

# *Quick review of Ch. 6 & 7*

*Quiz to follow*

## Energy and energy conservation

$$\text{Work: } W = Fs \cos \theta$$

$$\text{Kinetic Energy: } KE = \frac{1}{2}mv^2$$

$$\text{Work changes kinetic energy: } W = KE_f - KE_0 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$$

$$\text{Conservative forces} \Rightarrow \text{Potential Energy}$$

$$\text{Gravitational Potential Energy: } PE = mgh$$

$$\text{Total Energy: } E = KE + PE$$

$$\begin{aligned} &\text{Work by non-conservative forces (friction, humans, explosions)} \\ &\text{changes total energy: } W_{\text{NC}} = (KE_f - KE_0) + (PE_f - PE_0) \end{aligned}$$

If  $W_{\text{NC}} = 0$ , there is total energy conservation:

$$E_f = E_0 \Rightarrow KE_f + PE_f = KE_0 + PE_0$$

$$\text{Average power} = \text{Work/time} = (\text{Energy change})/\text{time} = F \bar{v}$$

## Momentum and momentum conservation

$$\text{Impulse: } \vec{\mathbf{J}} = \vec{\mathbf{F}}t$$

$$\text{Momentum: } \vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

Net average impulse changes momentum:

$$\sum \vec{\mathbf{F}}\Delta t = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_0 = m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_0$$

$$\text{Momentum of 2 masses in collision: } \vec{\mathbf{P}} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2$$

No net external force, momentum is conserved:

$$\vec{\mathbf{P}}_f = \vec{\mathbf{P}}_0 \Rightarrow m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f} = m_1\vec{\mathbf{v}}_{1o} + m_2\vec{\mathbf{v}}_{2o}$$

$$\text{Center of mass position: } x_{\text{cm}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}, \text{ and velocity } \vec{\mathbf{v}}_{\text{cm}} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1 + m_2}$$

$$\text{Conservation of momentum} \Rightarrow \vec{\mathbf{v}}_{\text{cm}} \text{ remains constant}$$

# Quiz 7

1. **C&J page 176 (bottom), Check Your Understanding #16: “Engine A ...”**

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1. **C&J page 176 (bottom), Check Your Understanding #16: “Engine A ...”**
2. **C&J page 198 (top), Check Your Understanding #10: “A canoe ...”**

# Quiz 7

1. **C&J page 176 (bottom), Check Your Understanding #16: “Engine A ...”**
2. **C&J page 198 (top), Check Your Understanding #10: “A canoe ...”**
3. **A space probe travels with a momentum of  $+6.0 \times 10^7 \text{ kg} \cdot \text{m} / \text{s}$  . A retrorocket fires for 10 s, with a force magnitude of  $2.0 \times 10^6 \text{ N}$ , that slows the speed of the probe. What is the momentum of the probe after the rocket ceases to fire?**
  - a)  $2.0 \times 10^7 \text{ kg} \cdot \text{m} / \text{s}$
  - b)  $4.0 \times 10^7 \text{ kg} \cdot \text{m} / \text{s}$
  - c)  $6.0 \times 10^7 \text{ kg} \cdot \text{m} / \text{s}$
  - d)  $8.0 \times 10^7 \text{ kg} \cdot \text{m} / \text{s}$
  - e)  $10.0 \times 10^7 \text{ kg} \cdot \text{m} / \text{s}$

**4. A mass with a momentum of  $+10.0 \text{ kg} \cdot \text{m} / \text{s}$ , collides with a mass twice as big with a momentum of  $-6.0 \text{ kg} \cdot \text{m} / \text{s}$ , and they stick together. What is the momentum of the combined system after the collision?**

- a)  $-2.0 \text{ kg} \cdot \text{m} / \text{s}$**
- b)  $+2.0 \text{ kg} \cdot \text{m} / \text{s}$**
- c)  $+4.0 \text{ kg} \cdot \text{m} / \text{s}$**
- d)  $+6.0 \text{ kg} \cdot \text{m} / \text{s}$**
- e)  $+16.0 \text{ kg} \cdot \text{m} / \text{s}$**

- 4. A mass with a momentum of  $+10.0 \text{ kg} \cdot \text{m} / \text{s}$ , collides with a mass twice as big with a momentum of  $-6.0 \text{ kg} \cdot \text{m} / \text{s}$ , and they stick together. What is the momentum of the combined system after the collision?**
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  - c)  $+4.0 \text{ kg} \cdot \text{m} / \text{s}$**
  - d)  $+6.0 \text{ kg} \cdot \text{m} / \text{s}$**
  - e)  $+16.0 \text{ kg} \cdot \text{m} / \text{s}$**
- 5. Two identical 2.0-kg objects are involved in a collision. The objects are on a frictionless track. The initial velocity of object A is  $+12 \text{ m} / \text{s}$  and the initial velocity of object B is  $-6.0 \text{ m} / \text{s}$ . What are the final velocities of the two objects?**
- a) Object A moves at  $-6.0 \text{ m} / \text{s}$  and B moves at  $+12 \text{ m} / \text{s}$ .**
  - b) Object A moves at  $+9.0 \text{ m} / \text{s}$  and B moves at  $+9.0 \text{ m} / \text{s}$ .**
  - c) Object A moves at  $-6.0 \text{ m} / \text{s}$  and B moves at  $+6.0 \text{ m} / \text{s}$ .**
  - d) Object A moves at  $-12 \text{ m} / \text{s}$  and B moves at  $+6.0 \text{ m} / \text{s}$ .**
  - e) This cannot be determined without more information.**

# Quiz 7

**1. C&J page 176 (bottom), Check Your Understanding #16: “Engine A ...”**

a) Engine A has greater power  $\Rightarrow$  greater (work/ $t$ )  $\Rightarrow$  same work in less time.

**2. C&J page 198 (top), Check Your Understanding #10: “A canoe ...”**

a) Conserves total momentum:  $(-60 + 170)\text{kg} \cdot \text{m/s} = +110 \text{ kg} \cdot \text{m/s}$

**3. A space probe travels with a momentum of  $+6.0 \times 10^7 \text{ kg} \cdot \text{m/s}$ . A retrorocket fires for 10 s, with a force magnitude of  $2.0 \times 10^6 \text{ N}$ , that slows the speed of the probe. What is the momentum of the probe after the rocket ceases to fire?**

a)  $2.0 \times 10^7 \text{ kg} \cdot \text{m/s}$

**b)  $4.0 \times 10^7 \text{ kg} \cdot \text{m/s}$**

c)  $6.0 \times 10^7 \text{ kg} \cdot \text{m/s}$

d)  $8.0 \times 10^7 \text{ kg} \cdot \text{m/s}$

e)  $10.0 \times 10^7 \text{ kg} \cdot \text{m/s}$

$$\mathbf{F}\Delta t = \mathbf{p}_f - \mathbf{p}_0$$

$$\mathbf{p}_f = \mathbf{p}_0 + \mathbf{F}\Delta t$$

$$\begin{aligned}\mathbf{p}_f &= (+6.0 \times 10^7 \text{ N} \cdot \text{s}) + (-2.0 \times 10^6 \text{ N})(10 \text{ s}) \\ &= +4.0 \times 10^7 \text{ N} \cdot \text{s}\end{aligned}$$

4. A mass with a momentum of  $+10.0 \text{ kg} \cdot \text{m} / \text{s}$ , collides with a mass twice as big with a momentum of  $-6.0 \text{ kg} \cdot \text{m} / \text{s}$ , and they stick together. What is the momentum of the combined system after the collision?

- a)  $-2.0 \text{ kg} \cdot \text{m} / \text{s}$
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- c)  $+4.0 \text{ kg} \cdot \text{m} / \text{s}$
- d)  $+6.0 \text{ kg} \cdot \text{m} / \text{s}$
- e)  $+16.0 \text{ kg} \cdot \text{m} / \text{s}$

Momentum is conserved

$$\begin{aligned} \mathbf{p}_{(1+2)f} &= \mathbf{p}_{1o} + \mathbf{p}_{2o} \\ &= (+10.0 \text{ kg} \cdot \text{m} / \text{s}) + (-6.0 \text{ kg} \cdot \text{m} / \text{s}) \\ &= 4.0 \text{ kg} \cdot \text{m} / \text{s} \end{aligned}$$

5. Two identical 2.0-kg objects are involved in a collision. The objects are on a frictionless track. The initial velocity of object A is  $+12 \text{ m} / \text{s}$  and the initial velocity of object B is  $-6.0 \text{ m} / \text{s}$ . What is the final velocity of the two objects?

- a) Object A moves at  $-6.0 \text{ m} / \text{s}$  and B moves at  $+12 \text{ m} / \text{s}$ .
- b) Object A moves at  $+9.0 \text{ m} / \text{s}$  and B moves at  $+9.0 \text{ m} / \text{s}$ .
- c) Object A moves at  $-6.0 \text{ m} / \text{s}$  and B moves at  $+6.0 \text{ m} / \text{s}$ .
- d) Object A moves at  $-12 \text{ m} / \text{s}$  and B moves at  $+6.0 \text{ m} / \text{s}$ .
- e) This cannot be determined without more information.

5. Two identical 2.0-kg objects are involved in a collision. The objects are on a frictionless track. The initial velocity of object A is +12 m/s and the initial velocity of object B is – 6.0 m/s. What is the final velocity of the two objects?

Momentum is conserved

$$\vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f} = \vec{\mathbf{p}}_{1o} + \vec{\mathbf{p}}_{2o}$$

$$m\vec{\mathbf{v}}_{1f} + m\vec{\mathbf{v}}_{2f} = m\vec{\mathbf{v}}_{1o} + m\vec{\mathbf{v}}_{2o}$$

$$\vec{\mathbf{v}}_{1f} + \vec{\mathbf{v}}_{2f} = (+12\text{m/s}) + (-6\text{m/s}) = +6\text{m/s}$$

Only know the sum:  $\vec{\mathbf{v}}_{1f} + \vec{\mathbf{v}}_{2f}$  (Need to know what happens to energy)

5. Two identical 2.0-kg objects are involved in a collision. The objects are on a frictionless track. The initial velocity of object A is +12 m/s and the initial velocity of object B is – 6.0 m/s. What is the final velocity of the two objects?

Momentum is conserved

$$\vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f} = \vec{\mathbf{p}}_{1o} + \vec{\mathbf{p}}_{2o}$$

$$m\vec{\mathbf{v}}_{1f} + m\vec{\mathbf{v}}_{2f} = m\vec{\mathbf{v}}_{1o} + m\vec{\mathbf{v}}_{2o}$$

$$\vec{\mathbf{v}}_{1f} + \vec{\mathbf{v}}_{2f} = (+12\text{m/s}) + (-6\text{m/s}) = +6\text{m/s}$$

Only know the sum:  $\vec{\mathbf{v}}_{1f} + \vec{\mathbf{v}}_{2f}$  (Need to know what happens to energy)

If energy  
is conserved

$$v_{1f}^2 + v_{2f}^2 = v_{1o}^2 + v_{2o}^2 = 144\text{m}^2/\text{s}^2 + 36\text{m}^2/\text{s}^2$$

$$v_{1f}^2 + (6\text{m/s} - v_{1f})^2 = 2v_{1f}^2 - (12\text{m/s})v_{1f} + 36\text{m}^2/\text{s}^2$$

$$\text{Quadratic Equation: } v_{1f}^2 - (6\text{m/s})v_{1f} - 72\text{m}^2/\text{s}^2 = 0$$

$$v_{1f} = \left(+3 \pm \sqrt{9 + 72}\right)\text{m/s} = \left(+3 \pm 9\right)\text{m/s}$$

$$v_{1f} = -6\text{ m/s} \quad (v_{1f} = +12\text{ m/s, no collision})$$

# *Chapter 8*

## ***Rotational Kinematics***

## 8.1 *Rotational Motion and Angular Displacement*

Why are there 360 degrees in a circle?

Why are there 60 minutes in an hour?

Why are there 60 seconds in a minute?

Because the Greeks, who invented these units were enamored with numbers that are divisible by most whole numbers, 12 or below (except 7 and 11).

Strange, because later it was the Greeks who discovered that the ratio of the radius to the circumference of a circle was a number known as  $2\pi$ .

A new unit, radians, is really useful for angles.

## 8.1 Rotational Motion and Angular Displacement

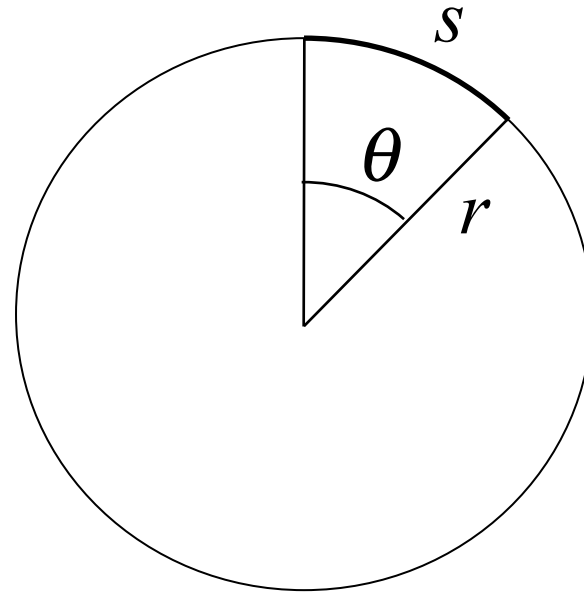
A new unit, radians, is really useful for angles.

### Radian measure

$$\theta(\text{radians}) = \frac{s \text{ (arc length)}}{r \text{ (radius)}}$$

$$s = r\theta$$

( $s$  in same units as  $r$ )



### Full circle

$$\begin{aligned}\theta &= \frac{s}{r} = \frac{2\pi r}{r} \\ &= 2\pi \text{ (radians)}\end{aligned}$$

### Conversion of degree to radian measure

$$\begin{aligned}\theta(\text{rad}) &= \theta(\text{deg.}) \left( \frac{2\pi \text{ rad}}{360 \text{ deg.}} \right) \\ \left( \frac{2\pi \text{ rad}}{360 \text{ deg.}} \right) &= 1\end{aligned}$$

## 8.1 Rotational Motion and Angular Displacement

### Example 1 Adjacent Synchronous Satellites

Synchronous satellites are put into an orbit whose radius is  $4.23 \times 10^7 \text{ m}$ .

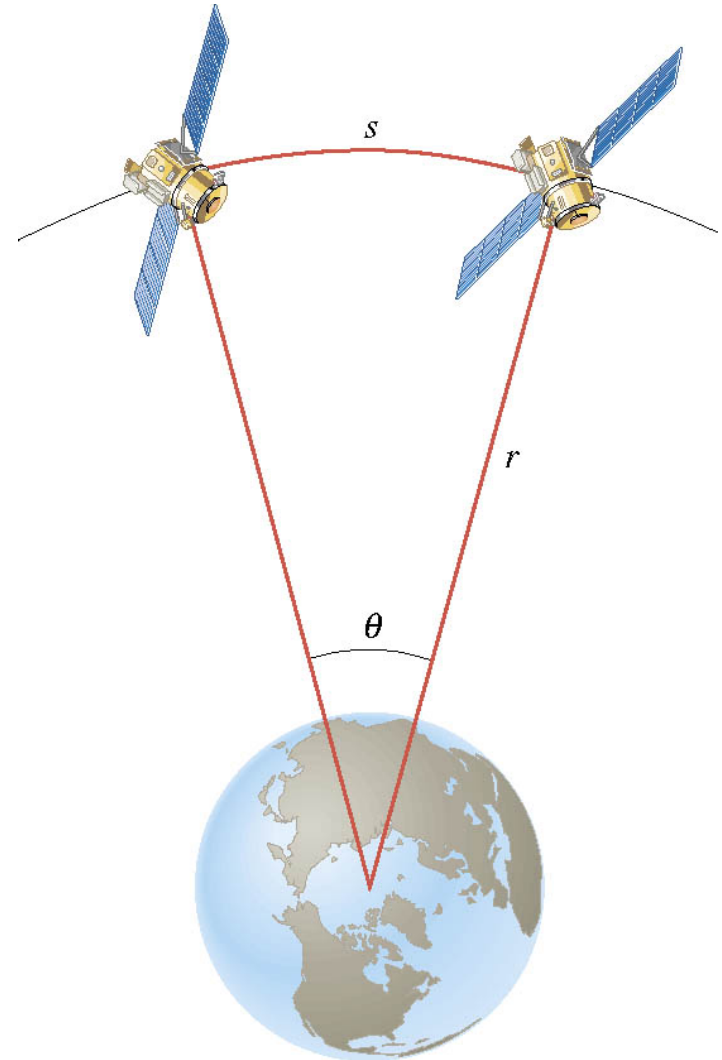
If the angular separation of the two satellites is 2.00 degrees, find the arc length that separates them.

Convert degree to radian measure

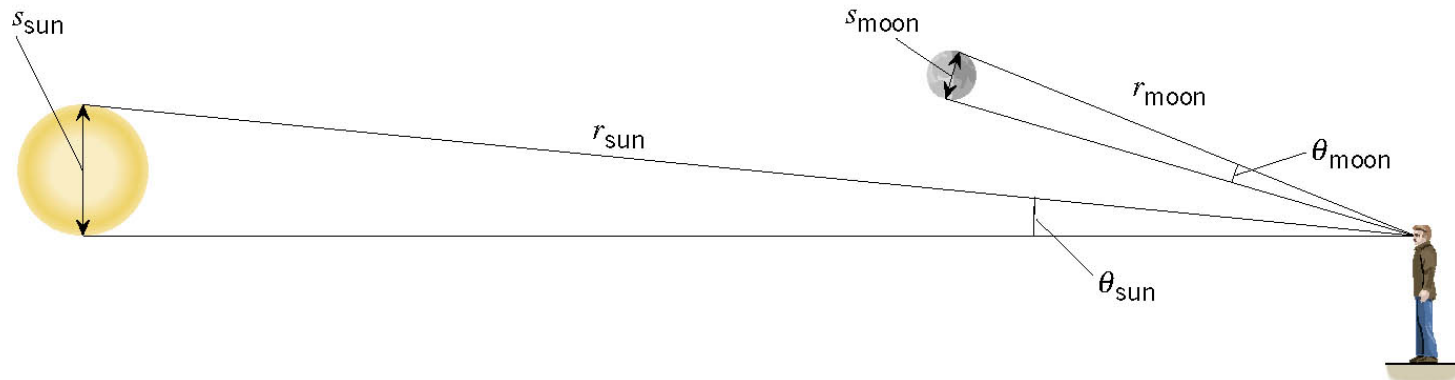
$$2.00 \text{ deg} \left( \frac{2\pi \text{ rad}}{360 \text{ deg}} \right) = 0.0349 \text{ rad}$$

Determine arc length

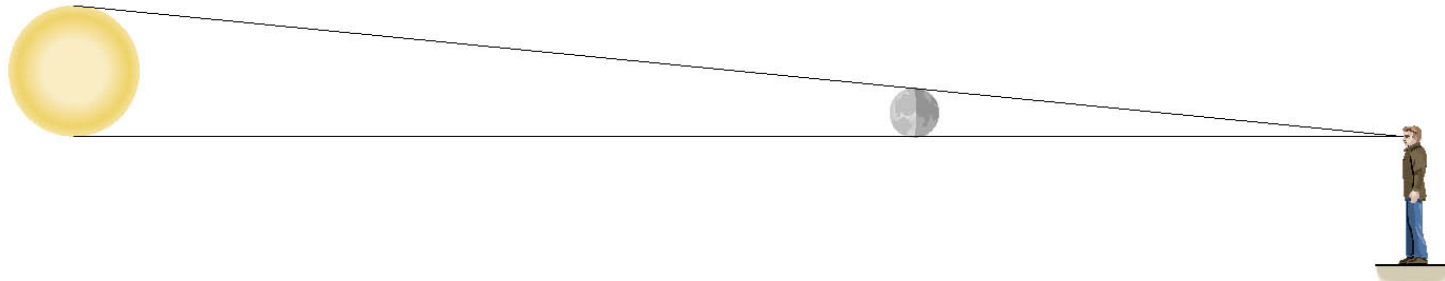
$$\begin{aligned} s &= r\theta = (4.23 \times 10^7 \text{ m})(0.0349 \text{ rad}) \\ &= 1.48 \times 10^6 \text{ m} \quad (920 \text{ miles}) \end{aligned}$$



## 8.1 Rotational Motion and Angular Displacement



(a)



(b)

For an observer on the earth, an eclipse can occur because angles subtended by the sun and the moon are the same.

$$\theta = \frac{s_{\text{Sun}}}{r_{\text{Sun}}} \approx \frac{s_{\text{Moon}}}{r_{\text{Moon}}} \approx 9.3 \text{ mrad}$$

## 8.1 Rotational Motion and Angular Displacement

The angle through which the object rotates is called the **angular displacement vector**

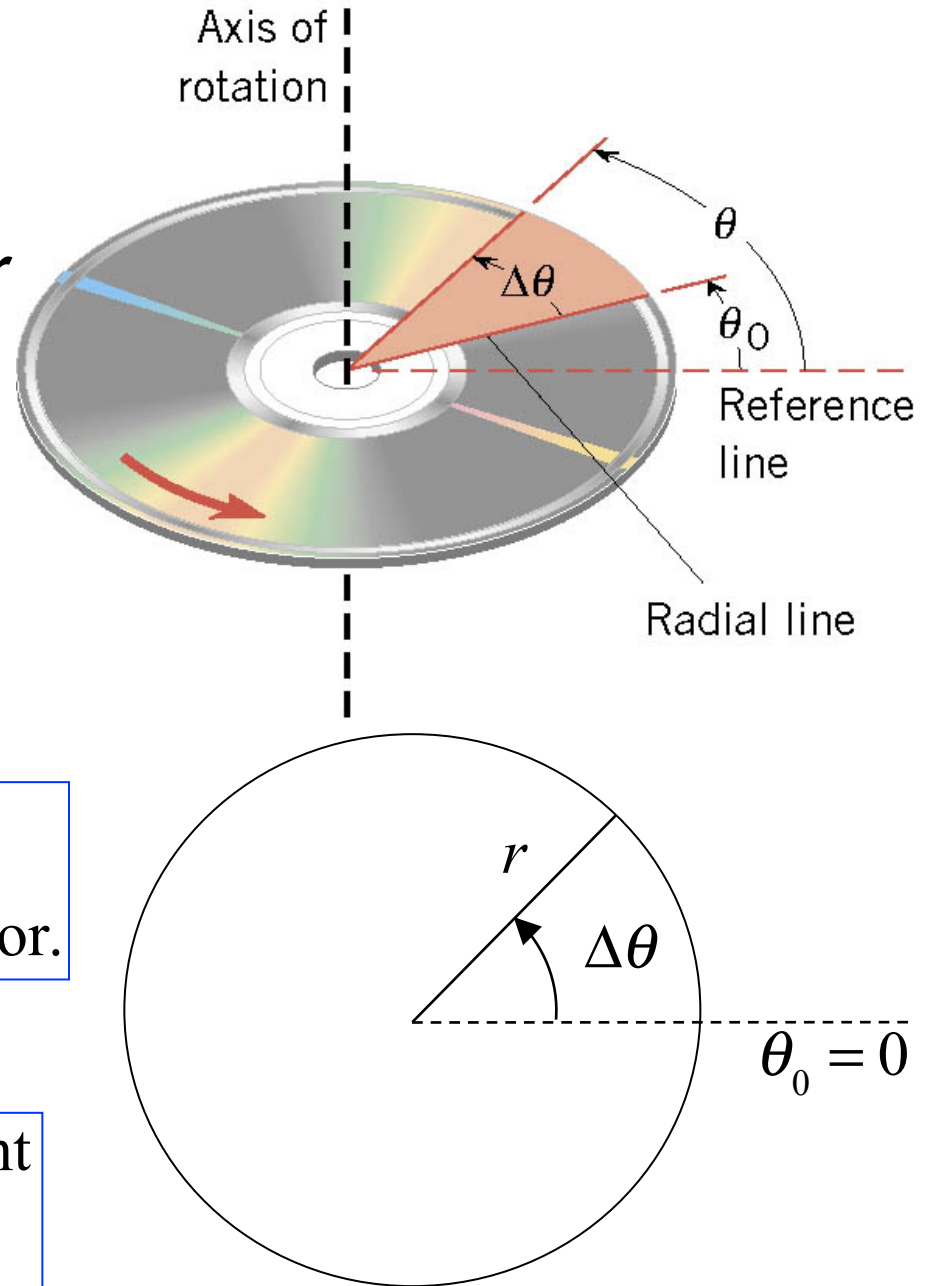
$$\Delta\theta = \theta - \theta_o$$

**SI** unit of angular displacement, radian (rad)

Simplified using  $\theta_o = 0$ , and  $\Delta\theta = \theta$ , angular displacement vector.

**Vector**

Counter-clockwise is + displacement  
Clockwise is – displacement



## Clicker Question 8.1 Radian measure for angles

Over the course of a day (twenty-four hours), what is the angular displacement of the second hand of a wrist watch in radians?

- a) 1440 rad
- b) 2880 rad
- c) 4520 rad
- d) 9050 rad
- e) 543,000 rad

## Clicker Question 8.1 Radian measure for angles

Over the course of a day (twenty-four hours), what is the angular displacement of the second hand of a wrist watch in radians?

- a) 1440 rad
- b) 2880 rad
- c) 4520 rad
- d) 9050 rad**
- e) 543,000 rad

Second hand makes 1 rev. in 1 minute.

$$\begin{aligned} 1 \text{ day} &= (1 \text{ day})(60 \text{ min./hr})(24 \text{ hr/day}) \\ &= 1440 \text{ min.} \end{aligned}$$

$$\# \text{ rev.} = (1 \text{ rev./min.})(1440 \text{ min.}) = 1440 \text{ rev.}$$

$$1 \text{ rev.} = 2\pi \text{ radians}$$

$$\begin{aligned} \text{Total } \theta \text{ displacement} &= (2\pi \text{ rad/min.})(1440 \text{ min.}) \\ &= 9048 \text{ rad} \end{aligned}$$

## 8.2 Angular Velocity and Angular Acceleration

### DEFINITION OF AVERAGE ANGULAR VELOCITY

$$\text{Average angular velocity} = \frac{\text{Angular displacement}}{\text{Elapsed time}}$$

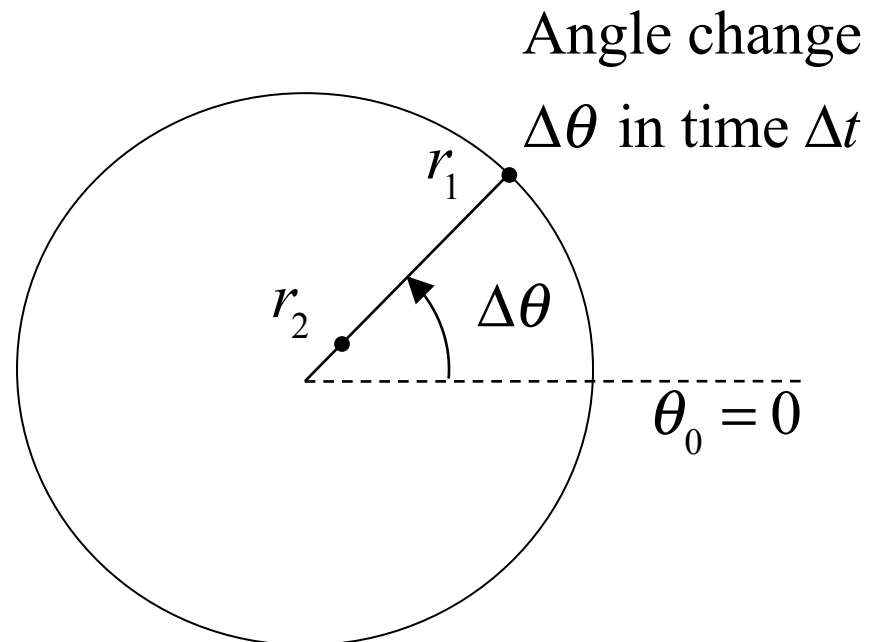
$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} \quad \text{where } \Delta t = t - t_o$$

**SI Unit of Angular Velocity:** radian per second (rad/s)

$\Delta\theta$  is the same at all radii.

$\Delta t$  is the same at all radii.

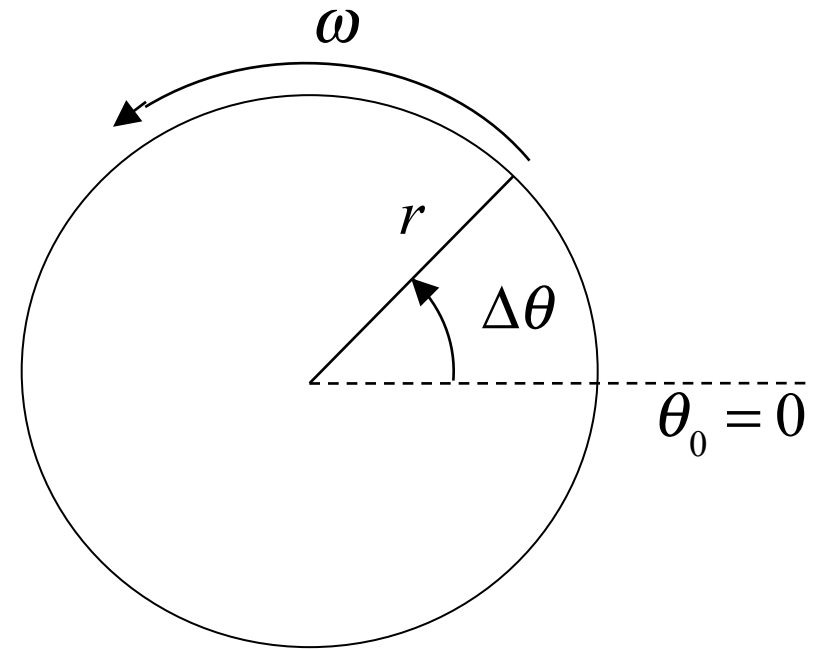
$$\omega = \frac{\Delta\theta}{\Delta t} \text{ is the same at all radii.}$$



## 8.2 Angular Velocity and Angular Acceleration

**Case 1:** Constant angular velocity,  $\omega$ .

$$\omega = \frac{\Delta\theta}{\Delta t} \qquad \Delta\theta = \omega \Delta t$$



**Example:** A disk rotates with a constant angular velocity of  $+1$  rad/s.

What is the angular displacement of the disk in 13 seconds?

How many rotations has the disk made in that time?

$$\Delta\theta = \omega \Delta t = (+1 \text{ rad/s})(13 \text{ s}) = +13 \text{ rad}$$

$$2\pi \text{ radians} = 1 \text{ rotation} \Rightarrow 2\pi \text{ rad/rot.}$$

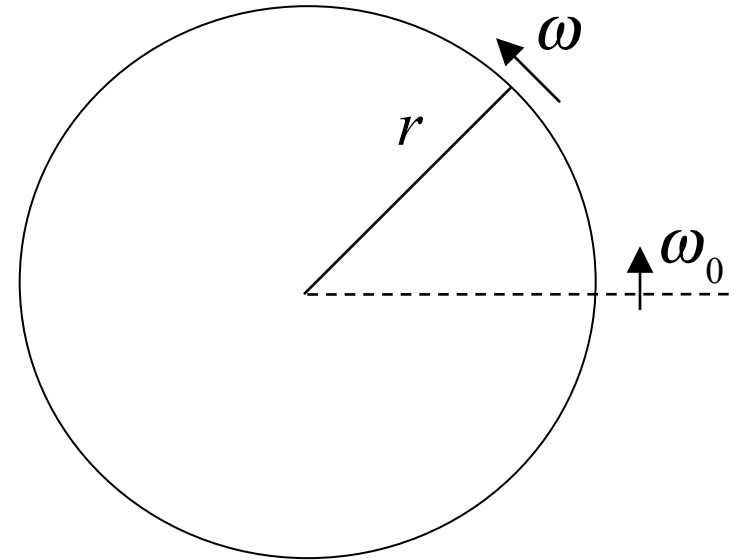
$$n_{\text{rot}} = \frac{\Delta\theta}{2\pi \text{ rad/rot.}} = \frac{13 \text{ rad}}{6.3 \text{ rad/rot}} = 2.1 \text{ rot.}$$

## 8.2 Angular Velocity and Angular Acceleration

**Case 2:** Angular velocity,  $\omega$ ,  
changes in time.

Instantaneous  
angular velocity  
at time  $t$ .

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t}$$



### DEFINITION OF AVERAGE ANGULAR ACCELERATION

Average angular acceleration =  $\frac{\text{Change in angular velocity}}{\text{Elapsed time}}$

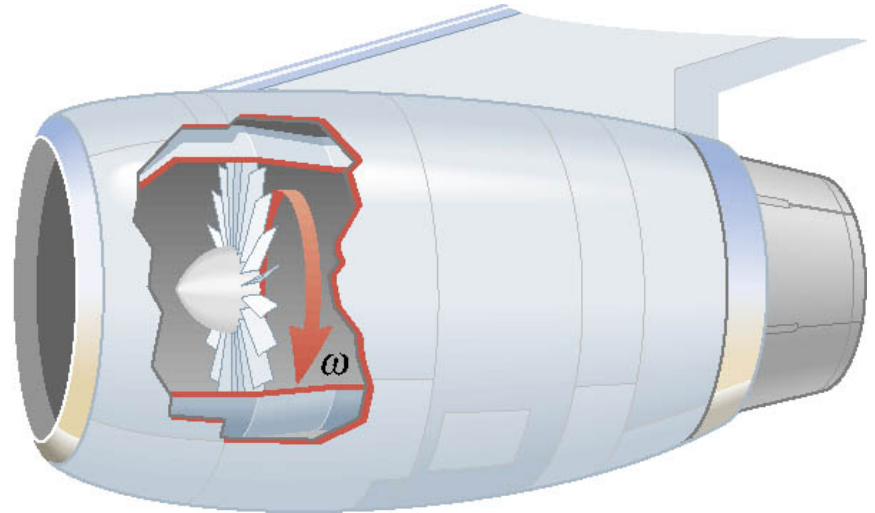
$$\bar{\alpha} = \frac{\omega - \omega_o}{t - t_o} = \frac{\Delta \omega}{\Delta t}$$

**SI Unit of Angular acceleration:** radian per second squared ( $\text{rad/s}^2$ )

## 8.2 Angular Velocity and Angular Acceleration

### Example 4 A Jet Revving Its Engines

As seen from the front of the engine, the fan blades are rotating with an angular speed of  $-110 \text{ rad/s}$ . As the plane takes off, the angular velocity of the blades reaches  $-330 \text{ rad/s}$  in a time of  $14 \text{ s}$ .



Rotation is clockwise (negative)


Find the angular acceleration, assuming it to be constant.

$$\bar{\alpha} = \frac{(-330 \text{ rad/s}) - (-110 \text{ rad/s})}{14 \text{ s}} = -16 \text{ rad/s}^2$$

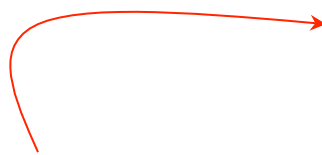
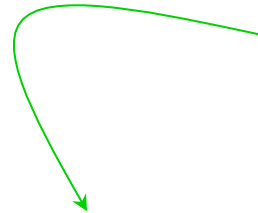
### 8.3 *The Equations of Rotational Kinematics*

The equations of rotational kinematics for constant angular acceleration:

ANGULAR VELOCITY

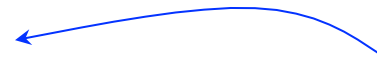

$$\omega = \omega_o + \alpha t$$

ANGULAR ACCELERATION



$$\theta = \frac{1}{2} (\omega_o + \omega) t$$

TIME



ANGULAR DISPLACEMENT

$$\omega^2 = \omega_o^2 + 2\alpha\theta$$

$$\theta = \omega_o t + \frac{1}{2} \alpha t^2$$

## 8.3 The Equations of Rotational Kinematics

**Table 8.2 Symbols Used  
in Rotational and Linear Kinematics**

Rotational Motion	Quantity	Linear Motion
$\theta$	Displacement	$x$
$\omega_0$	Initial velocity	$v_0$
$\omega$	Final velocity	$v$
$\alpha$	Acceleration	$a$
$t$	Time	$t$

**Table 8.1 The Equations of Kinematics  
for Rotational and Linear Motion**

Rotational Motion ( $\alpha = \text{constant}$ )		Linear Motion ( $a = \text{constant}$ )	
$\omega = \omega_0 + \alpha t$	(8.4)	$v = v_0 + at$	(2.4)
$\theta = \frac{1}{2}(\omega_0 + \omega)t$	(8.6)	$x = \frac{1}{2}(v_0 + v)t$	(2.7)
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	(8.7)	$x = v_0 t + \frac{1}{2}at^2$	(2.8)
$\omega^2 = \omega_0^2 + 2\alpha\theta$	(8.8)	$v^2 = v_0^2 + 2ax$	(2.9)

## 8.3 *The Equations of Rotational Kinematics*

### Reasoning Strategy

1. Make a drawing.
2. Decide which directions are to be called positive (+) and negative (−).
3. Write down the values that are given for any of the five kinematic variables.
4. Verify that the information contains values for at least three of the five kinematic variables. Select the appropriate equation.
5. When the motion is divided into segments, remember that the final angular velocity of one segment is the initial angular velocity for the next.
6. Keep in mind that there may be two possible answers to a kinematics problem.

## Clicker Question 8.2 Rotational motion kinematics

Given the initial and final angular velocity of a disk, and the total angular displacement of the disk, with which single equation can the angular acceleration of the disk be obtained?

a)  $\omega = \omega_0 + \alpha t$

b)  $\theta = \frac{1}{2}(\omega + \omega_0)t$

c)  $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

d)  $\omega^2 = \omega_0^2 + 2\alpha\theta$

e) none of the above

## Clicker Question 8.2 Rotational motion kinematics

Given the initial and final angular velocity of a disk and the total angular displacement of the disk, the angular acceleration of the disk can be obtained with which single equation?

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c)  $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

d)  $\omega^2 = \omega_0^2 + 2\alpha\theta$

e) none of the above

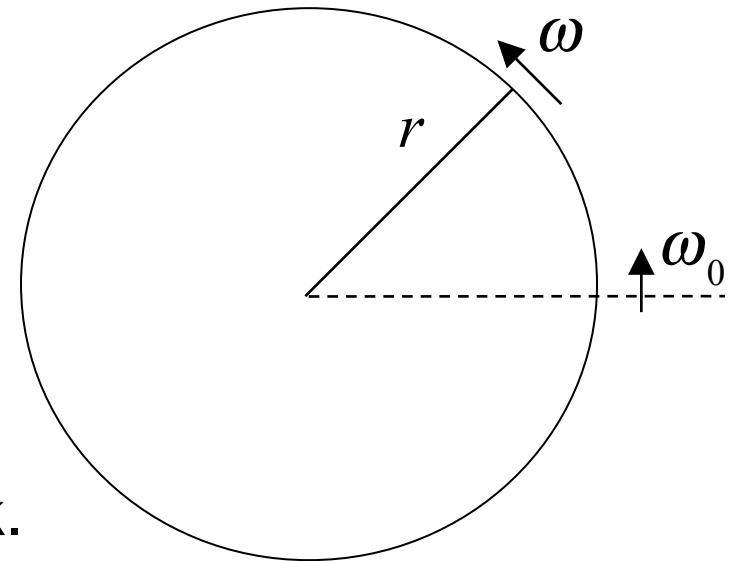
No time given!

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$
$$\alpha = \frac{\omega^2 - \omega_0^2}{2\theta}$$

### 8.3 The Equations of Rotational Kinematics

**Example:** A disk has an initial angular velocity of  $+375 \text{ rad/s}$ . The disk accelerates and reaches a greater angular velocity after rotating through an angular displacement of  $+44.0 \text{ rad}$ .

If the angular acceleration has a constant value of  $+1740 \text{ rad/s}^2$ , find the final angular velocity of the disk.



Given:  $\omega_0 = +375 \text{ rad/s}$ ,  $\theta = +44 \text{ rad}$ ,  $\alpha = 1740 \text{ rad/s}^2$

Want: final angular velocity,  $\omega$ .

No time!

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$= (375 \text{ rad/s})^2 + 2(1740 \text{ rad/s}^2)(+44 \text{ rad})$$

$$\omega = 542 \text{ rad/s}$$

## 8.4 Angular Variables and Tangential Variables

$\omega$  = angular velocity is the **same at all radii**

$\alpha$  = angular acceleration is the **same at all radii**

$\vec{v}_T$  = tangential velocity is **different at each radius**

$\vec{a}_T$  = tangential acceleration is **different at each radius**

Direction is tangent to circle at that  $\theta$

$$\vec{v}_T = \omega r$$

$$\vec{a}_T = \alpha r$$

$$\vec{v}_T \text{ (m/s)}$$

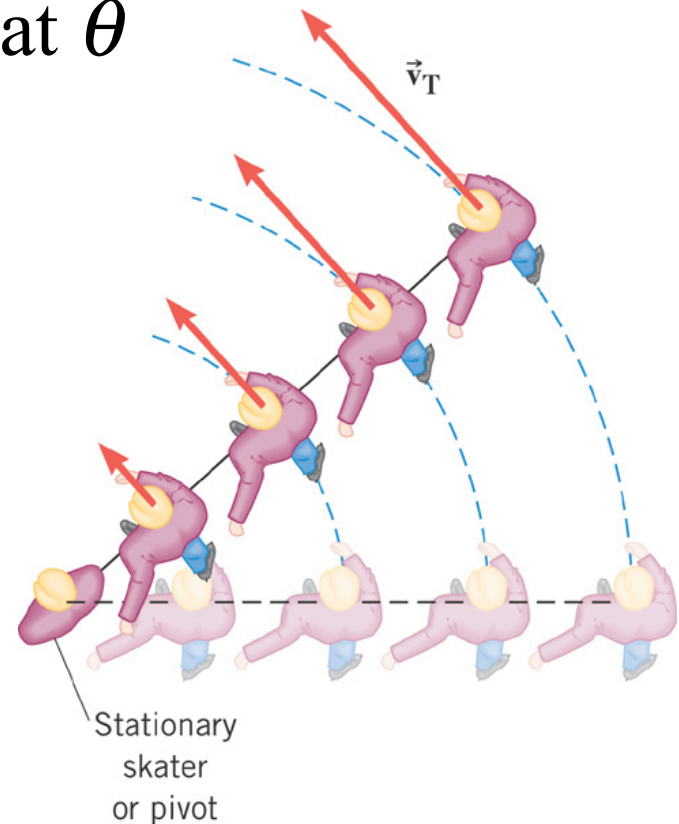
$$\vec{a}_T \text{ (m/s}^2\text{)}$$

$$\omega \text{ (rad/s)}$$

$$\alpha \text{ (rad/s}^2\text{)}$$

$$r \text{ (m)}$$

$$r \text{ (m)}$$

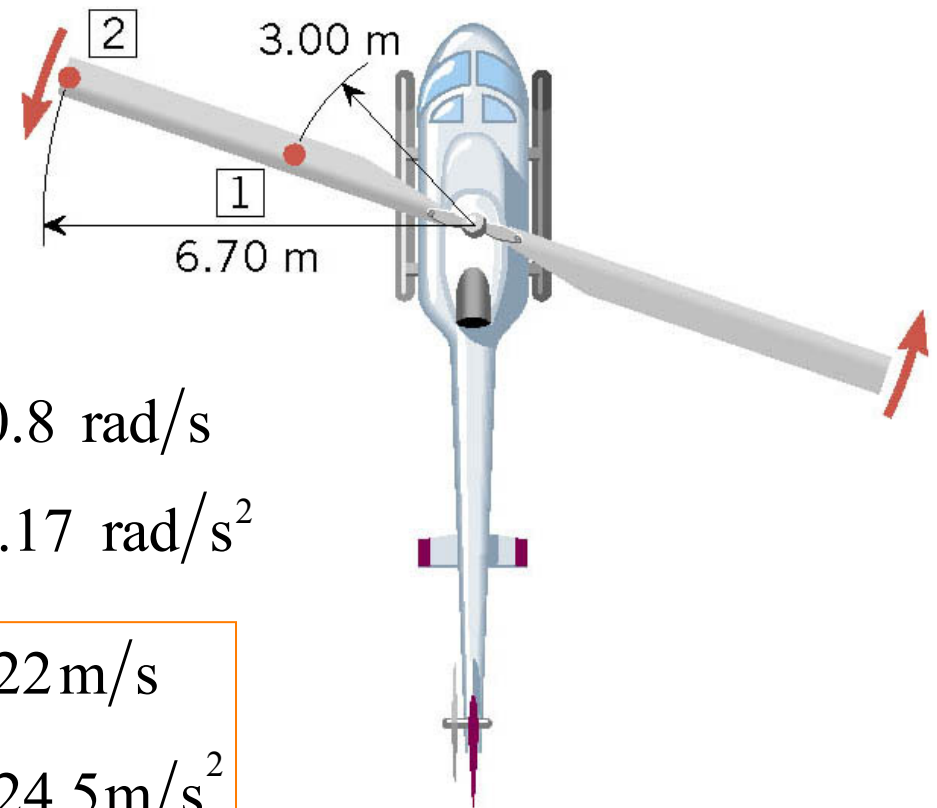


## 8.4 Angular Variables and Tangential Variables

### Example 6 A Helicopter Blade

A helicopter blade has an angular speed of  $6.50 \text{ rev/s}$  and an angular acceleration of  $1.30 \text{ rev/s}^2$ .

For point 1 on the blade, find the magnitude of (a) the tangential speed and (b) the tangential acceleration.



Convert revolutions to radians

$$\omega = (6.50 \text{ rev/s})(2\pi \text{ rad/rev}) = 40.8 \text{ rad/s}$$

$$\alpha = (1.30 \text{ rev/s}^2)(2\pi \text{ rad/rev}) = 8.17 \text{ rad/s}^2$$

$$v_T = \omega r = (40.8 \text{ rad/s})(3.00 \text{ m}) = 122 \text{ m/s}$$

$$a_T = \alpha r = (8.17 \text{ rad/s}^2)(3.00 \text{ m}) = 24.5 \text{ m/s}^2$$

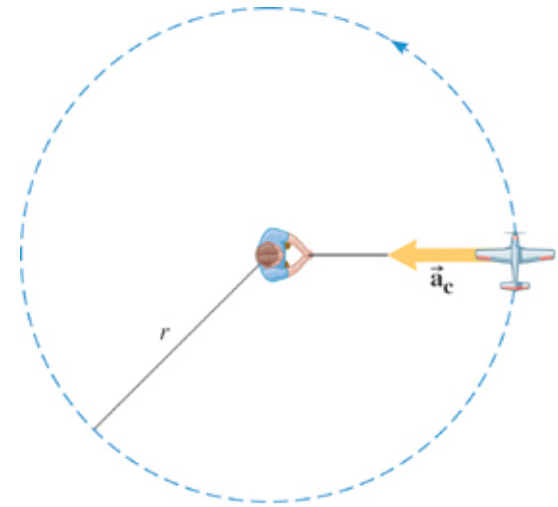
## 8.5 Centripetal Acceleration and Tangential Acceleration

$$a_c = \frac{v_T^2}{r} \quad (\text{Chapter 5})$$

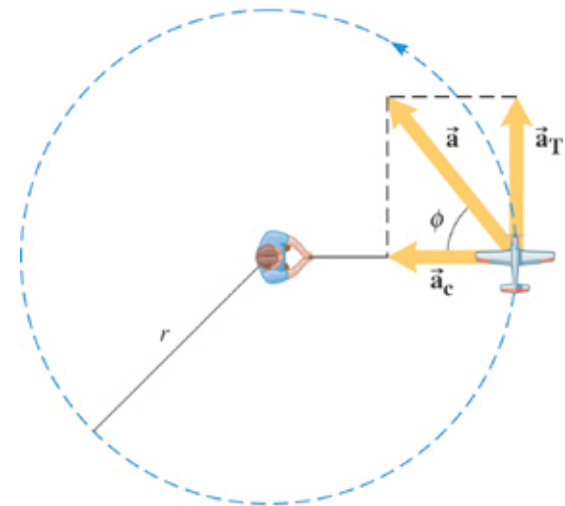
$$= \frac{(\omega r)^2}{r} = \omega^2 r \quad (\omega \text{ in rad/s constant})$$

$$a_T = \alpha r$$

$$a_{total} = \sqrt{a_c^2 + \alpha^2 r^2}$$



(a) Uniform circular motion



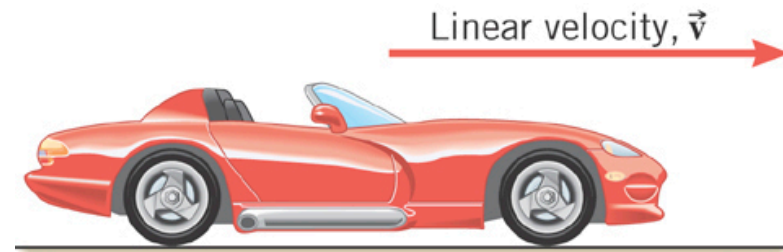
(b) Nonuniform circular motion

## 8.6 Rolling Motion

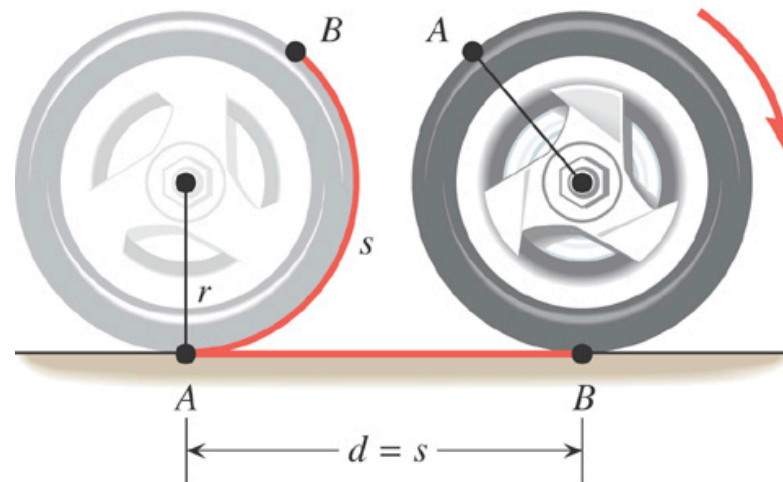
The tangential speed of a point on the outer edge of the tire is equal to the speed of the car over the ground.

$$v = v_T = \omega r$$

$$a = a_T = \alpha r$$



(a)



(b)

## 8.7 The Vector Nature of Angular Variables

**Right-Hand Rule:** Grasp the axis of rotation with your right hand, so that your fingers circle the axis in the same sense as the rotation.

Your extended thumb points along the axis in the direction of the angular velocity.

