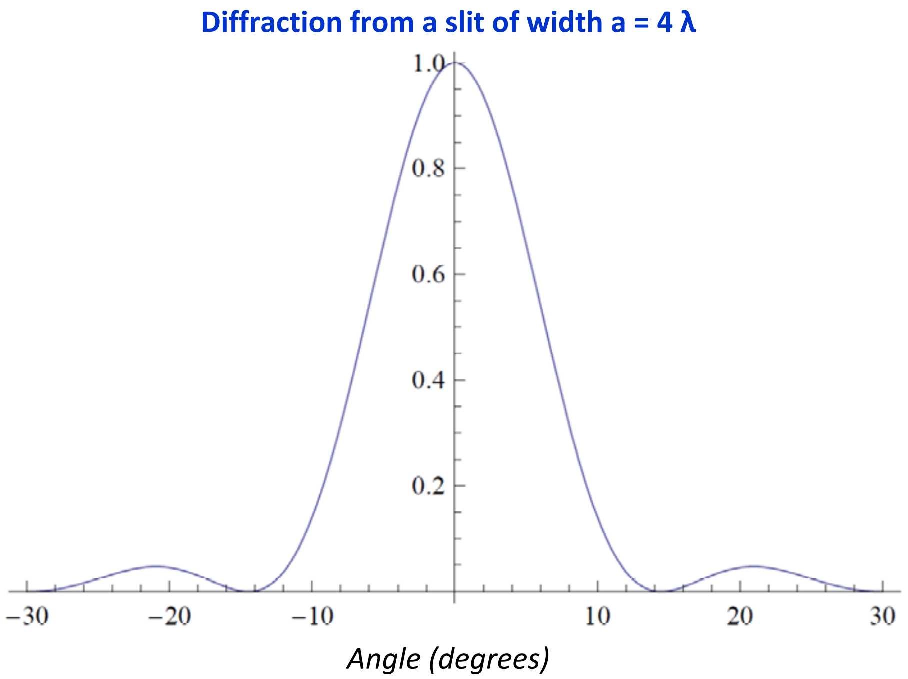

Fig. 5b: Diffraction from a single slit

The intensity distribution resulting from the combination of diffraction and interference is given by the product of Equations 3 and 6 and is displayed in Figure 5c.

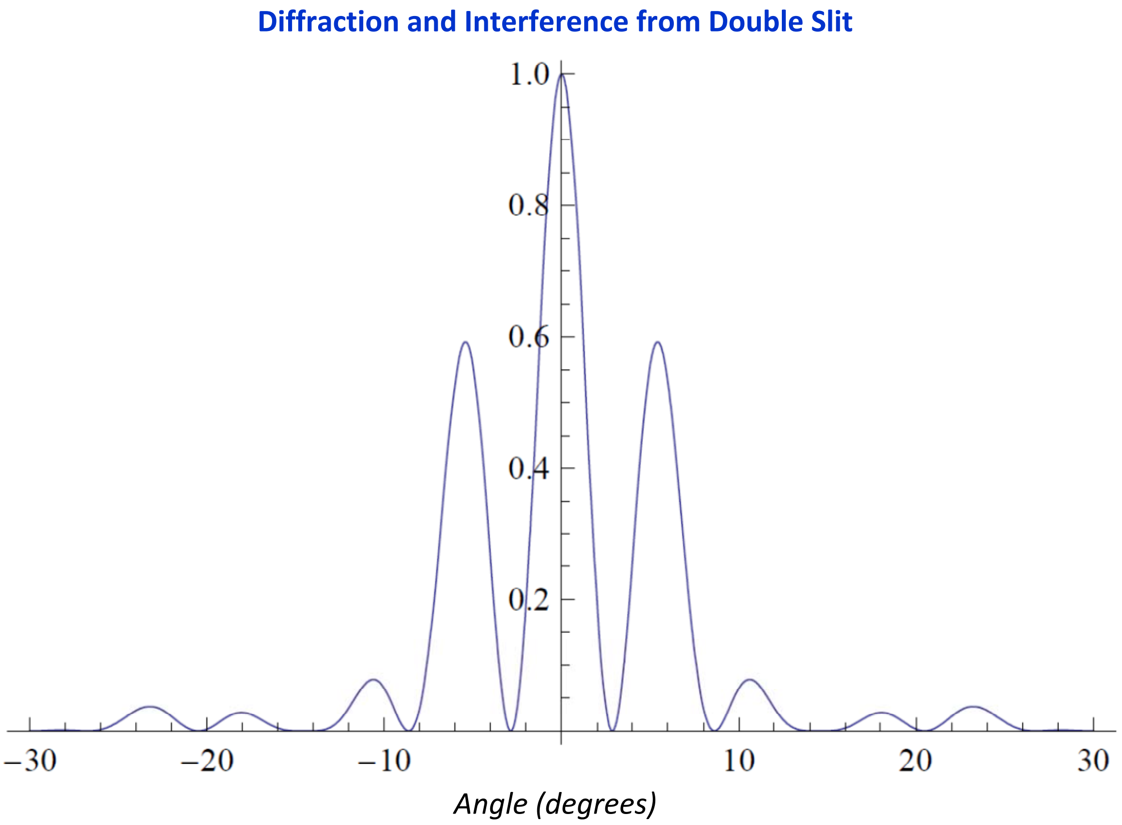

Fig. 5c: Diffraction and Interference from Double Slit
For slit separation = 10 λ and Slit Width = 4 λ

The envelope of the distribution is determined by diffraction, whereas, the internal structure is due to interference. Note, however, that the envelope depends critically on the width of the slit and, as expected, the influence of diffraction can be minimized by choosing very narrow slits (see Figure 3). In our example, diffraction broadens the minimum in the region of 14°, as you can see by comparing the narrow minimum of Figure 5a with the broader minimum of 5b and thus in 5c.

Multiple Slits

A logical extension of the double slit experiment is the multiple slit experiment where the number of slits is increased from two to some large number N. A particular multiple slit arrangement where the number of slits can exceed 10^3/mm is called a diffraction grating. With increasing N, the interference fringes within the central diffraction envelope become narrower. The condition for maximum is still given by Eq. 5, but now "d" has become very small.

Experiment:

Part 1: Diffraction and Single Slits

A laser produces monochromatic light (λ = 632.8 nm red; 532 nm green) which is well collimated and coherent. Make sure your eyes are never exposed to direct laser light or its reflections.

1.1 Single Slit Diffraction

Set up a projection screen by clipping a piece of paper to the stand on the optical beam. Use the single slit wheel, which contains several single slits of different widths, in front of the laser so that a diffraction pattern is produced on the screen. Do the best compromise between autocollimation and centering you can (autocollimation attempts to make the slit wheel perpendicular to the beam). Record the patterns for the four available single slits. Then for each width, record the locations of the minima ($\geq$ 5 on each side).

1.1.1 From the pattern, the distance D from slit to screen and from λ, compute the slit width "a" and estimate your error in "a".

1.1.2 Compare your measurement with the value indicated on the slit.

1.1.3 Do the general features of the observed patterns and their dependence on the slit width agree with the predictions of Figure 3?

1.1.4 Observe and describe qualitatively the results of using the variable slit.

1.1.5 **Extra Credit**: Describe what you see for of the one of the 2-d patterns or circular apertures. How is it related to the physical pattern on the disc?

1.2 Diffraction from a line and a slit

Now use the line/slit patterns. From the diffraction pattern determine the thickness of the line using the single diffraction formula for the location of the minima. Is the pattern the same as the slit of the same width? Again estimate errors and compare to the given width. Use this method and a sample holder to measure the diameter of your hair (no error estimate needed).

Question 1: If you shine the laser beam on a hair, wire or line, why do you expect to see a diffraction pattern instead of just a shadow? In particular, why could there be a bright spot directly behind the obstacle? Think about the wire as being complementary to a slit.

Part 2 Interference and Diffraction: Double Slits

Switch holders to the double slits. Repeat the procedure of part 1 for this set of four double slits: quantitatively for one of the slits (.04 or .08 mm x .25 mm works well), and qualitatively for the rest. From the observed pattern for the selected double slit, calculate both slit width and slit separation and their uncertainties for one of the double slits. Measure minima for diffraction, maxima for interference (you'll need 5 or more mins and maxes on each side of 0). For the other 3 double slits, compare the observed patterns with your expectations based on a knowledge of "a" and "d". Describe the effects of the variable double slit, and use it to explain your results qualitatively.

Part 3: Multiple Slits

Repeat the procedure qualitatively for the set of multiple slits with N = 2, 3, 4, 5. Comment on the spacing, width and brightness of the principal maxima for these 4 cases.

Part 4: Diffraction Grating

Measure the bright spots for two different "diffraction" gratings. In the lab, there should be gratings with 500 lines/mm and 1000 lines/mm. You may have to reduce D to observe the patterns, and probably won't be able to find 5 bright spots on each side. In any case, the angles involved may not be small, so you'll have to do the trigonometry without making any small angle approximations. From the patterns calculate the grating spacing, "d", for each grating.

Manufacturer's Tolerance

In your analysis of results, the manufacturer's error tolerances are useful to know. For the Pasco slits on the rotating holder, use the following:

slit width tolerance: ±.005 mm
slit separation tolerance
±.005 mm if separation is .125 mm
±.010 mm otherwise

These specifications are best interpreted, not as a standard deviation, but as giving a range of values within which the manufacturer claims the true value will lie. One way to handle this is to find a t value for the nominal slit parameters from the manufacturer, and then, if this t value indicates incompatibility, to check whether there is any value within the manufacturer's range which would give an acceptable t value.

Question 2: Which is more precise: your ability to measure slit parameters with this technique (using an accurately-know laser wavelength)? Or the manufacturer's ability to fabricate the slits?