Chapter 7

Impulse and Momentum

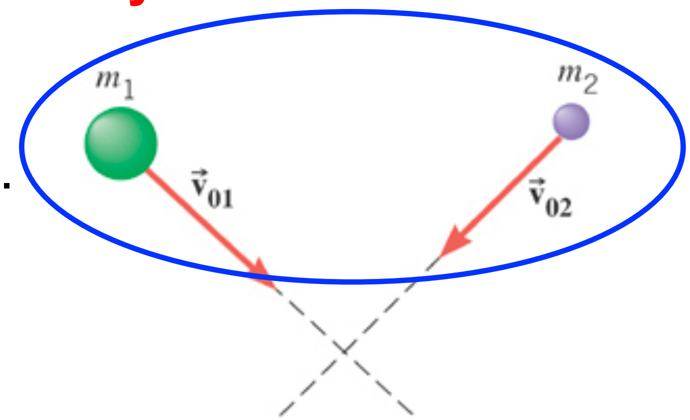
continued

7.2 *The Principle of Conservation of Linear Momentum*

System of two masses

External forces **–** Forces exerted on the objects by agents external to the system. Net force changes the velocity (and momentum) of a mass.

Newton's 2nd Law $\vec{\mathbf{W}}_1$ & $\vec{\mathbf{W}}_2$



(*a*) Before collision

Internal forces **–** Forces that objects **within the system** exert on each other. These forces have equal magnitudes and opposite directions.

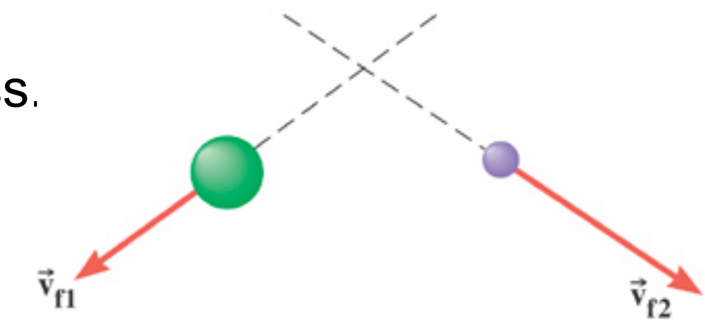
Newton's 3rd Law

Also $\vec{\mathbf{W}}_1$ & $\vec{\mathbf{W}}_2$

(*b*) During collision

External forces **–** Forces exerted on the objects by agents external to the system. Net force changes the velocity (and momentum) of a mass.

Newton's 2nd Law $\vec{\mathbf{W}}_1$ & $\vec{\mathbf{W}}_2$



(*c*) After collision

7.2 *The Principle of Conservation of Linear Momentum*

$\ell.c.$ p — initial momentum of mass 1 and mass 2

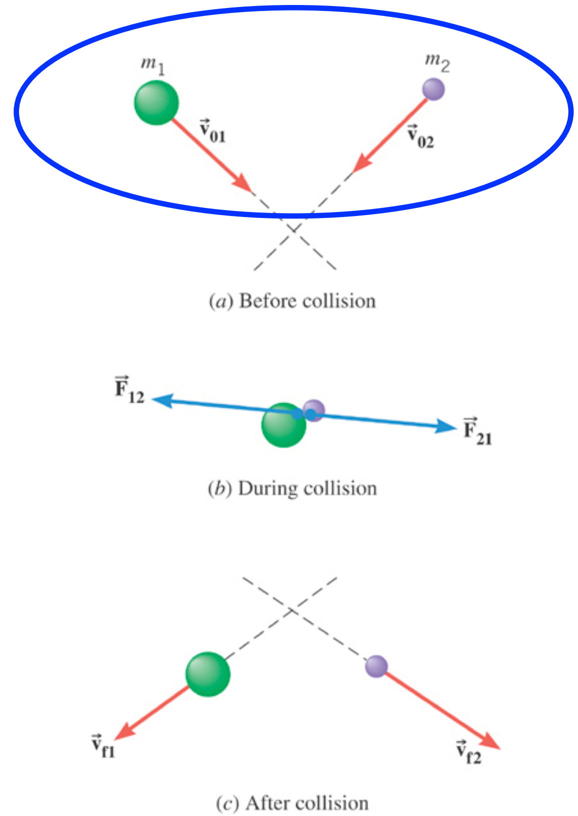
$$\vec{\mathbf{p}}_{\mathbf{o}1} = m_1 \vec{\mathbf{v}}_{\mathrm{o}1} \qquad \vec{\mathbf{p}}_{\mathbf{o}2} = m_2 \vec{\mathbf{v}}_{\mathrm{o}2}$$

Capital P

$$\vec{\mathbf{P}}_{\mathbf{o}} = m_1 \vec{v}_{\mathrm{o}1} + m_2 \vec{v}_{\mathrm{o}2} + \ldots$$ initial momentum sum

$$\vec{\mathbf{P}}_{\mathbf{f}} = m_1 \vec{v}_{\mathrm{f}1} + m_2 \vec{v}_{\mathrm{f}2} + \ldots$$ final momentum sum

System of two masses



(*a*) Before collision

(*b*) During collision

(*c*) After collision

Net External Force $\neq 0$ — Changes the momentum sum

$$\left(\sum \vec{\overline{\mathrm{F}}}_{\mathrm{external}} \right) \Delta t = \vec{\mathbf{P}}_{\mathbf{f}} - \vec{\mathbf{P}}_{\mathbf{o}}$$

Most Important case

Net External Force $= 0$ $\quad 0 = \vec{\mathbf{P}}_{\mathbf{f}} - \vec{\mathbf{P}}_{\mathbf{o}}$ — No change in momentum sum

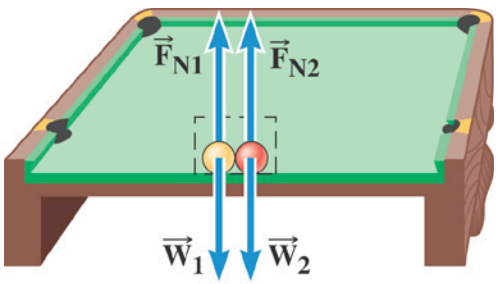
$$\boxed{\vec{\mathbf{P}}_{\mathbf{f}} = \vec{\mathbf{P}}_{\mathbf{o}}}$$ Conservation of Linear Momentum Sum

7.2 *The Principle of Conservation of Linear Momentum*

PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

The total linear momentum of an isolated system is constant (conserved). An isolated system is one for which the sum of the average external forces acting on the system is zero.

In the picture, the net external force on the system is zero.



Sum of Weight force and Normal Force on each mass is zero, and no net external force.

However,

If the cue of a pool player hits one of the balls, the force of the cue on the ball is an external force and in the collision the momentum sum changes from zero to non-zero.

7.2 The Principle of Conservation of Linear Momentum

Why is adding momentum vectors of different masses together useful at all?

$$\vec{\mathbf{P}}_{\mathbf{o}} = m_1\vec{\mathbf{v}}_{o1} + m_2\vec{\mathbf{v}}_{o2} + \ldots$$

initial momentum sum

$$\vec{\mathbf{P}}_{\mathbf{f}} = m_1\vec{\mathbf{v}}_{f1} + m_2\vec{\mathbf{v}}_{f2} + \ldots$$

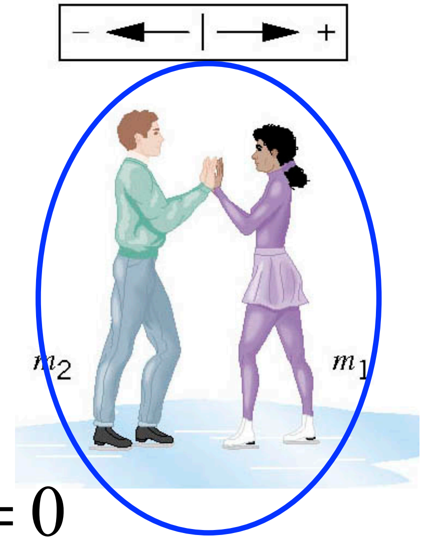
final momentum sum

Net External Force on system of two skaters is zero.

$$\vec{\mathbf{P}}_{f} = \vec{\mathbf{P}}_{\mathbf{o}}$$

$$\vec{\mathbf{P}}_{f} = 0$$

System of two masses



$$\vec{\mathbf{P}}_{\mathbf{o}} = 0$$

Before

Still not so clear why adding momentum vectors is useful. But,

$$\vec{\mathbf{P}}_{\mathbf{f}} = m_1\vec{\mathbf{v}}_{f1} + m_2\vec{\mathbf{v}}_{f2} = 0$$

Momentum vector of mass 2 $\quad m_2\vec{\mathbf{v}}_{f2} = -m_1\vec{\mathbf{v}}_{f1} \quad$ is opposite to Momentum vector of mass 1

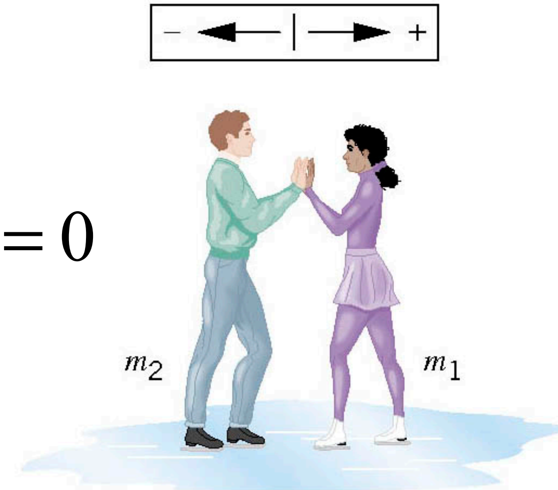
7.2 *The Principle of Conservation of Linear Momentum*

Example 6 Ice Skaters

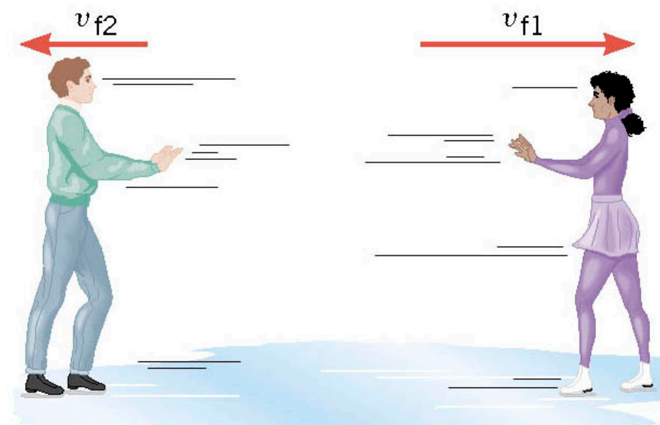
Starting from rest, two skaters push off against each other on ice where friction is negligible.

One is a 54-kg woman and one is a 88-kg man. The woman moves away with a speed of +2.5 m/s. Find the recoil velocity of the man.

$$\vec{\mathbf{P}}_{\mathbf{o}} = 0$$



Before



After

$$\vec{\mathbf{P}}_{\mathbf{f}} = \vec{\mathbf{P}}_{\mathbf{o}}$$

$$m_1\vec{\mathbf{v}}_{f1} + m_2\vec{\mathbf{v}}_{f2} = 0 \quad \Rightarrow \quad m_2\vec{\mathbf{v}}_{f2} = -m_1\vec{\mathbf{v}}_{f1}$$

$$\vec{\mathbf{v}}_{f2} = -\frac{m_1}{m_2}\vec{\mathbf{v}}_{f1} = -\frac{54}{88}(+2.5\ \text{m/s}) = -1.5\ \text{m/s}$$

Clicker Question 7.1

A 9-kg object is at rest. Suddenly, it explodes and breaks into two pieces. The mass of one piece is 6 kg and the other is a 3-kg piece. Which one of the following statements concerning these two pieces is correct?

a) The speed of the 6-kg piece will be one eighth that of the 3-kg piece.

b) The speed of the 3-kg piece will be one fourth that of the 6-kg piece.

c) The speed of the 6-kg piece will be one forth that of the 3-kg piece.

d) The speed of the 3-kg piece will be one half that of the 6-kg piece.

e) The speed of the 6-kg piece will be one half that of the 3-kg piece.

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d) The speed of the 3-kg piece will be one half that of the 6-kg piece.

e) The speed of the 6-kg piece will be one half that of the 3-kg piece.

$$m_3 v_3 + m_6 v_6 = 0$$

$$v_3 = -\frac{m_6}{m_3} v_6 \quad \text{speeds} \quad v_3 = \frac{m_6}{m_3} v_6$$

$$v_3 = 2v_6 \text{ or } v_6 = \tfrac{1}{2} v_3$$

Applying the Principle of Conservation of Linear Momentum

1. Decide which objects are included in the system.

2. Relative to the system, identify the internal and external forces.

3. Verify that the system is isolated.

4. Set the final momentum of the system equal to its initial momentum. Remember that momentum is a vector.

7.3 *Collisions in One Dimension*

In a collision of a puck with a wall, the speed of the puck hitting the wall is the same as the speed of the puck coming off the wall.

Clicker Question 7.2

Which statement below is true about this collision?

a) momentum of the puck is conserved

b) the system consists of the puck before and after the collision

c) kinetic energy is not conserved in the collision

d) total energy is not conserved in the collision

e) momentum is conserved in a system containing the earth and puck.

7.3 Collisions in One Dimension

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Clicker Question 7.1

Which statement below is true about this collision?

a) momentum of the puck is conserved

b) the system consists of the puck before and after the collision

c) kinetic energy is not conserved in the collision

d) total energy is not conserved in the collision

e) momentum is conserved in a system containing the earth and puck.

a) momentum before is $-mv$, while after it is $+mv$ (not conserved)

b) a system must be defined before the collision

c) kinetic energy is the same before and after collision (conserved)

d) total energy is the same before and after the collision (conserved)

e) no external forces are acting in a system containing earth and puck

7.3 Collisions in One Dimension

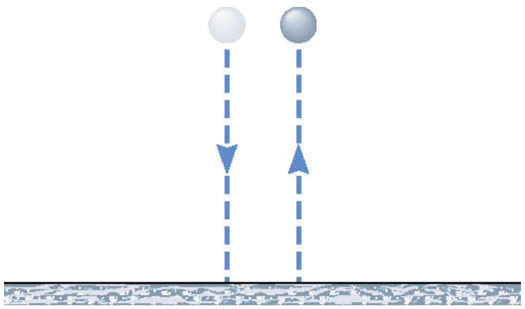
The total linear momentum is conserved when two objects collide, provided they constitute an isolated system.

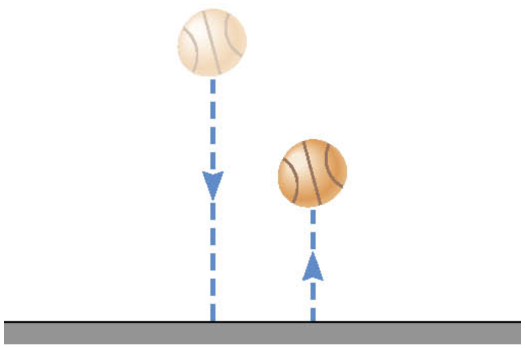
Isolated system is the ball **and** the earth.

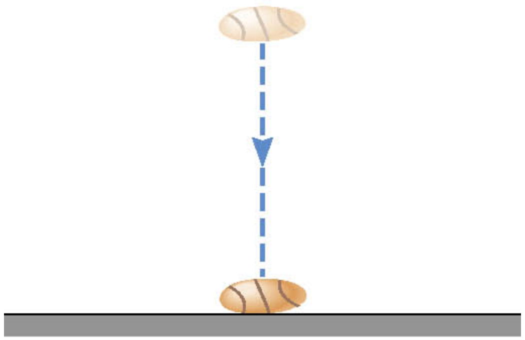
Energy considerations in collisions

Elastic collision -- One in which the total kinetic energy of the system after the collision is equal to the total kinetic energy before the collision.

Inelastic collision -- One in which the total kinetic energy of the system after the collision is *not* equal to the total kinetic energy before the collision; if the objects stick together after colliding, the collision is said to be completely inelastic.


(*a*) Elastic collision

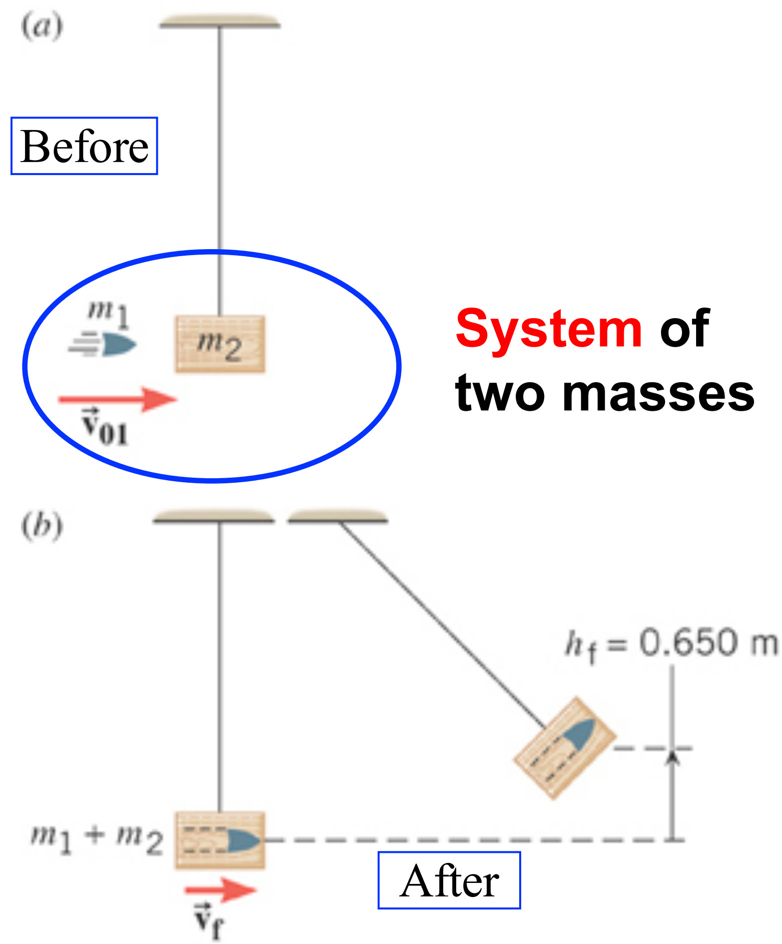

(*b*) Inelastic collision


(*c*) Completely inelastic collision

Example 8 A Ballistic Pendulim

The mass of the block of wood is 2.50-kg and the mass of the bullet is 0.0100-kg. The block swings to a maximum height of 0.650 m above the initial position.

Find the initial speed of the Bullet, v_{o1}.

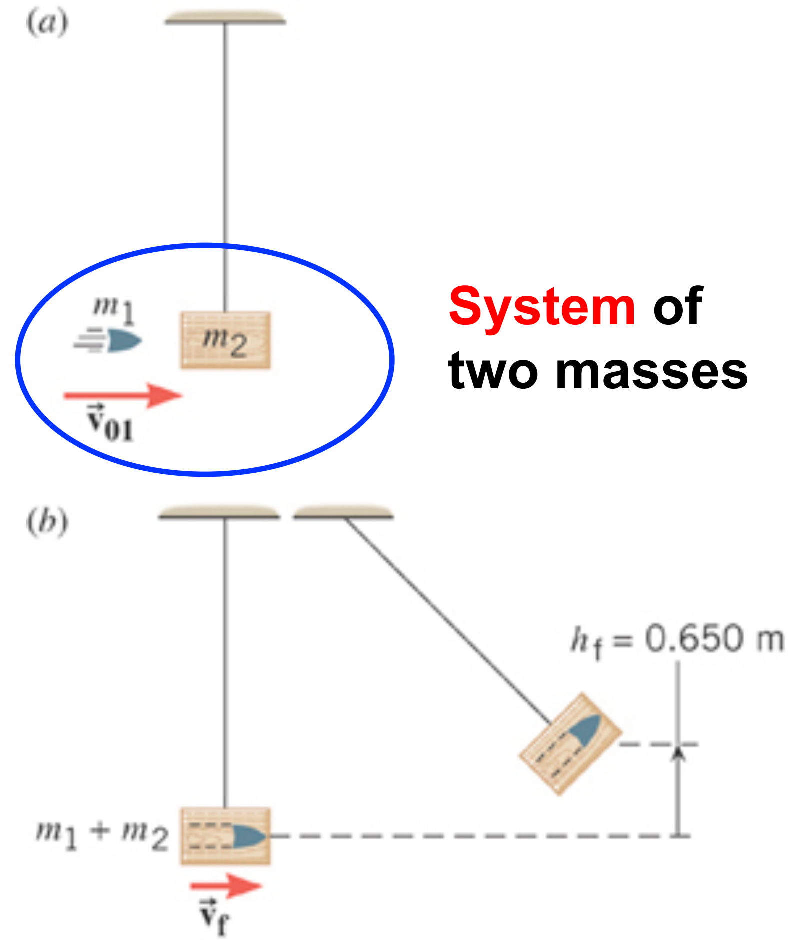


7.3 Collisions in One Dimension

In the collision of bullet and block, tension force of the string and weight of the mass are external forces, but the net external force is zero.

Therefore, momentum is conserved.

But, the bullet is stopped in the block by friction, therefore, energy is not conserved in collision.



But, after the collision, only gravity (a conservative force) does work.

Therefore, total energy is conserved.

7.3 Collisions in One Dimension

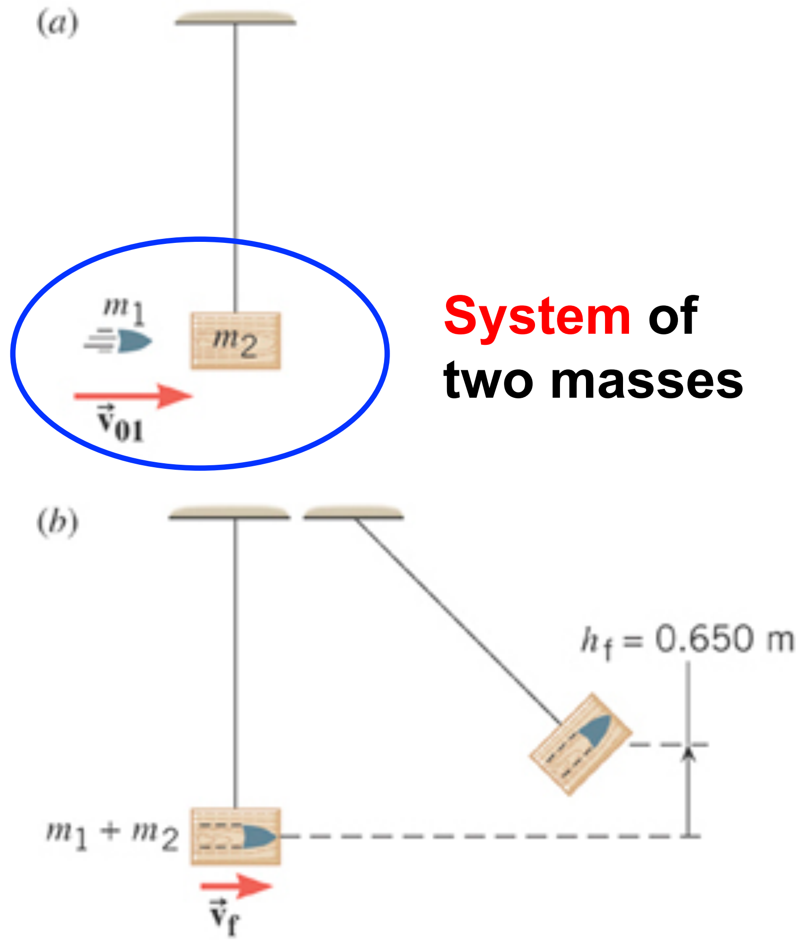
Apply conservation of momentum to the collision:

$$m_1\vec{v}_{f1} + m_2\vec{v}_{f2} = m_1\vec{v}_{o1} + m_2\vec{v}_{o2}$$

$$\left(m_1 + m_2\right)\vec{v}_f = m_1\vec{v}_{o1}$$

$$\vec{v}_{o1} = \frac{\left(m_1 + m_2\right)}{m_1}\vec{v}_f \quad \text{need } \vec{v}_f$$

To determine the speed of the bullet + block after collision use conservation of energy in the swing. collision v_f = swing v_0



System of two masses

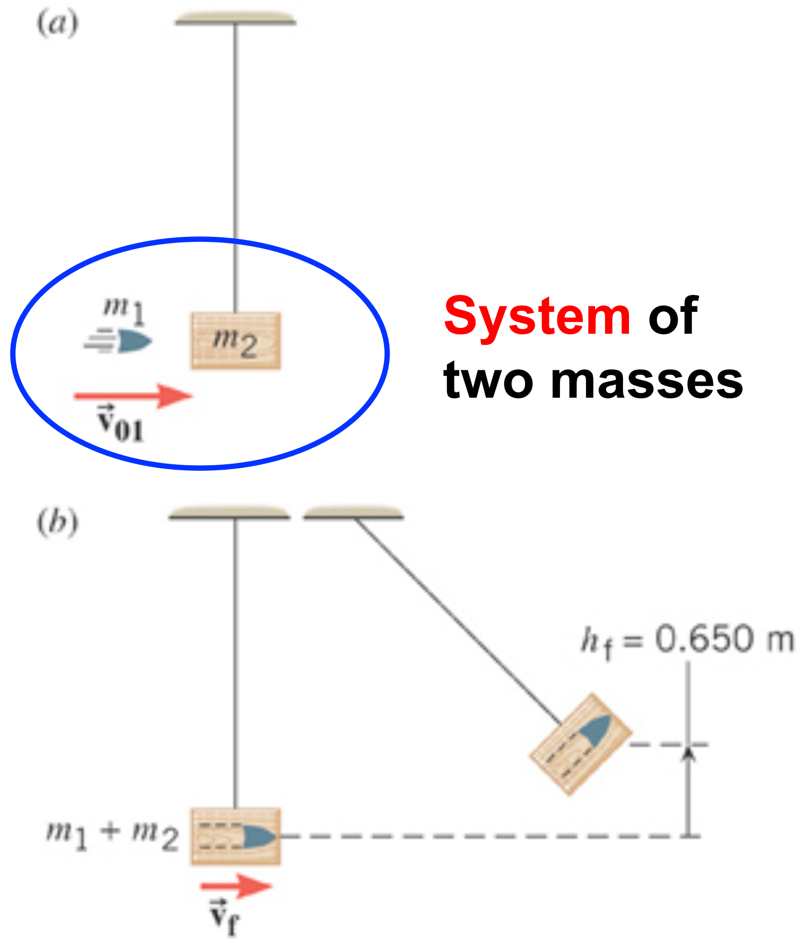
7.3 Collisions in One Dimension

Apply conservation of momentum to the collision:

$$m_1\vec{v}_{f1} + m_2\vec{v}_{f2} = m_1\vec{v}_{o1} + m_2\vec{v}_{o2}$$

$$(m_1 + m_2)\vec{v}_f = m_1\vec{v}_{o1}$$

$$\vec{v}_{o1} = \frac{(m_1 + m_2)}{m_1}\vec{v}_f \quad \text{need } \vec{v}_f$$



To determine the speed of the bullet + block after collision use conservation of energy in the swing. collision v_f = swing v_0

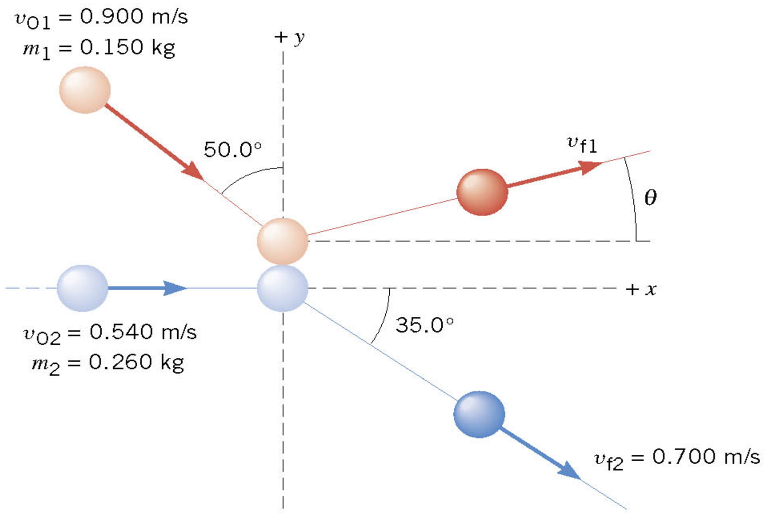
$$\mathrm{KE}_0 + \mathrm{PE}_0 = \mathrm{KE}_f + \mathrm{PE}_f$$

$$\tfrac{1}{2}mv_0^2 + 0 = mgh + 0$$

$$v_0 = \sqrt{2gh}, \; \vec{v}_0 = +\sqrt{2gh}$$

$$\vec{v}_{o1} = \frac{(m_1 + m_2)}{m_1}\sqrt{2gh} = +896\,\mathrm{m/s}$$

7.4 Collisions in Two Dimensions



Momentum conserved in each of the two dimensions, x and y.

Use these masses below.

$$m_1 = 0.150 \text{ kg}, \quad m_2 = 0.260 \text{ kg}$$

x-components: $m_1\vec{v}_{f1} + m_2\vec{v}_{f2} = m_1\vec{v}_{o1} + m_2\vec{v}_{o2}$;

$\vec{v}_{o1} = +0.900\sin 50^\circ$ m/s , $\vec{v}_{o2} = +0.540$ m/s, $\vec{v}_{f2} = +0.700\cos 35^\circ$ m/s

y-components: $m_1\vec{v}_{f1} + m_2\vec{v}_{f2} = m_1\vec{v}_{o1} + m_2\vec{v}_{o2}$;

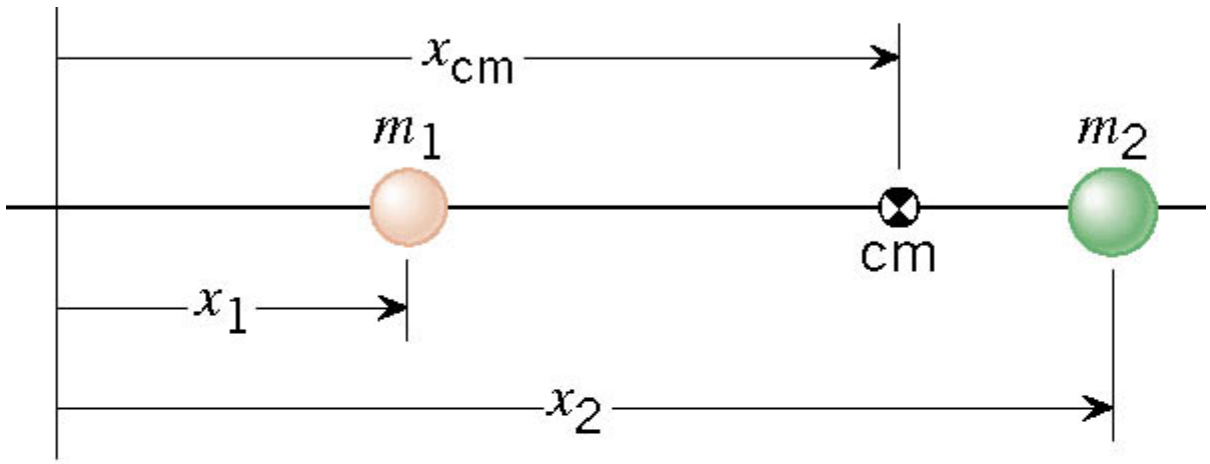
$\vec{v}_{o1} = -0.900\cos 50^\circ$ m/s $\vec{v}_{o2} = 0$, $\vec{v}_{f2} = -0.700\sin 35^\circ$ m/s

final $x : v_{1x} = +0.63$ m/s final $y : v_{1y} = +0.12$ m/s

$$v_1 = \sqrt{v_{1x}^2 + v_{1y}^2} = +0.64 \text{ m/s}; \quad \theta = \tan^{-1}\left(v_{1y}/v_{1x}\right) = 11^\circ$$

7.5 Center of Mass

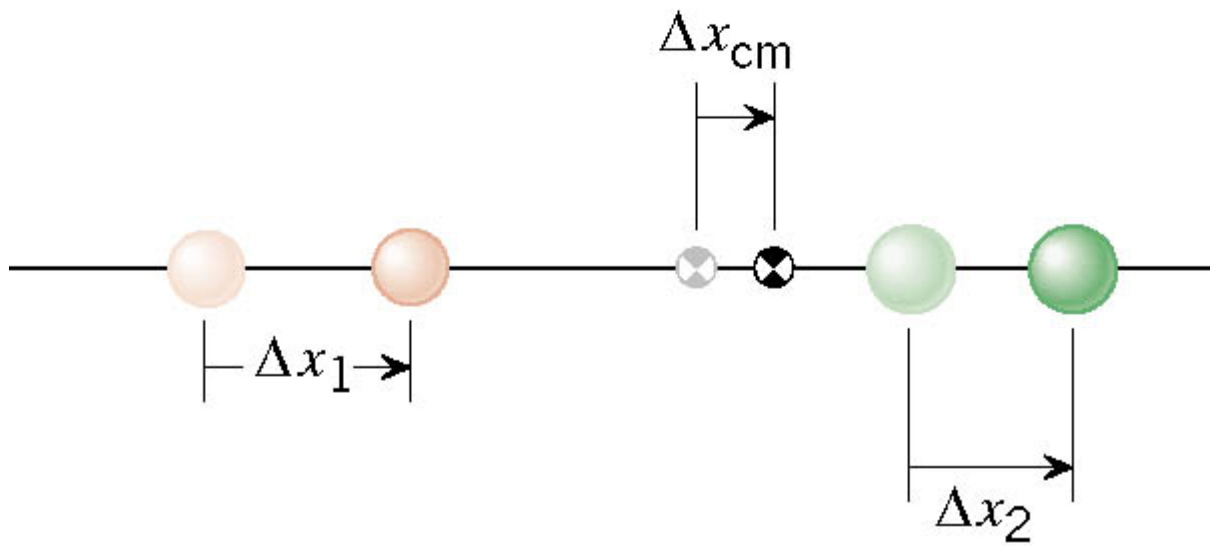
The center of mass is a point that represents the average location for the total mass of a system.



The center of mass: $$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

The center of mass can move if the objects are moving.

If masess are moving



$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2} \Rightarrow \vec{v}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$$

7.5 Center of Mass

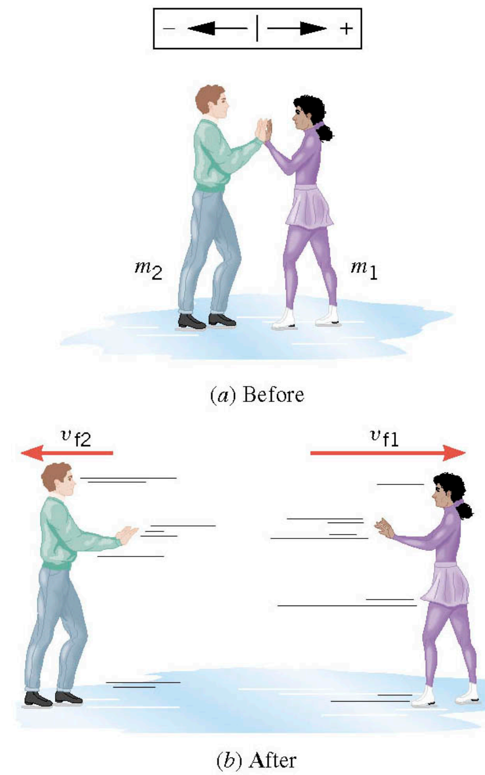
$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

In an isolated system, the total linear momentum does not change, therefore the velocity of the center of mass does not change.

7.5 Center of Mass

BEFORE

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = 0$$



AFTER

$$\vec{v}_{cm} = \frac{(88\text{ kg})(-1.5\text{m/s}) + (54\text{ kg})(+2.5\text{m/s})}{88\text{ kg} + 54\text{ kg}} = 0.002 \approx 0$$