Chapter 1

Introduction and Mathematical Concepts

Continued

Clicker Question 1.1: Using the dimensions given for the variables in the table, determine which one of the following expressions is correct.

A) $f = \frac{k}{m}$

B) $f = \sqrt{mk}$

C) $f = \frac{1}{\sqrt{mk}}$

D) $2\pi f = \sqrt{k/m}$

E) $f = \frac{1}{2\pi}\sqrt{m/k}$

variable	dimension
f	$\frac{1}{[\mathrm{T}]}$
m	$[\mathrm{M}]$
k	$\frac{[\mathrm{M}]}{[\mathrm{T}]^2}$

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variable	dimension
f	$\frac{1}{[\mathrm{T}]}$
m	$[\mathrm{M}]$
k	$\frac{[\mathrm{M}]}{[\mathrm{T}]^2}$

Left side Right side

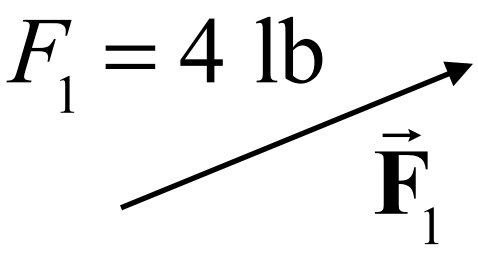
$$\frac{1}{[\mathrm{T}]} = \sqrt{\left(\frac{[\mathrm{M}]}{[\mathrm{T}]^2}\right)\Big/[\mathrm{M}]} = \sqrt{\frac{1}{[\mathrm{T}]^2}} = \frac{1}{[\mathrm{T}]}$$

1.5 Scalars and Vectors

Directions of vectors $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ appear to be the same.

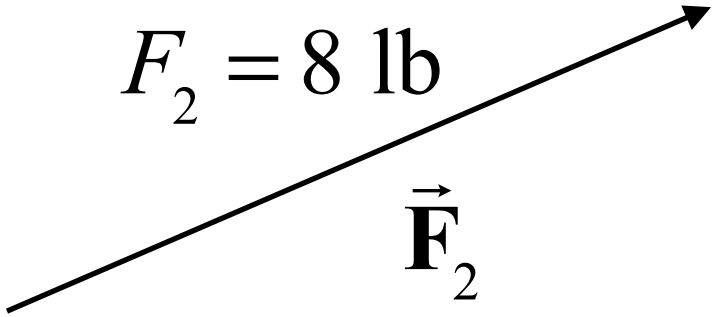
Vector $\vec{\mathbf{F}}_1$, (bold + arrow over it)

has 2 parts:
- magnitude = F_1 *(italics)*
- direction = up & to the right



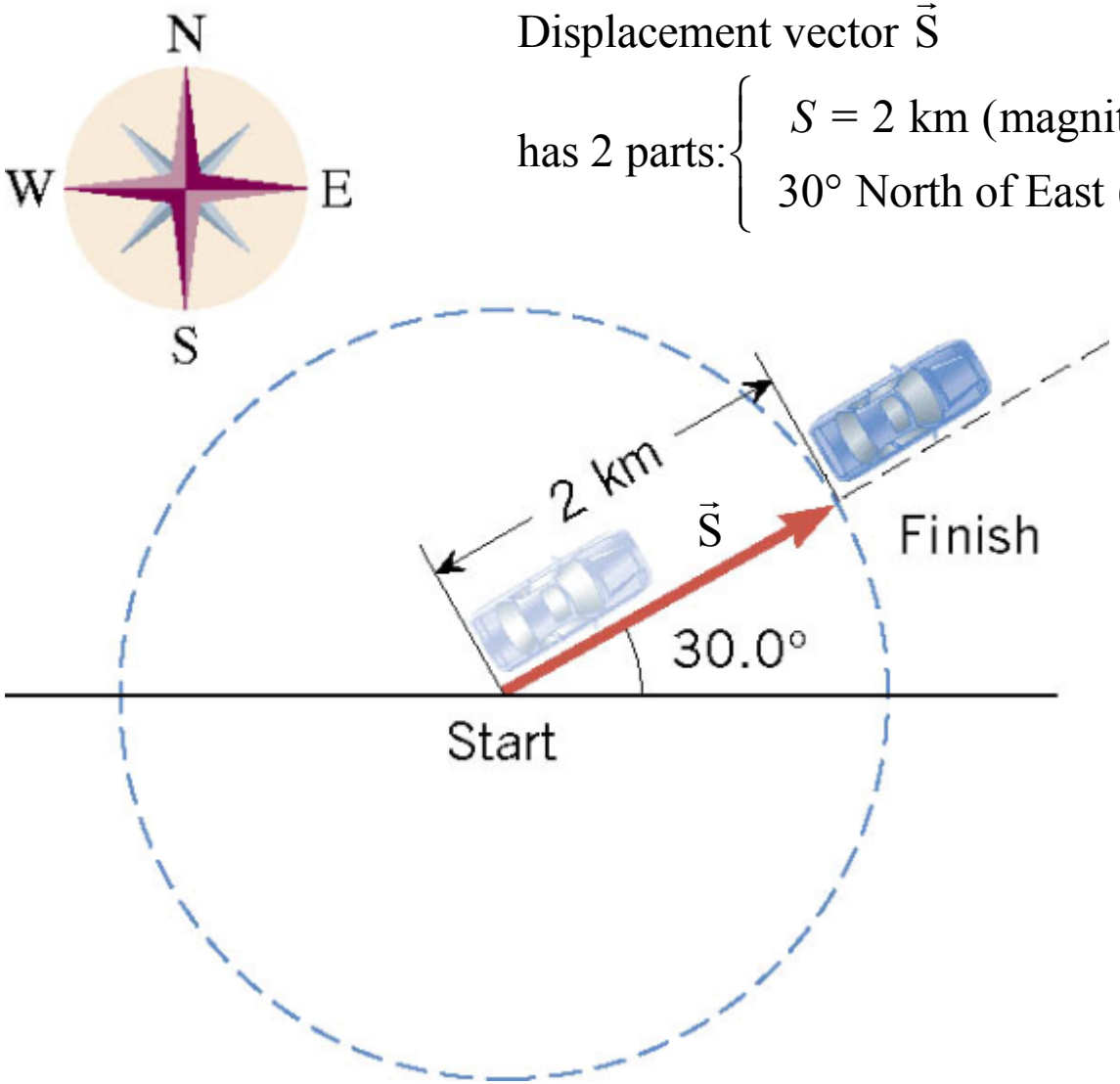
Vector $\vec{\mathbf{F}}_2$, (bold + arrow over it)

has 2 parts:
- magnitude = F_2 *(italics)*
- direction = up & to the right



Directions of vectors $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ appear to be the same.

1.5 Scalars and Vectors

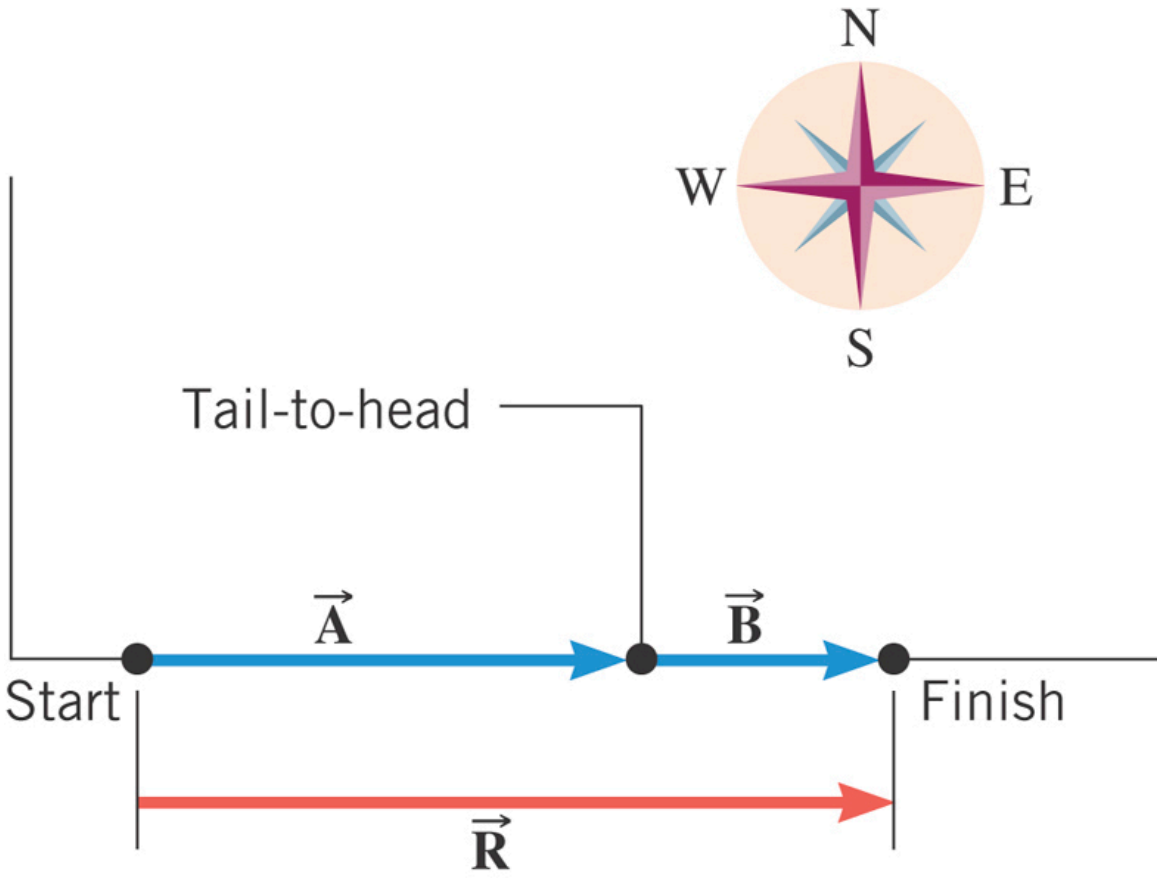


Displacement vector $\vec{S}$

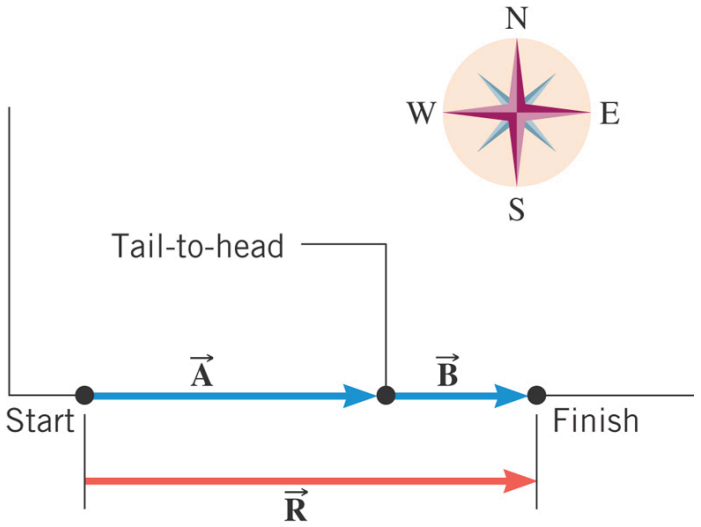
has 2 parts: { S = 2 km (magnitude) , and
30° North of East (direction)

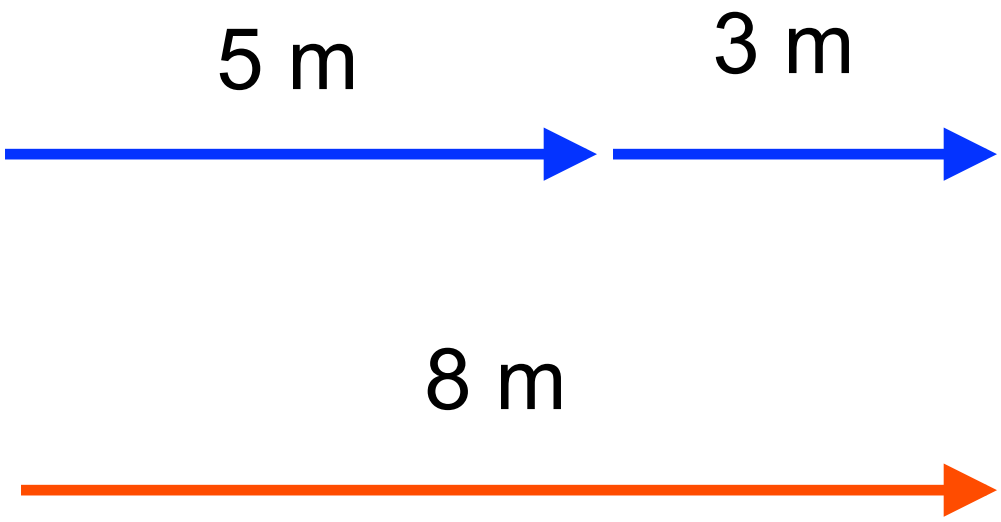
1.6 Vector Addition and Subtraction

Often it is necessary to add one vector to another.



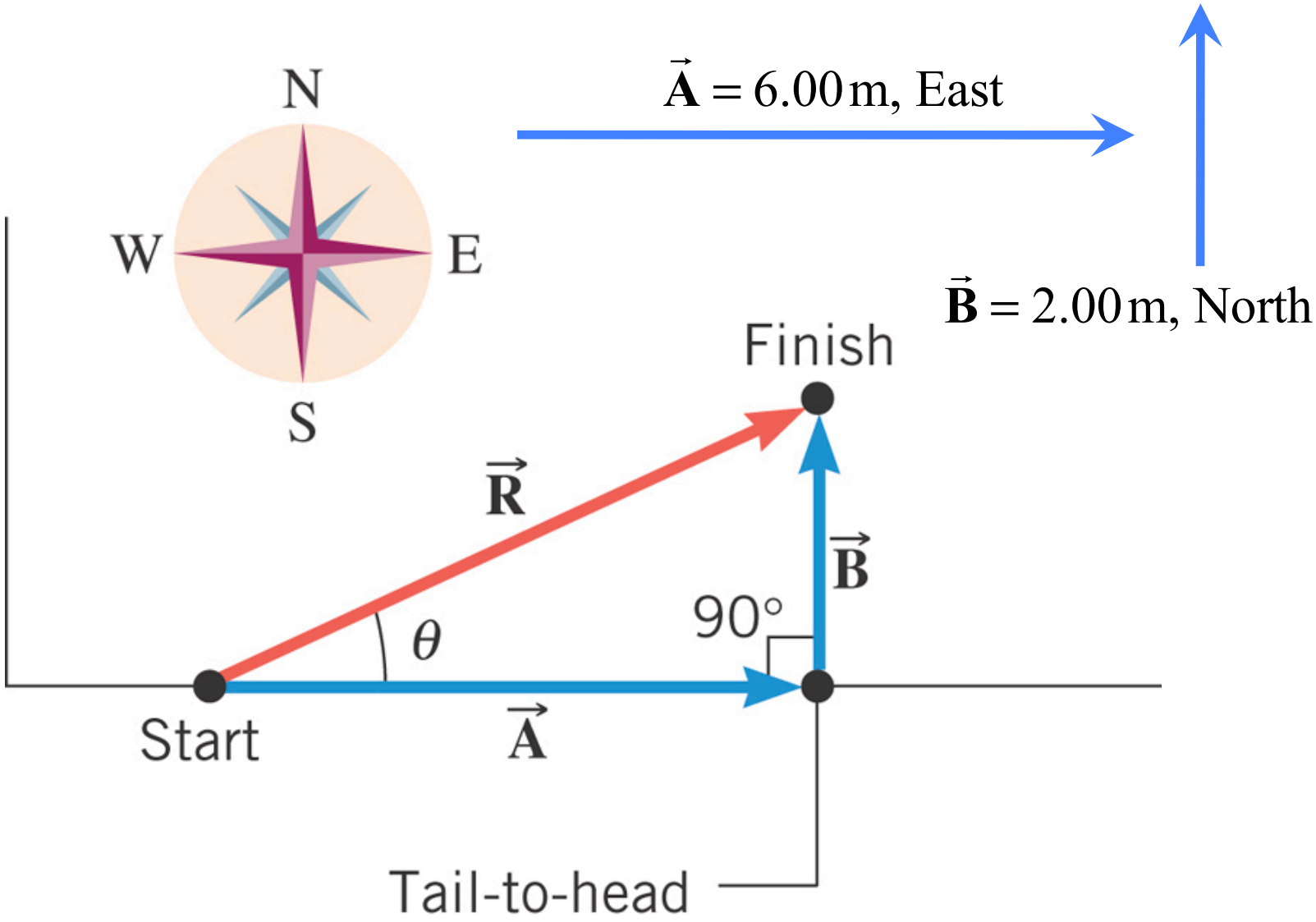
1.6 Vector Addition and Subtraction



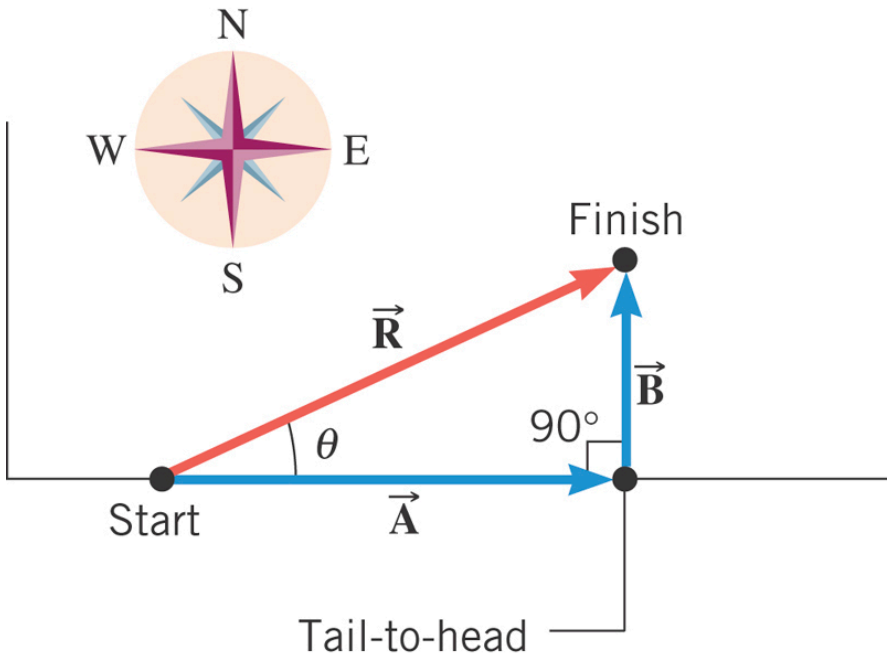


1.6 Vector Addition and Subtraction

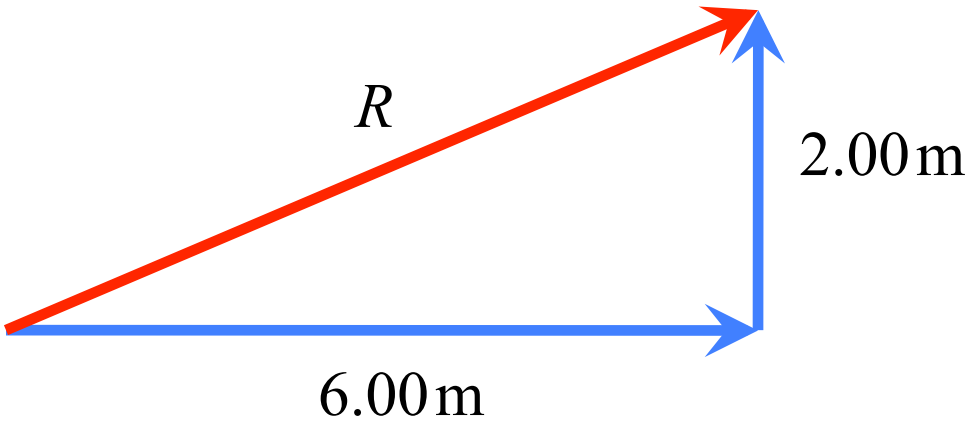
Two displacement vectors



1.6 Vector Addition and Subtraction



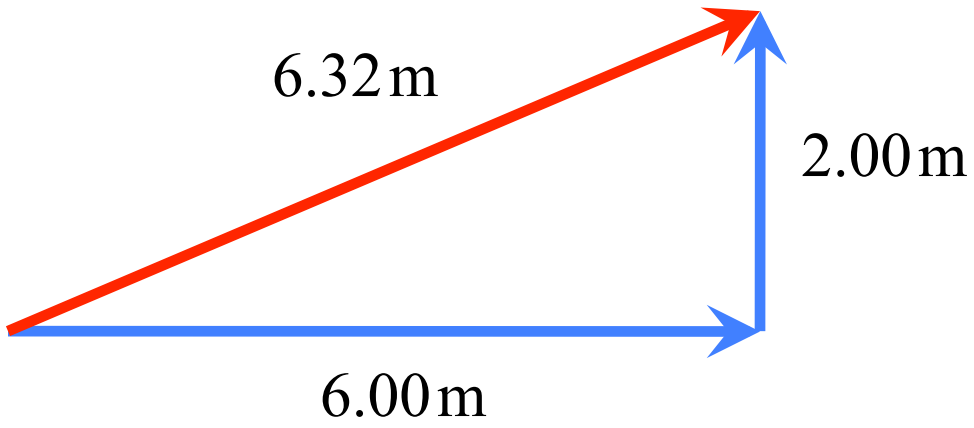
Shown here are the magnitudes of the displacement vectors



1.6 Vector Addition and Subtraction

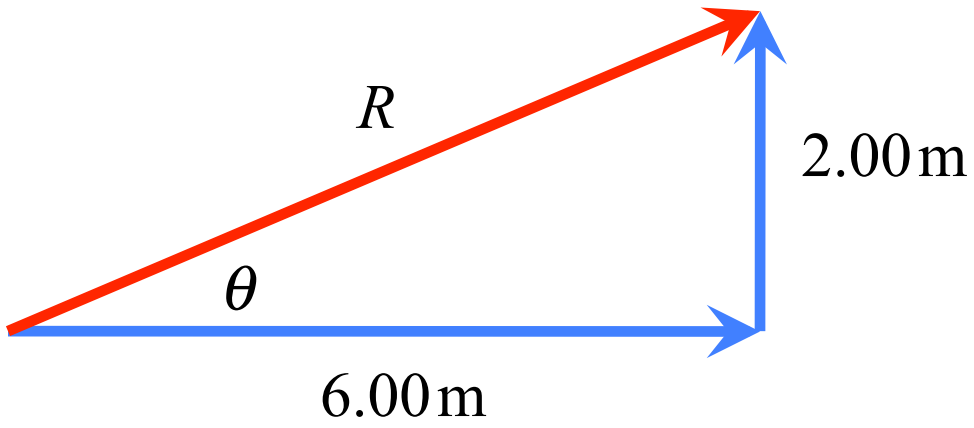
$$R^2 = (2.00\,\text{m})^2 + (6.00\,\text{m})^2$$

$$R = \sqrt{(2.00\,\text{m})^2 + (6.00\,\text{m})^2} = 6.32\text{m}$$



$$\tan\theta = 2.00/6.00$$

$$\theta = \tan^{-1}(2.00/6.00) = 18.4^\circ$$



Clicker Question 1.2 Given three vectors: $\vec{\mathbf{A}}, \vec{\mathbf{B}},$ and $\vec{\mathbf{C}}$, what choice below is always equal to $\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$?

A) $\vec{\mathbf{C}} + \vec{\mathbf{B}} + \vec{\mathbf{A}}$

B) $\vec{\mathbf{B}} + \vec{\mathbf{A}} + \vec{\mathbf{C}}$

C) $\vec{\mathbf{C}} + \vec{\mathbf{A}} + \vec{\mathbf{B}}$

D) All of the above

E) None of the above

Clicker Question 1.2

Given three vectors: $\vec{\mathbf{A}}, \vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$, what choice below is always equal to $\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$?

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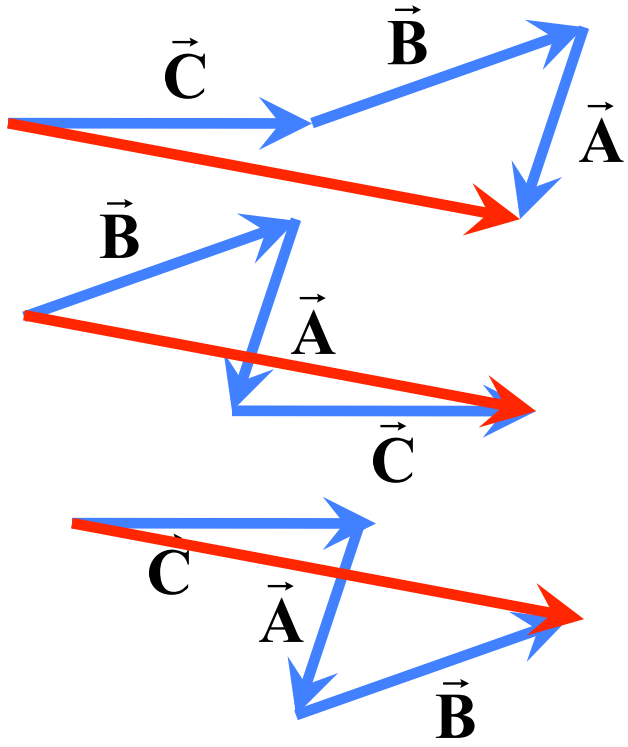
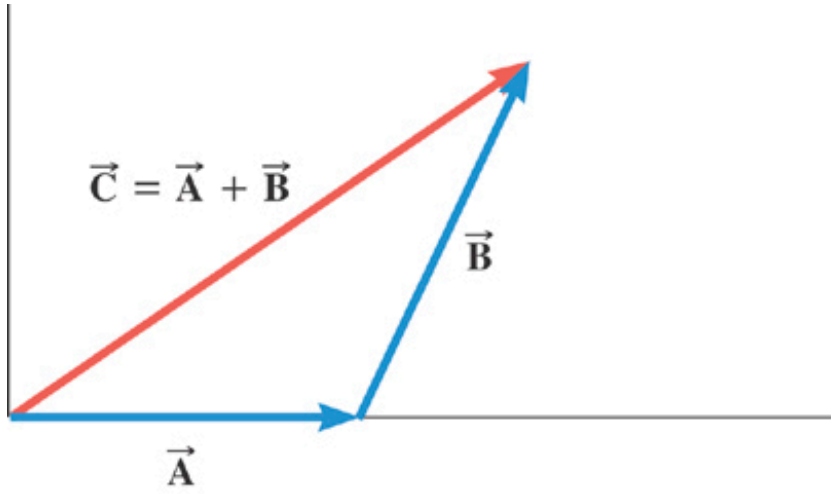
C) $\vec{\mathbf{C}} + \vec{\mathbf{A}} + \vec{\mathbf{B}}$

D) All of the above

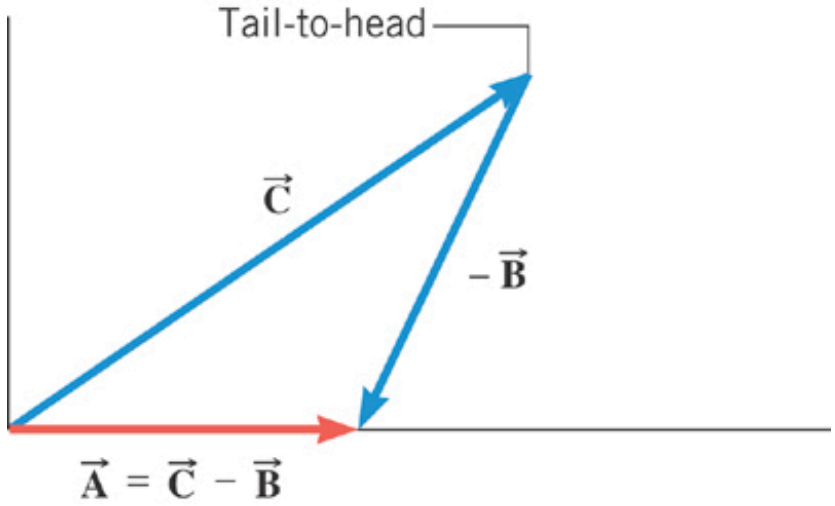
E) None of the above



1.6 Vector Addition and Subtraction



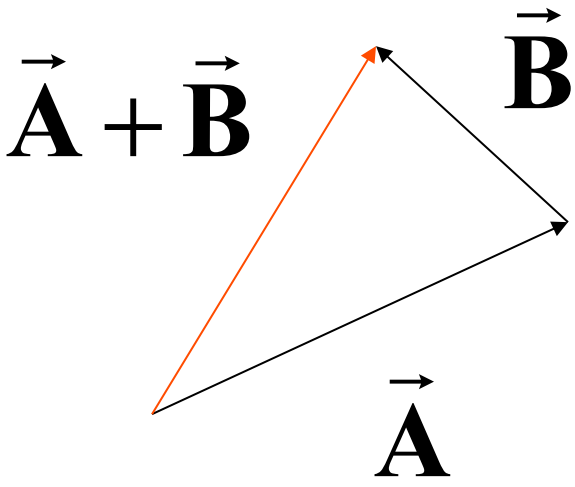
(a)

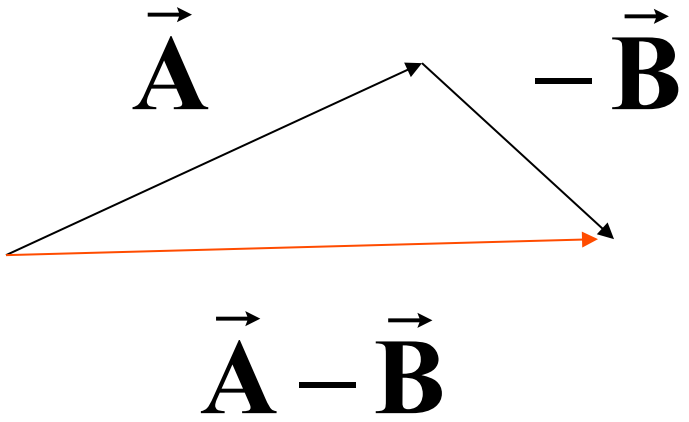


(b)

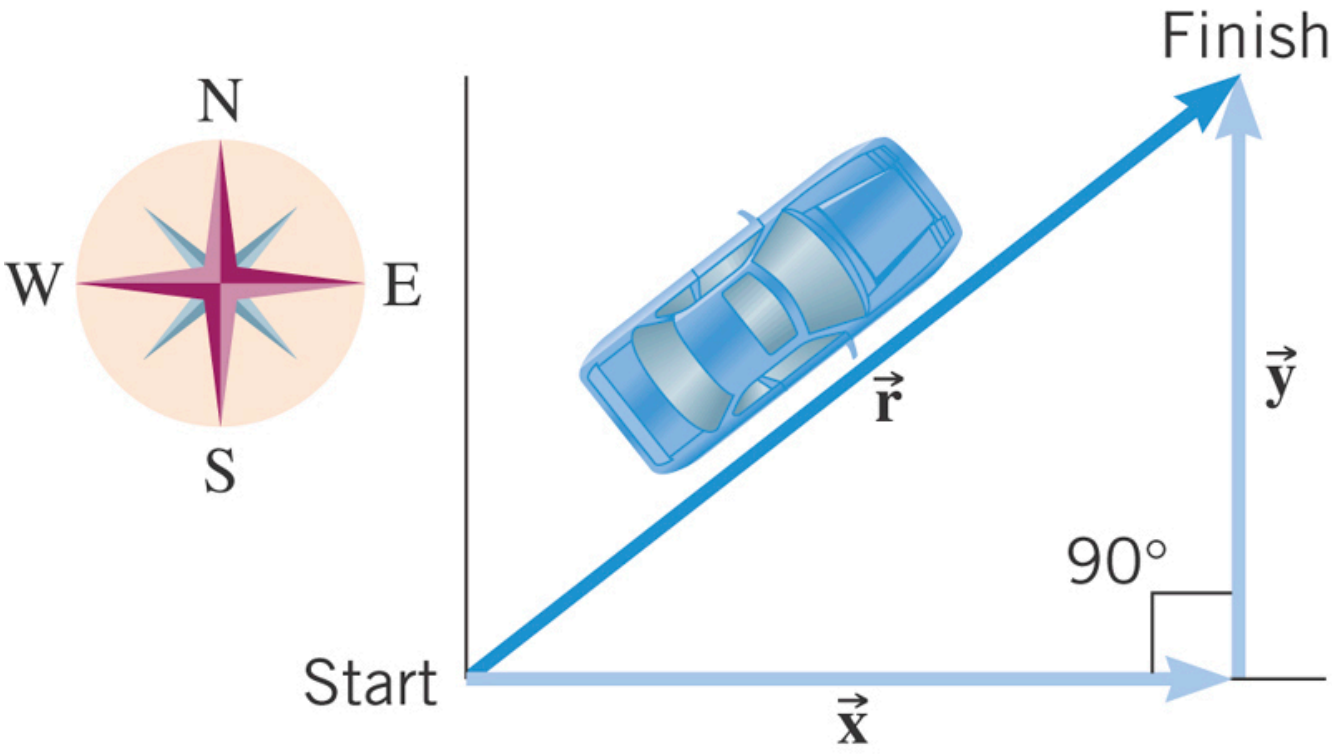
When a vector is multiplied by -1, the magnitude of the vector remains the same, but the direction of the vector is reversed.

1.6 Vector Addition and Subtraction



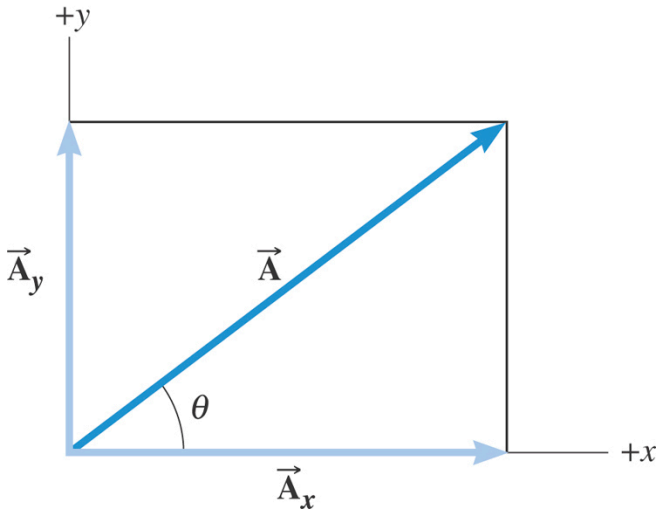


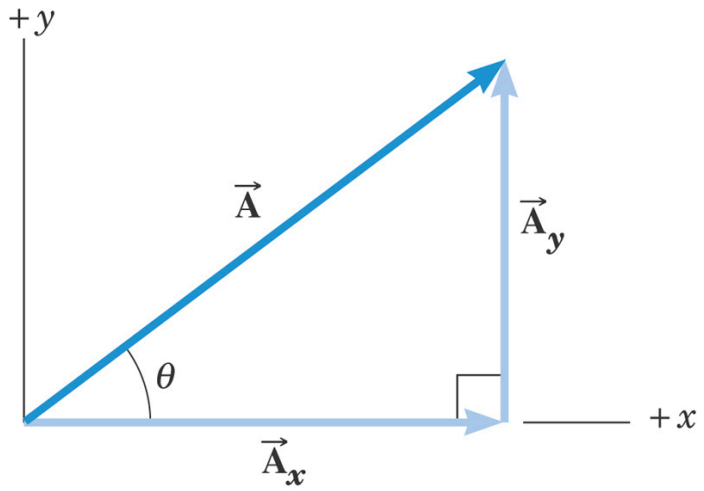
1.7 The Components of a Vector



$\vec{x}$ and $\vec{y}$ are called the x vector component and the y vector component of $\vec{r}$.

1.7 The Components of a Vector



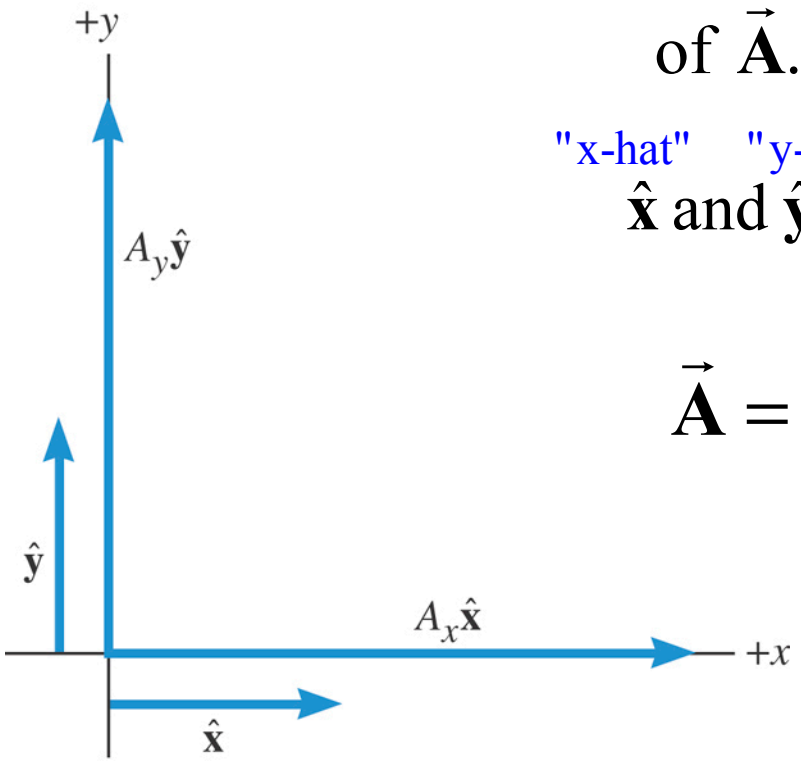


The vector components of $\vec{\mathbf{A}}$ are two perpendicular vectors $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$ that are parallel to the x and y axes, and add together vectorially so that $\vec{\mathbf{A}} = \vec{\mathbf{A}}_\mathbf{x} + \vec{\mathbf{A}}_\mathbf{y}$.

1.7 The Components of a Vector

It is often easier to work with the **scalar components** rather than the vector components.

A_x and A_y are the scalar components of $\vec{\mathbf{A}}$.



"x-hat" "y-hat"

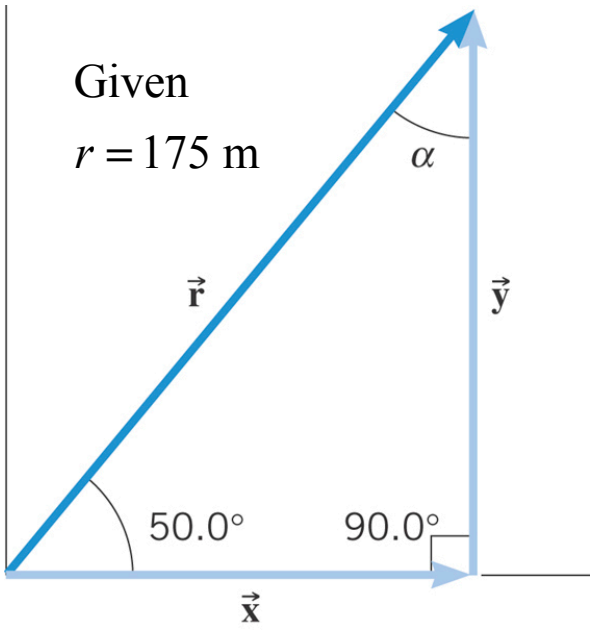
$\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are unit vectors with magnitude 1.

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}}$$

1.7 The Components of a Vector

Example

A displacement vector has a magnitude of 175 m and points at an angle of 50.0 degrees relative to the *x* axis. Find the *x* and *y* components of this vector.



vector $\vec{\mathbf{x}}$ has magnitude *x*

$$\sin\theta = y/r$$

y-component of the vector $\vec{\mathbf{r}}$

$$y = r\sin\theta = (175\text{ m})(\sin 50.0^\circ) = 134\text{ m}$$

$$\cos\theta = x/r$$

x-component of the vector $\vec{\mathbf{r}}$

$$x = r\cos\theta = (175\text{ m})(\cos 50.0^\circ) = 112\text{ m}$$

$$\vec{\mathbf{r}} = (112\text{ m})\hat{\mathbf{x}} + (134\text{ m})\hat{\mathbf{y}}$$

Clicker Question 1.3

$\vec{\mathbf{A}} = A_x \hat{\mathbf{x}} + A_y \hat{\mathbf{y}}$, where $A_x = 3\text{m}$, and $A_y = 4\text{m}$.

What is the magnitude of the vector $\vec{\mathbf{A}}$?

A) $7\,\text{m}$

B) $5\,\text{m}$

C) $\sqrt{7}\,\text{m}$

D) $6\,\text{m}$

E) $25\,\text{m}$

Clicker Question 1.3

$\vec{\mathbf{A}} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}}$, where $A_x = 3\text{m}$, and $A_y = 4\text{m}$.

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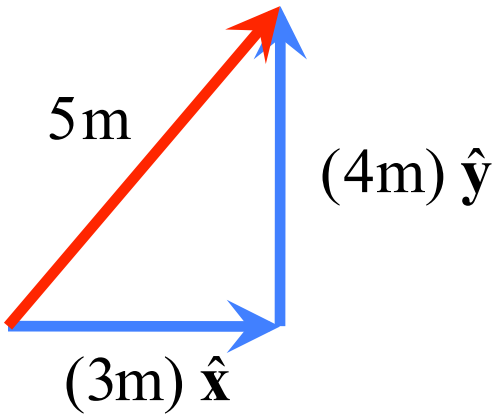
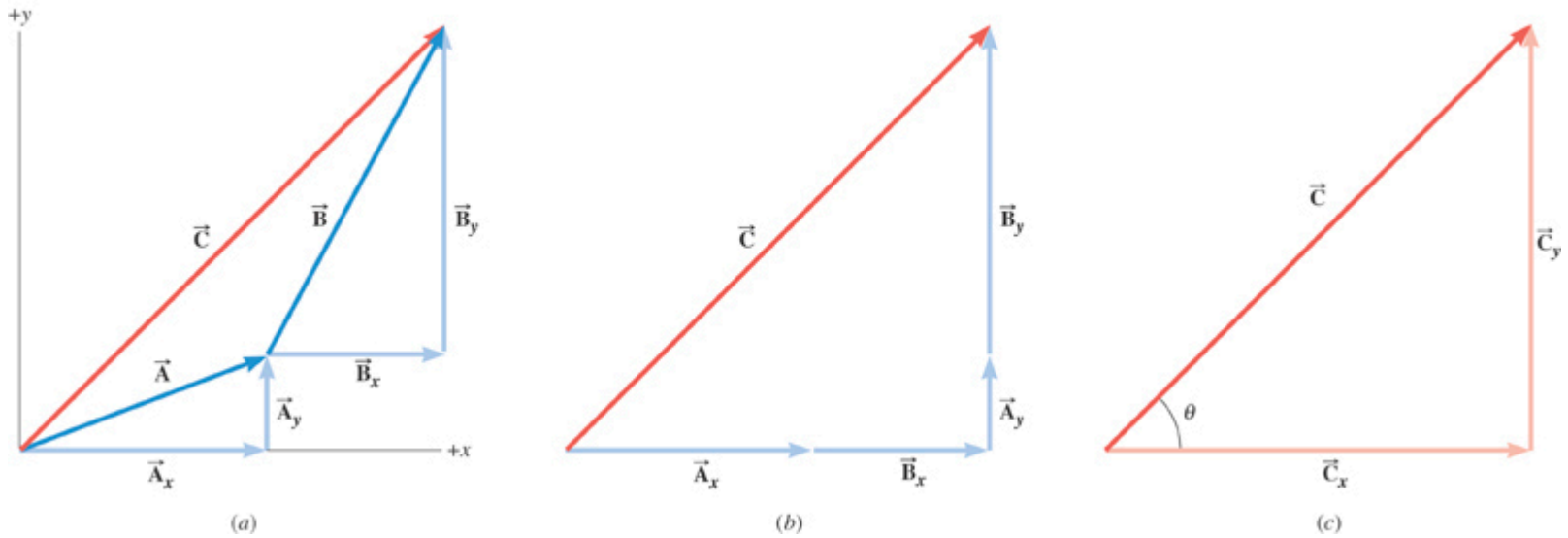


$$\vec{\mathbf{A}} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}}$$

magnitude of $\vec{\mathbf{A}}$, $A = \sqrt{A_x^2 + A_y^2} = \sqrt{(3\text{m})^2 + (4\text{m})^2}$

$$= \left(\sqrt{9+16}\right)\text{m} = \sqrt{25}\ \text{m} = 5\text{m}$$

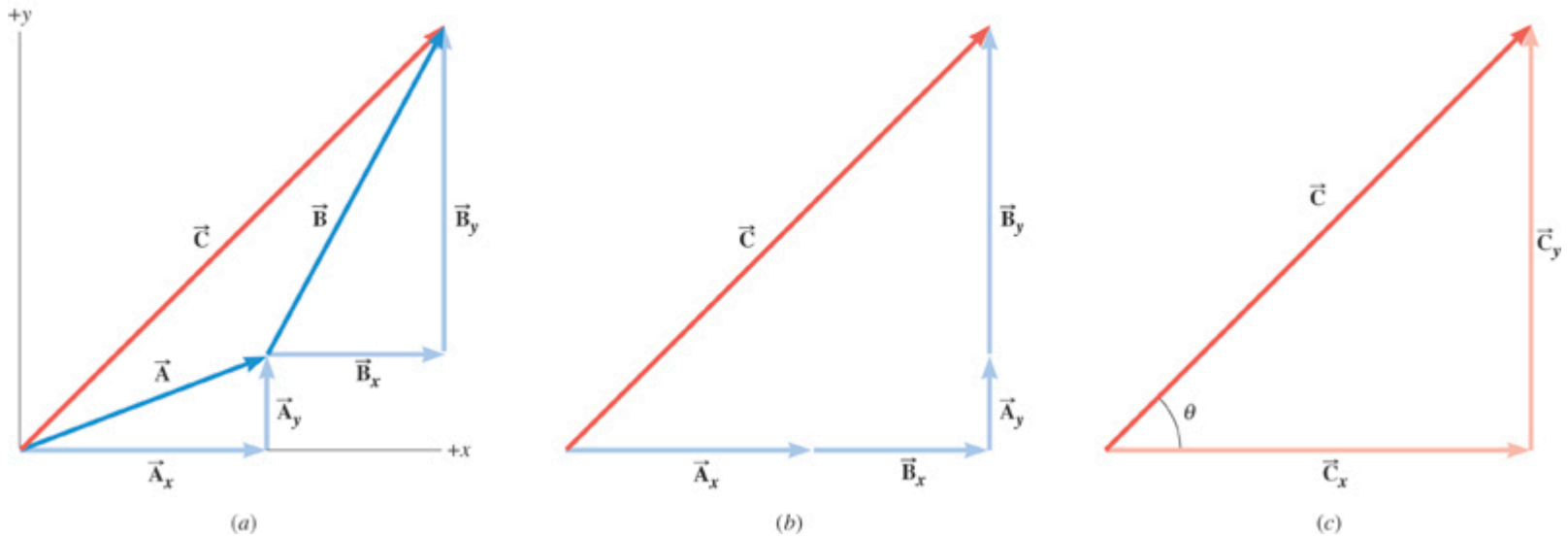
1.8 Addition of Vectors by Means of Components



$$\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} \qquad \vec{\mathbf{B}} = B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}}$$

1.8 Addition of Vectors by Means of Components



$$\vec{\mathbf{C}} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}}$$

$$= \left(A_x + B_x\right)\hat{\mathbf{x}} + \left(A_y + B_y\right)\hat{\mathbf{y}}$$

$$C_x = A_x + B_x \qquad\qquad C_y = A_y + B_y$$