31 Jan 2012—Cosmology
Hubble's Law. Robertson-Walker metric. Derivation of Hubble's Law

2 Feb 2012—Friedmann's equation

7 Feb 2012—Cosmological measurements

9 Feb 2012—Flux of a standard candle

16 Feb 2012—Evidence for acceleration of the expansion.
Discovery of the radiation from the Big Bang.

21 Feb 2012—Cosmic background radiation determines the early history of the universe.

23 Feb 2012—Anisotropy of the Cosmic Background Radiation

28 Feb 2012—Sound waves before recombination

1 March 2012—Paradoxes of the Big-Bang. Inflation

- Dicke's paradoxes
  - Flatness: Density parameter requires fine tuning.
  - The horizon: CBR is acausal.
- Another paradox: Too few magnetic monopoles.
- Inflation solves these paradoxes.

Flatness paradox: Density parameter requires fine tuning

Friedmann's equation

$$\left(\frac{d\,a}{H_0\,d\,t}\right)^2 = \left(\Omega_{k0} + \Omega_{m0}\,a^{-1} + \Omega_{r0}\,a^{-2} + \Omega_{v0}\,a^2\right)$$

$(1 - \Omega_{m0} - \Omega_{r0} - \Omega_{v0}) \equiv \Omega_{k0}$ Letter "k" stands for curvature.
Recall r_0 is the radius of curvature. $H_0^2\,r_0^2 = -(1 - \Omega_{m0} - \Omega_{r0} - \Omega_{v0}) = -\Omega_{k0}$

Hubble's constant changes as

$$\left(\frac{H}{H_0}\right)^2 \equiv \frac{1}{H_0^2}\left(\frac{1}{a}\,\frac{da}{dt}\right)^2 = \Omega_{r0}\,a^{-4} + \Omega_{m0}\,a^{-3} + \Omega_{v0} + (1 - \Omega_{r0} - \Omega_{m0} - \Omega_{v0})\,a^{-2}$$

The density parameter $\Omega \equiv \Omega_m + \Omega_r + \Omega_v$ changes as

$$\Omega - 1 \equiv \frac{8\pi}{3}\,\frac{G\rho}{H^2} - 1 = \left(\Omega_{r0}\,a^{-4} + \Omega_{m0}\,a^{-3} + \Omega_{v0}\right)\left(\frac{H_0}{H}\right)^2 - 1.$$

Substitute F's equation to get

$$(\Omega - 1) = (\Omega_{r0} + \Omega_{m0} + \Omega_{v0} - 1)\,a^{-2}\left(\frac{H_0}{H}\right)^2$$

or

$$(\Omega - 1) = (\Omega_0 - 1)\,a^{-2}\left(\frac{H_0}{H}\right)^2$$

In the early universe, radiation dominates. $\Omega_{r0} = 1.2\times 10^{-5}$.

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{r0}\, a^{-4}$$

When radiation dominated the mass density,

$$(\Omega - 1) = (\Omega_0 - 1)\, a^{-2}\left(\frac{H_0}{H}\right)^2$$
$$= a^2(\Omega_0 - 1)\big/\Omega_{r0}$$

Consider the time when helium formed. $a = 3\, K/2\, \text{MeV} \approx 10^{-10}$.

$$(\Omega - 1) = 10^{-20}\, (\Omega_0 - 1)\big/\left(1.2\times 10^{-5}\right)$$
$$= 10^{-15}\, (\Omega_0 - 1)$$

If $(\Omega_0 - 1) = 0.1$, then at the time of helium formation $(\Omega - 1) = 10^{-14}$.

Statement 1: For reasonable, non-zero values of Ω_0, the universe has to have been born with a value of Ω that is unreasonably close to 1.

Statement 2: If the universe was born with some reasonable value for $\Omega > 1$ at helium formation, then Ω_0 is very large, and the universe will have already collapsed.

Recall the radius of curvature r_0

$$(H_0\, r_0)^2 = (\Omega_0 - 1)^{-1}$$

Saying Ω_0 must be 1 is equivalent to saying $r_0^2 \to \infty$ or the universe is flat.
Statement 1': The universe has to have been born extremely flat.

Horizon paradox: the isotropy of the CMR "violates causality."

For definiteness let $\Omega_{m0} = 1$, $\Omega_{v0} = 0$, and $\Omega_{r0} = 0$.

Friedman's equation becomes

$$\frac{1}{H_0^2}\left(\frac{1}{a}\,\frac{da}{dt}\right)^2 = a^{-3}$$

and the metric is

$$d\,s^2 = -d\,t^2 + a(t)^2\left(d\,r^2 + r^2\, d\,\theta^2 + \sin^2\theta\, d\,\phi^2\right).$$

The time from the Big Bang is

$$t(a) = \frac{2}{3}\, H_0^{-1}\, a^{3/2}$$

and the comoving coordinate of a light ray that is emitted at a and gets to us is

$$r(a) = \int a^{-1}\, d\,t = 2\, H_0^{-1}\left(1 - a^{1/2}\right)$$

The age of the universe at recombination (for $H_0^{-1} = 13$ Byr and $a_R = .001$) is

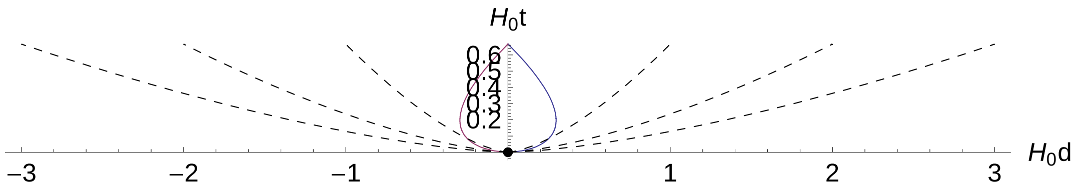
```
t[a_] := 2/3 13*^3 "Myr" a^(3/2)
t[.001]
0.274064 Myr
```

A region 0.3MLy in size has had time to reach the same temperature. However, our horizon is $r = 2\, H_0^{-1}$. At recombination, the size of our horizon was

$$0.001 \times 2\, H_0^{-1} = 26\,\text{MLy}.$$

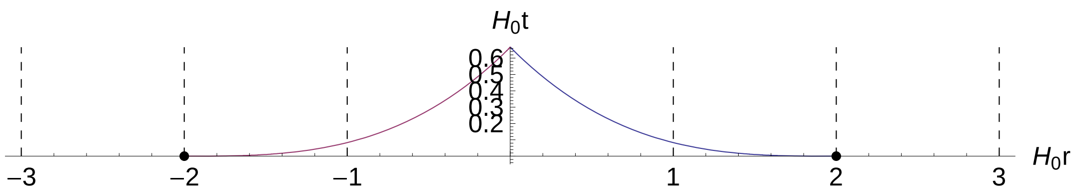
This is 100 times the horizon at recombination.

The paradox is that the temperature of the radiation is uniform over scales that are 100 times larger than the horizon.

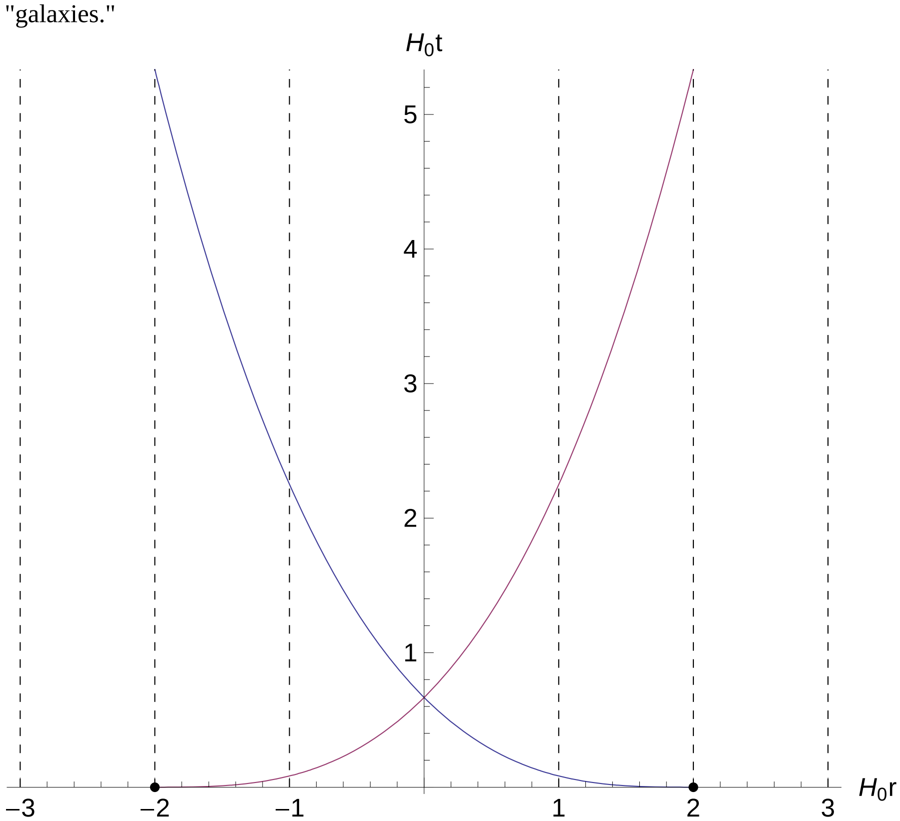


Caption: Spacetime diagram for $\Omega_{m0} = 1$, $\Omega_r = 0$, and $\Omega_v = 0$ Two solid lines show the paths of light pulses (solid) that reach us. Dashed lines show the world lines of "galaxies."

Q: Why do the paths of the light pulses change directions?



Caption: Spacetime diagram of comoving coordinate vs time. Two solid lines show the paths of light pulses (solid) that reach us. Dashed lines show the world lines of "galaxies."



The temperature of the CBR is uniform to a part in 10^5, but the two regions that emitted the CBR that reached us today from opposite directions were never able to signal their temperatures to each other until $H_0\, t = 5.3$, which is in the future.

- **Generate plots**

Magnetic monopoles

Grand unified theories: At high energies, strong, weak, and electromagnetic forces are unified. When the universe cools to 10^{16} GeV, the three forces become distinct.

At this time, radiation dominated and space was flat.

$$H^2 = \frac{1}{a^2}\left(\frac{da}{dt}\right)^2 = \frac{8\pi}{3}\, G\,\rho = \frac{8\pi}{3}\, G\, a_B\, T^4\, c^{-2}$$

where the Stefan-Boltzmann constant

$$a_B = 8\,\pi^5\, k_B^4 \big/ \left(15\, h^3\, c^3\right)$$

The age of the universe

$$t \approx H^{-1} \approx \left(G\, a_B\, T^4\, c^{-2}\right)^{-1/2}$$

Aside: Newton's gravitational constant can be converted into a length or energy

$$L_{\text{Planck}} = \left(G\,\hbar \big/ c^3\right)^{1/2} = 1.62\times 10^{-33}\ \text{cm}$$

$$E_{\text{Planck}} = \left(\hbar\, c^5 \big/ G\right)^{1/2} = 1.22\times 10^{19}\ \text{GeV}$$

or time or density or temperature.

The field has magnetic monopoles. They annihilate if they can find an opposite charge. Unpaired monopoles are left. There is one left-over monopole every horizon length.

At this time,

$$k_B\, T = M\, c^2.$$

$$t \approx \left(G\, a_B\, T^4\, c^{-2}\right)^{-1/2}$$

$$= \left[G\, 8\,\pi^5\, M^4\, c^8 \big/ \left(15\, h^3\, c^5\right)\right]^{-1/2}$$

Therefore the monopole number density is

$$n_{\text{monopole}} = 1 \big/ t_U^3 = \left(\frac{G\, c^3}{h^3}\, M^4\right)^{3/2} \propto M^6,$$ where M is the mass of the monopole.

The photon density is $0.370\, \frac{a_B}{k_B}\, T^3$. The photon number density is proportional to M^3.

For $M = 10^{16}$ GeV, there are 10^9photons for every monopole. If the monopoles do not find another to annihilate, then there are the same number of monopoles as baryons. We would have learned div $B = 4\,\pi \rho_B$ in E&M class.

Searches for monopoles set a limit $n_{\text{monopole}} < 10^{-30}\, n_{\text{nucleon}}$.

- **Check units**

Inflation

Consider the case $\Omega_v = 1$ and $\Omega_r = 0$, and $\Omega_m = 0$. The vacuum energy dominates.

Friedman's equation

$$\frac{1}{H_0^2}\left(\frac{1}{a}\frac{da}{dt}\right)^2 = \Omega_{r0}\, a^{-4} + \Omega_{m0}\, a^{-3} + \Omega_{v0} + (1 - \Omega_{r0} - \Omega_{m0} - \Omega_{v0})\, a^{-2}$$

becomes

$$\frac{1}{H_0^2}\left(\frac{1}{a}\frac{da}{dt}\right)^2 = \Omega_{v0} = 1.$$

Integrate to find

$$a(t) = a_0\, e^{H_0 t}$$

Here a_0 and H_0 refer to some point in time, not the present.

Distances change exponentially. This is very different from the case where matter or radiation dominates, where the distances expand as a power of t.

For matter, $t(a) = \frac{3}{2} H_0^{-1} a^{3/2}$ or $a(t) = \left(\frac{2}{3} H_0\, t\right)^{2/3}$.

For radiation, $t(a) = \frac{1}{2} H_0^{-1} a^2$ or $a(t) = (2\, H_0\, t)^{1/2}$

How inflation solves the horizon problem

1. The temperature of the universe is equal over small regions where $L < t$, where t is the age of the universe and H is Hubble's constant. The size is small enough that the entire region comes to thermal equilibrium.
2. At some time, the universe becomes dominated by the vacuum energy density. The universe expands by a large amount. Two regions that are H_I^{-1}apart before inflation are $H_I^{-1}\, e^{H_I\, \Delta t}$ apart.

To solve the horizon paradox, we need $e^{H_I\, \Delta t} > 100$. The universe need to be dominated by the vacuum for a modest $5\, H_I$.

How inflation solves the flatness problem

1. The radius of curvature of the universe r_0 is some reasonable, non infinite value.
2. At some time, the universe becomes dominated by the vacuum energy density. The universe expands by a large amount. Two points that are r_0 apart before inflation are $r_0\, e^{H_I\, \Delta t}$ apart. Two points of interest to us, for instance two galaxies, are separated by much less that the radius of curvature.

How inflation solves the monopole problem

1. At $T = 10^{16}$ GeV, magnetic monopoles are produced.
2. At some time, the universe becomes dominated by the vacuum energy density. The universe expands by a large amount e^N. The number density of monopoles drops by e^{3N}.

Why are there baryons but no monopoles?

If baryons are created before inflation, the baryon density must be very low. The creation of baryons must occur after inflation.

History

1. At $T = 10^{16}$ GeV, magnetic monopoles are produced.
2. There is some field that has a value that is not the true minimum. The vacuum energy density of the field drives inflation. Hubble's constant is constant during inflation. After $N\, H^{-1}$, the universe is larger by a factor of e^N.
3. The field decays to the true minimum, and inflation stops. The energy of the field becomes the energy of photons and other particles.
4. Baryons are created. Above $T = 1$ GeV, nucleons have not formed. There were quarks and antiquarks in about the same number as photons. ($N_{\text{baryons}} = \frac{1}{3}\left(N_{\text{quark}} - N_{\text{antiquark}}\right)$). The temperature cools, and quarks bind to become baryons. The number of

baryons is much smaller than the number of photons, because there must be a small assymetry between quarks and antiquarks.
5. At $T = 2.2$ MeV, deuterium becomes stable and helium forms.