

1. Bianchi identity.

- (a) (3 pts.) Explain how carrying a vector A_μ around the faces of a cube can give us a *differential* relationship for the Riemann Curvature Tensor.
- (b) (3 pts.) Explain why the equation $R^{\mu\nu} = -8\pi GT^{\mu\nu}$, which was Einstein's first attempt for his field equation, is incorrect. $R^{\mu\nu}$ is the Ricci tensor and $T^{\mu\nu}$ is the stress-energy tensor.

2. The stress-energy tensor for a perfect gas in flat space at rest is

$$T^{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix},$$

where ρ is the mass-energy density and P is the pressure. (Recall that the speed of light is 1.)

- (a) (3 pts.) Compute numerical values of the stress-energy tensor of a perfect gas of galaxies for which the mass density is 1 hydrogen atom per cubic meter and the velocity is 200 km/s. Assume the Minkowski metric.
- (b) (3 pts.) Redo part (a) with the Robertson-Walker metric. Assume $\Omega = 0$ and $H_0^{-1} = 4000$ Mpc. Explain why the stress-energy tensor is not changed substantially.
3. The Ricci tensor of a homogeneous and isotropic 3-dimensional space is $\tilde{R}_{ij} = -2r_0^{-2}\tilde{g}_{ij}$, where \tilde{g}_{ij} is the 3-dimensional metric, and r_0 is a constant. (Class of 5 April.)
- (a) (3 pts.) Find the curvature scalar.
- (b) (3 pts.) Is $\tilde{R}_{ij} = -2r_0^{-2}\tilde{g}_{ij}$ true in 2 dimensions?
4. (6 pts.) For a homogeneous and isotropic 3-dimensional space (no time), the Riemann curvature tensor

$$R_{\lambda\rho\sigma\nu} = \frac{1}{6}R (g_{\nu\rho}g_{\lambda\sigma} - g_{\sigma\rho}g_{\lambda\nu}),$$

where g is the metric and R is the curvature scalar. Suppose you carry a vector around a unit square. How much does it change?

5. (5 pts.) Answer the questions posed in class on 3 April. Submit your answer on angel. The link is Lessons—Hwk8B. This and the next question are due by 2:40 on 10 April.
6. (5 pts.) Answer the questions posed in class on 5 April. Submit your answer on angel. The link is Lessons—Hwk8B.