PHY321 Practice Midterm 2

Formulas pertaining to the material:

Rocket equation: $v = -gt + u \log(m_0/m)$

Damped harmonic oscillator equation: $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$

General solution: $x(t) = \mathrm{e}^{-\beta t}\left[A_1 \exp\left(\sqrt{\beta^2 - \omega_0^2}t\right) + A_2 \exp\left(-\sqrt{\beta^2 - \omega_0^2}t\right)\right]$

Driven harmonic oscillator equation: $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = A\cos\omega t$

Amplitude of stationary oscillations: $D = \frac{A}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4\omega^2\beta^2}}$

Phase lag: $\delta = \tan^{-1}\left(\frac{2\omega\beta}{\omega_0^2 - \omega^2}\right)$

1. A rocket is ascending against gravity. The rocket engine delivers a thrust of $\tau = 1.50 \times 10^5\,\mathrm{N}$.

   (a) [1 pt] At what rate is the fuel being burnt, if the speed of the exhaust gas is $u = 3050\,\mathrm{m/s}$?

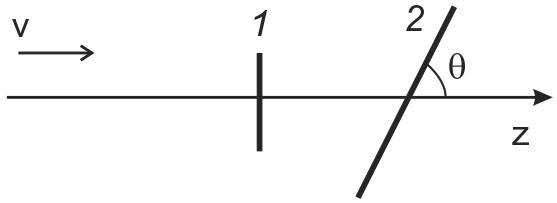
   (b) [2 pts] What is the maximal mass m_0 that this rocket could have for the take-off to occur right away?

   (c) [3 pts] How long would it take for the rocket with the maximal mass m_0 to reach the mass ratio of $m_0/m = 3.50$?

2. Two circular plates *1*, of radius $R_1 = 11.0\,\mathrm{cm}$, and *2*, of radius $R_2 = 26.0\,\mathrm{cm}$, are concentric with the z-axis. Plate *1* is perpendicular to the z-axis and plate 2 is inclined to the axis at an angle $\theta = 60°$. Pointlike projectile particles are incident onto the plates along the z-axis as shown.

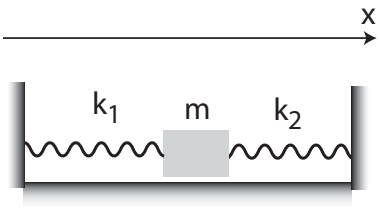
   (a) [1 pt] What is the cross section for projectile particles hitting plate *1*?

   (b) [4 pts] What is the cross section for projectile particles hitting any of the plates?



(Over)

3. A small object of mass m is positioned on a smooth plane and attached to walls on its two sides with stretched massless horizontal springs of spring constants k_1 and k_2, as displayed in the figure.



(a) [2 pts] What is the net force acting on the mass m when it is in its equilibrium position? What is the net force acting on the mass m when it is displaced by x from the equilibrium position, expanding by that distance one of the springs and compressing the other?

(b) [2 pts] What is the angular frequency of oscillations about the equilibrium position, in absence of friction? Compute the value for $m = 2.50\,\mathrm{kg}$, $k_1 = 6.50\,\mathrm{N/m}$ and $k_2 = 3.50\,\mathrm{N/m}$.

(c) [2 pts] If a friction force is further applied on the mass, opposite and proportional to the velocity, $F_f = -b\,\dot{x}$, with a proportionality constant of $b = 0.80\,\mathrm{kg/s}$, does the motion become underdamped, overdamped or critically damped?

(Over)

4. A mass $m = 3\,\text{kg}$, attached to a spring of spring constant $k = 5\,\text{N/m}$ and subject to a resistance force $F_r = -b\,\dot{x}$, where $b = 3\,\text{kg/s}$, is driven by a force changing sinusoidally with time $F_d = F_0 \cos \omega t$, where $F_0 = 7\,\text{N}$.

   (a) [1 pt] What is the period of steady-state oscillations of the mass, when the driving frequency is $\omega = 4\,\text{rad/s}$?

   (b) [4 pts] What is the amplitude of oscillations for the mass when $\omega = 4\,\text{rad/s}$?

   (c) [4 pts] How much work is done on the mass by the driving force above, during one period?

   (d) [2 pts] Describe in words and illustrate with a sketch how the phase lag changes when the driving frequency ω is changed from low to high values.