Pound-Rebka effect, Shapiro effect—12 Jan 2012

- Outline
    - Pound-Rebka effect
    - Shapiro effect

Pound-Rebka effect

“Apparent weight of photons,” 1960, PRL, 4, 337.

How much does the energy/frequency/wavelength of light change from the basement to the atic?

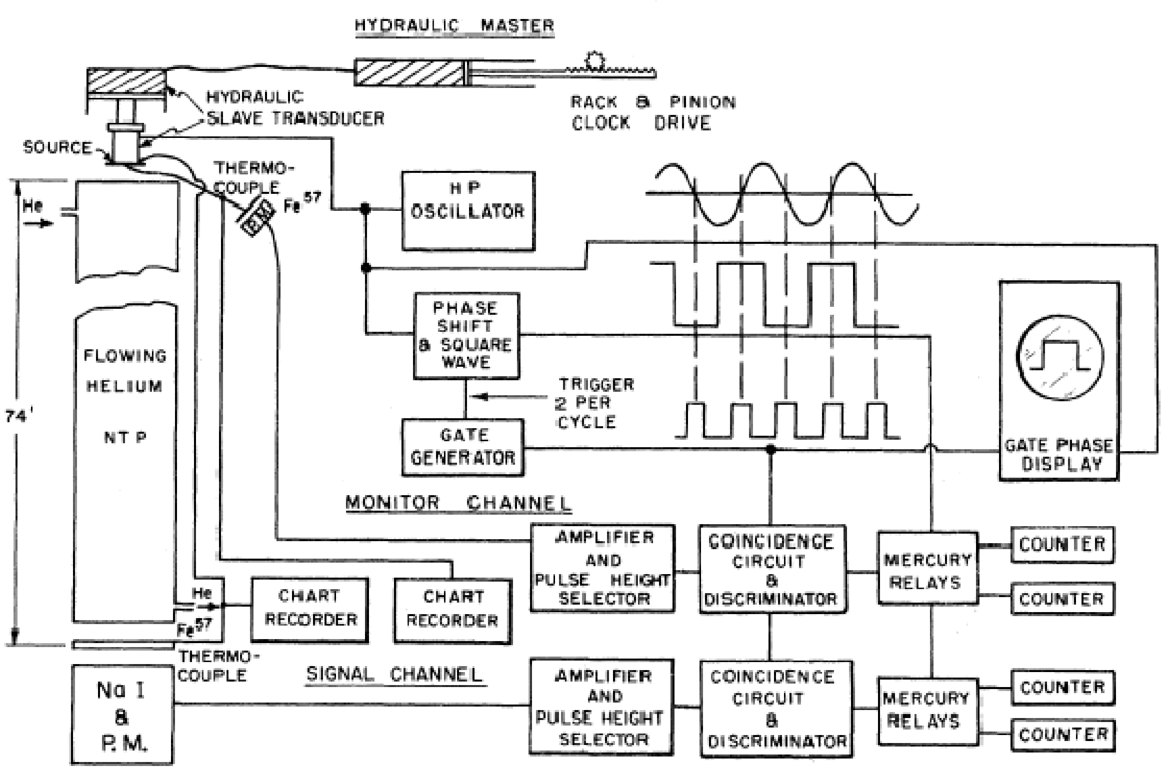

FIG. 1. A block diagram of the over-all experimental arrangement. The source and absorber-detector units were frequently interchanged. Sometimes a ferroelectric and sometimes a moving-coil magnetic transducer was used with frequencies ranging from 10 to 50 cps.

This measurement tests a component of the Schwartzschild metric.

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In[1026]:= Convert[GravitationalConstant SolarMass / SpeedOfLight^2, Kilo Meter]

Out[1026]= 1.47713 Kilo Meter

In[1031]:= massEarth = Convert[GravitationalConstant EarthMass / SpeedOfLight^2, Centimeter]

Out[1031]= 0.443652 Centimeter
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In[1032]:= rEarth = Convert[EarthRadius, 1. Kilo Meter]

Out[1032]= 6378.14 Kilo Meter
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Q: Estimate the size of the PoundRebka effect if the “atic” is at infinity.

Calculation

The Schwartzschild metric applies outside a star of mass M. The metric is

$$ds^2 = -\left(1 - 2\,\frac{M}{r}\right) dt^2 + \left[\left(1 - 2\,\frac{M}{r}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

Events

1. Source emits a crest.
2. Source emits a second crest.
3. First crest hits the receiver in the attic.
4. Second crest hits the receiver in the attic.

The basement is at $r_\oplus$. The attic is at $r_\oplus + h$.

Events occur at time coordinate t_1, t_2, etc.

Key idea

Q: Why is the time elapsed between events 1 and 2 not $(t_2 - t_1)$?

Calculation

Distance between events 1 and 2 is

$$1/\nu_e = (1 - 2\,M/r_\oplus)^{1/2}\,(t_2 - t_1)$$

Distance between events 3 and 4 is

$$1/\nu_r = (1 - 2\,M/(r_\oplus + h))^{1/2}\,(t_4 - t_3)$$

Distance between events 1 and 3 is 0. Why?

$$t_3 - t_1 = \int_{r_1}^{r_3} (1 - 2\,M/r)\,dr$$

For the same reasoning,

$$t_4 - t_2 = \int_{r_1}^{r_3} (1 - 2\,M/r)\,dr = t_3 - t_1.$$

Collect all to get

$$\nu_r/\nu_e = (1 - 2\,M/r_\oplus)^{1/2} \big/ (1 - 2\,M/(r_\oplus + h))^{1/2}$$
$$\approx 1 - M/r_\oplus + M/(r_\oplus + h)$$
$$\approx 1 - M\,h \big/ r_\oplus^2$$

Numerical value

For h=20m, $1 - \nu_r/\nu_e$ is

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In[1040]:= Convert[massEarth 20 Meter / rEarth^2, 1]

Out[1040]= 2.18115 × 10^-15
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What speed gives a Doppler shift of 2×10^{15}?

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In[1041]:= Convert[% SpeedOfLight, Micro Meter / Second]

Out[1041]= 0.653891 Meter Micro / Second
```

Pound & Rebka used γ rays from ^{57}Fe. The line width to frequency $\delta\nu/\nu = 10^{-12}$.

Shapiro effect

Shapiro, I., 1968, PRL, 13, 789
Shapiro et al., 1971, PRL, 26, 1132

Send a radar signal from Earth to Venus (or Mercury). Radar bounces off the planet and returns. Record the amount of time.

“The speed of propagation of a light ray decreases as it passes through a region of increasing gravitational potential.”

Q: Is Irwin Shipiro wrong? Isn’t the speed of light a constant? What does Shipiro mean?

■ More convenient form of the Schwartzschild metric

Let

$$r' = \frac{1}{2}\left[\left(r^2 - 2\,M\,r\right)^{1/2} + r - M\right]$$

or

$$r = r'[1 + M/(2\,r')]^2$$

At $r \gg M$, $r \approx r'\,(1 + M/r') = r' + M$. For large r, the radial coordinate shifts by M.

The new Schwartzschild metric (written without primes) is

$$d s^2 = -\left[\left(1 - \frac{M}{2\,r}\right)\Big/\left(1 + \frac{M}{2\,r}\right)\right]^2 d t^2 + \left(1 + \frac{M}{2\,r}\right)^4\left[d r^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right.$$

This looks complicated. What property makes this form of the Schwartzschild metric useful?

Consider the case $M/r << 1$. Note $M/r = -\Phi$, where Φ is the gravitational potential, not the angle. Then

$$d s^2 = -(1 + \Phi)^2\, d t^2 + (1 - \Phi)^2\, d l^2.$$

■ According to the new Schwartzschild metric, how is time distorted? How is space distorted?

The coordinate system $(t,\ x,\ y,\ z)$ or $(t,\ r,\ \theta,\ \phi)$ is defined so that coordinate differences are proper time and space differences at ∞.

A clock tics once every τ. When placed at r,

$$\tau = (1 + \Phi)\, d t_r = \left(1 - \frac{M}{r}\right) d t_r.$$

The same clock when placed at $r = \infty$ gives

$$\tau = d t_\infty$$

Therefore

$$d t_r = d t_\infty\left(1 + \frac{M}{r}\right)$$

Q: “The clock runs more slowly when it is near the sun.” “The clock tics once per τ seconds.” Reconcile these statements.

■ Compute the coordinate time for a light pulse to go from Earth to Venus.

The light passes within y_0 of the sun.

- **Events**

A: light emitted at $(x_A,\ y_0)$
B: Light passes nearest the sun at $(0,\ y_0)$. The sun is at $(0,\ 0)$.
C: Light hits Venus at $(x_C,\ y_0)$

- **Calculation**

$$t_{B\to C} = \int_B^C dt = \int_B^C \left(1 + \frac{2M}{r}\right) dx$$
$$= x_C + 2M \int_0^{x_C} \frac{dx}{\left(y_0^2 + x^2\right)^{1/2}}$$
$$= x_C + 2M \log\left[x + \left(y_0{}^2 + x^2\right)^{1/2}\right] \Big|_0^{x_C}$$
$$= x_C + 2M \log \frac{2x_C}{y_0}$$

Similarly

$$t_{A\to B} = x_A + 2M \log \frac{2x_A}{y_0}$$

Then

$$\boxed{t_{A\to C} = x_A + x_C + 2M \log \frac{4x_A x_C}{y_0^2}}$$

The presence of the sun increases the time by $2M \log \frac{4x_A x_C}{y_0^2}$.

For a pulse from Earth to Venus that grazes the sun, the extra time is 120μs.