

1. Suppose an alternative metric where time is distorted but space is flat.

$$ds^2 = -(1 - 2M/r)dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

- (a) (5 pts.) Is there a Pound-Rebka effect? Explain.
 (b) (5 pts.) Does the perihelion of Mercury precess? Explain.
2. Problem 9.2 in the textbook.
 3. Problem 9.6 in the textbook.
 4. Problem 9.8 in the textbook.
 5. The form of the Robertson-Walker metric that we have used is

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{(1 - r^2/r_0^2)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right].$$

An alternative metric is

$$ds^2 = -dt^2 + a(t)^2 [dr^2 + r_0^2 \sin^2(r/r_0)(d\theta^2 + \sin^2\theta d\phi^2)].$$

For both metrics, a galaxy that is moving with the expansion of the universe stays at the same (r, θ, ϕ) .

- (a) (1 pt.) Which term indicates that the distance between galaxies is increasing (or decreasing)?
 (b) (2 pts) Suppose $a(t) = 1$. Which coordinates measure proper time and proper distance?
 (c) (5 pts.) Consider a 4-vector $x^\mu = (dt, dr, d\theta, d\phi)$. Find x_μ for each metric. Why are some components of x^μ and x_μ the same and some different?