PHY321 Homework Set 9

1. [5 pts] Sketch the equipotential surfaces and the lines of force for: (a) single point mass, (b) two equal point masses separated by a distance d and (c) a uniform spherical mass of radius R. To the extent possible, the equipotential surfaces are normally drawn at constant decrements of the potential. In case (b), pay particular attention to aspects of lines and surfaces in close proximity to individual masses and far away from the masses. If you need formulas to help you with (c), you can use $\Phi(r)$ and $g(r)$ for a shell from the textbook, with inner shell radius equal to 0. In case (c), emphasize any differences between the lines and surfaces within and outside of the spherical mass.

2. [5 pts] Consider a spherically symmetric sphere of mass of radius R. Within the sphere the gravitational field vector points in the radial direction and has a constant magnitude g_0 .

   (a) Starting with the Gauss' law, arrive at the differential equation that $g(r)$ satisfies in a spherically system.

   (b) Find $\rho(r)$ for the constant g.

   (c) Find the net mass M of the sphere.

   (d) Determine the gravitational field $g(r)$ outside of the sphere, $r > R$. Is your field continuous at $r = R$?

3. [5 pts] An escape velocity is the minimal velocity an object must have to escape the effects of a gravitational field for a given location, without any further propulsion. Determine the escape velocity for an object leaving the surface of (a) Earth, (b) Moon, (c) Mars, (d) Jupiter and (e) Earth but including also the effects of the gravity of the Sun. For simplicity, in the latter case assume the Earth being at a fixed position relative to the Sun rather than circling around.

4. [10 pts] Consider a thin rod of mass M and length ℓ. The rod is concentric with the origin of a coordinate system and oriented along the z axis. (a) Find gravitational potential due to the rod at a location (r, z), where r is the distance away from the z-axis. Useful integral: $\int \mathrm{d}x/\sqrt{x^2+a^2} = \ln\left(x + \sqrt{x^2+a^2}\right)$. (b) Demonstrate that the obtained potential reduces to that for a point mass when the location is far away from the rod, $\sqrt{r^2+z^2} \gg \ell$. (c) Demonstrate that the obtained potential reduces to that for an infinite rod, familiar from electrostatics, when the location is very close to the finite rod, while away from the rod's ends.

5. [5 pts] Consider a model for Earth as a uniform sphere.

   (a) Use the Gauss' law for gravity to obtain gravitational field as a function of distance from Earth's center, both inside and outside of Earth. Verify that your results are consistent with the magnitude of the surface gravitational acceleration being about $9.81\,m/s^2$. Represent the field, both inside and outside, in terms of the surface magnitude, of approximately $9.81\,m/s^2$, multiplied by a factor that depends on position.

   (b) Imagine that a hole is drilled straight through the center of Earth. Demonstrate that, in absence of air resistance, a mass dropped into that hole would execute a simple harmonic motion about the Earth center.

   (c) Determine how long it would take for the mass to return once dropped into the hole.

6. [5 pts]

   (a) Use the Gauss' law for gravity to obtain the gravitational field in the vicinity of a sheet with uniform mass density σ.

   (b) What is the force exerted by the sheet on a uniform sphere of mass M and radius R at a distance $d > R$ from the sheet?

   (c) What is the force exerted by the sphere on the sheet?