Chapter 9

Rotational Dynamics

9.1 The Action of Forces and Torques on Rigid Objects

Chapter 8 developed the concepts of angular motion.

θ : angles and radian measure for angular variables

ω : angular velocity of rotation (same for entire object)

α : angular acceleration (same for entire object)

$v_T = \omega r$: tangential velocity

$a_T = \alpha r$: tangential acceleration

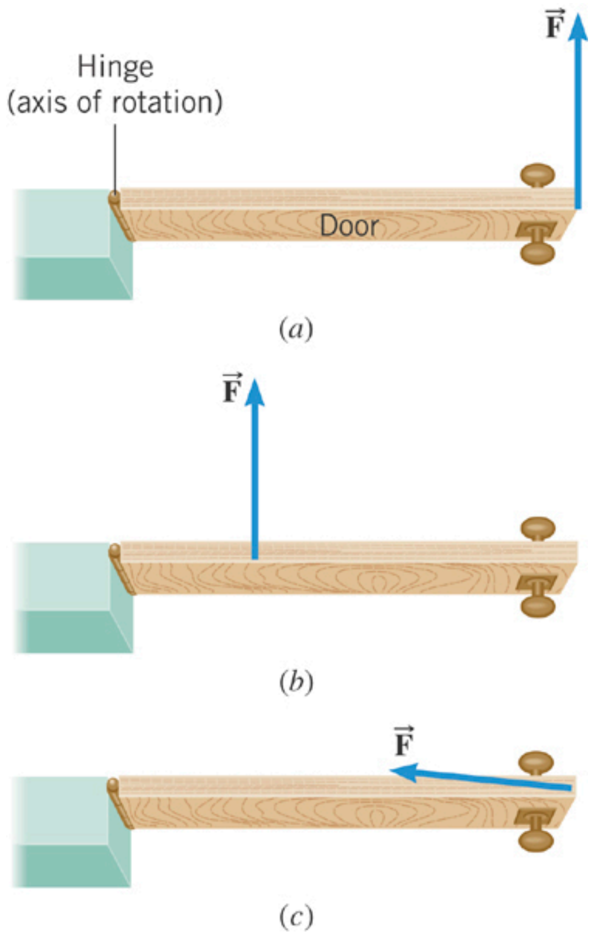
According to Newton's second law, a net force causes an object to have a ***linear acceleration***.

What causes an object to have an ***angular acceleration***?

TORQUE

9.1 The Action of Forces and Torques on Rigid Objects

The amount of torque depends on where and in what direction the force is applied, as well as the location of the axis of rotation.



Maximum rotational effect of the force F.

Smaller rotational effect of the force F.

Rotational effect of the force F is minimal; it compresses more than rotates the bar

9.1 The Action of Forces and Torques on Rigid Objects

DEFINITION OF TORQUE

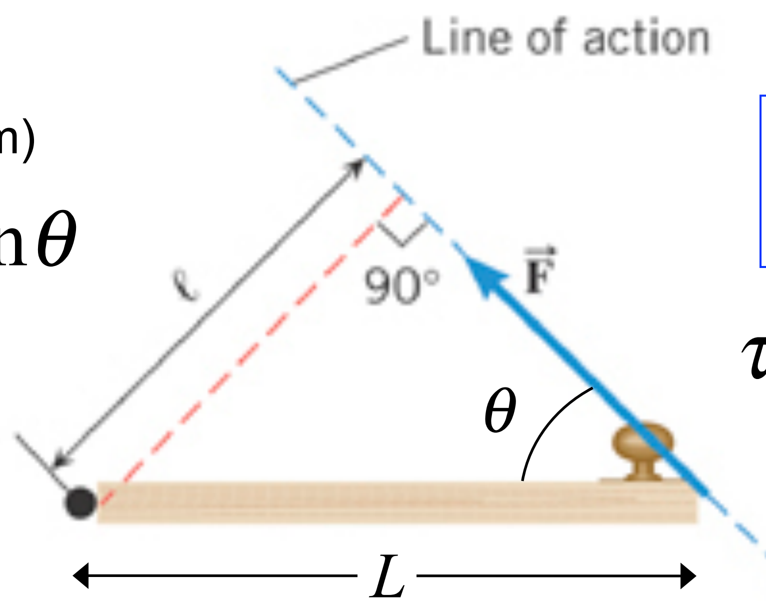
Magnitude of Torque = (Magnitude of the force) x (Lever arm)

$$\tau = F\ell$$

Direction: The torque is positive when the force tends to produce a counterclockwise rotation about the axis.
SI Unit of Torque: newton x meter (N·m)

(Lever arm)

$$\ell = L\sin\theta$$



θ is the angle between $\vec{\mathbf{F}}$ and L

$$\tau = F\ell = FL\sin\theta$$

9.1 The Action of Forces and Torques on Rigid Objects

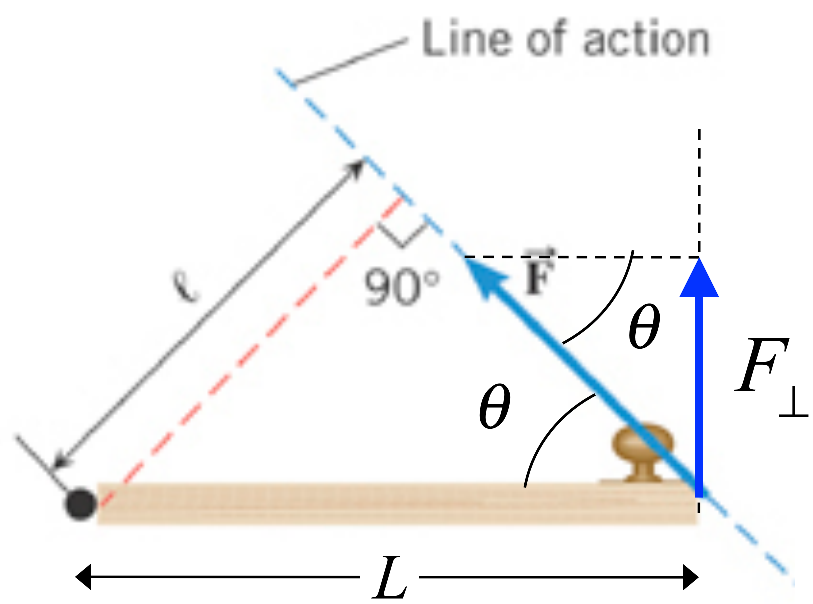
$$\text{Magnitude of Torque} = (\text{Magnitude of the force}) \times (\text{Lever arm})$$

$$\tau = F\ell = FL\sin\theta \qquad \ell = L\sin\theta$$

Alternate (Equivalent) Interpretation

$$\text{Magnitude of Torque} = (\text{Component of Force} \perp \text{ to } L) \times L$$

$$\tau = F_{\perp}L = (F\sin\theta)(L) = FL\sin\theta$$



$F_{\perp} = F\sin\theta$

θ is the angle between $\vec{\mathbf{F}}$ and L

Clicker Question 9.1 Torque

A 1.5-kg ball is tied to the end of a string. The ball is then swung at a constant angular velocity of 4 rad/s in a horizontal circle of radius 2.0 m. What is the torque on the stone?

a) $18\ \mathrm{N \cdot m}$

b) $29\ \mathrm{N \cdot m}$

c) $36\ \mathrm{N \cdot m}$

d) $59\ \mathrm{N \cdot m}$

e) zero $\mathrm{N \cdot m}$

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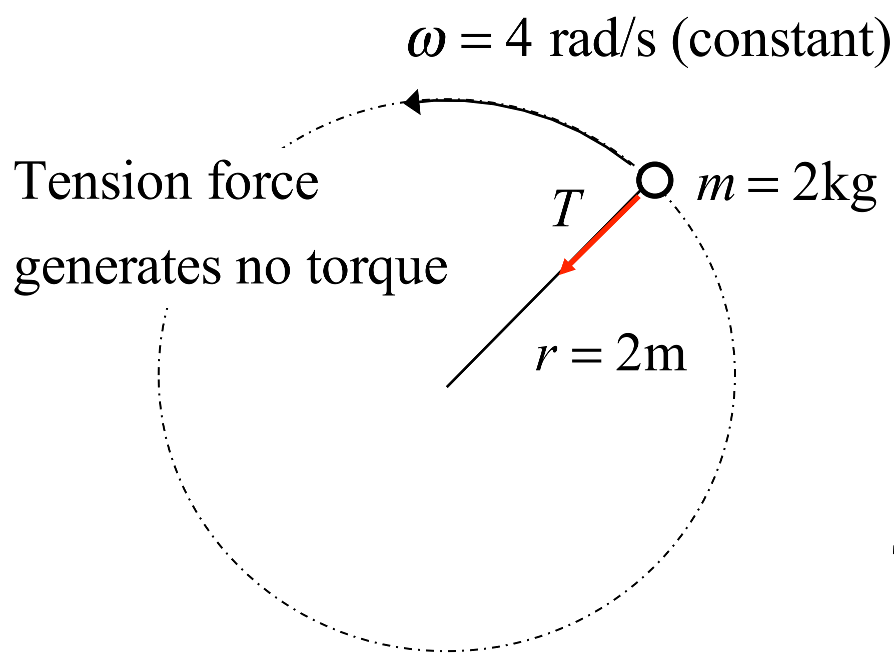
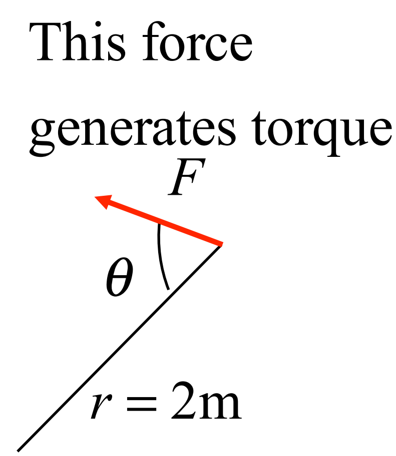


$\tau = F\ell$

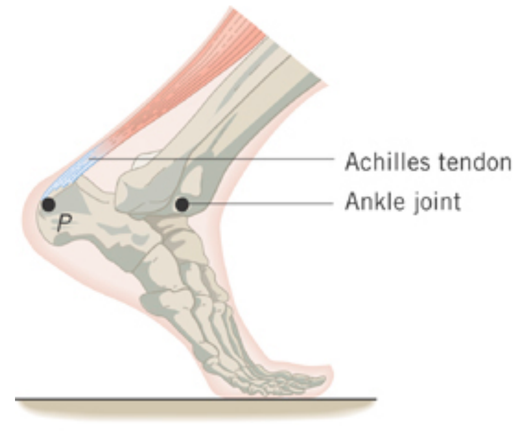
Angle between tension ($\vec{\mathbf{T}}$) and string is zero.

Lever arm is zero (or $\perp$ component of $\vec{\mathbf{T}}$ is zero).



Example 2 The Achilles Tendon

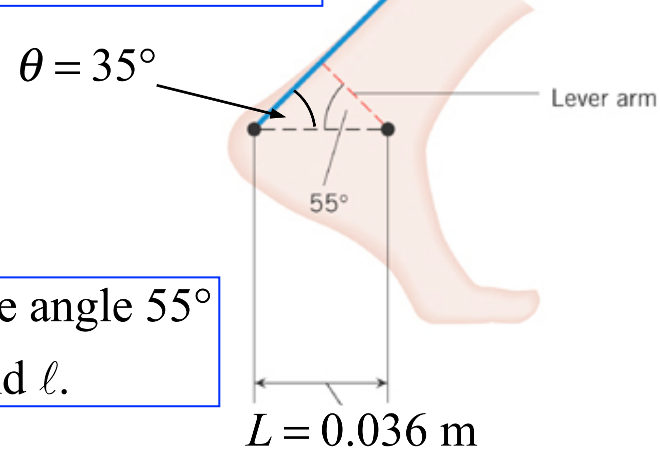
The tendon exerts a force of magnitude 720 N. Determine the torque (magnitude and direction) of this force about the ankle joint.



(a)

$$\tau = \left(F\sin\theta\right)L = (720\text{ N})(\sin 35°)(.036\text{ m})$$
$$= 15.0\text{ N}\cdot\text{m}$$

θ is the angle between $\vec{\mathbf{F}}$ and L



(b)

$$\tau = F\left(L\sin\theta\right) = (720\text{ N})(.036\text{ m})\sin 35°$$
$$= 15.0\text{ N}\cdot\text{m}$$

Example gave angle 55° between L and ℓ.

Direction is clockwise (–) around ankle joint
Torque vector $\tau = -15.0\text{ N}\cdot\text{m}$

9.2 Rigid Objects in Equilibrium

If a rigid body is in equilibrium, neither its linear motion nor its rotational motion changes.

$$a_x = a_y = 0 \qquad \alpha = 0$$

$$\sum F_x = 0 \quad \sum F_y = 0 \qquad \sum \tau = 0$$

All equilibrium problems use these equations – no net force and no net torque.

EQUILIBRIUM OF A RIGID BODY

A rigid body is in equilibrium if it has zero translational acceleration and zero angular acceleration. In equilibrium, the sum of the externally applied forces is zero, and the sum of the externally applied torques is zero.

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum \tau = 0$$

Note: constant linear speed or constant rotational speed are allowed for an object in equilibrium.

Reasoning Strategy

1. Select the object to which the equations for equilibrium are to be applied.

2. Draw a free-body diagram that shows all of the external forces acting on the object.

3. Choose a convenient set of *x*, *y* axes and resolve all forces into components that lie along these axes.

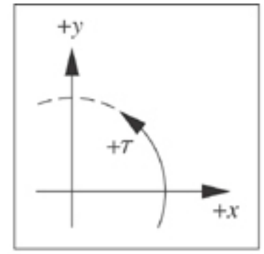
4. Apply the equations that specify the balance of forces at equilibrium. (Set the net force in the *x* and *y* directions equal to zero.)

5. Select a convenient axis of rotation. Set the sum of the torques about this axis equal to zero.

6. Solve the equations for the desired unknown quantities.

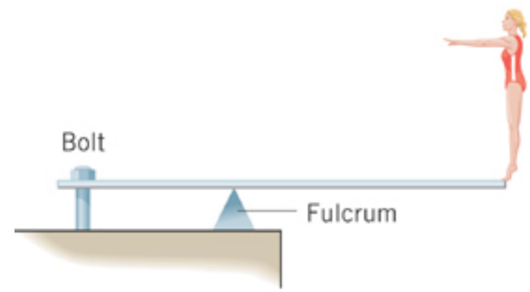
9.2 Rigid Objects in Equilibrium

Example 3 A Diving Board

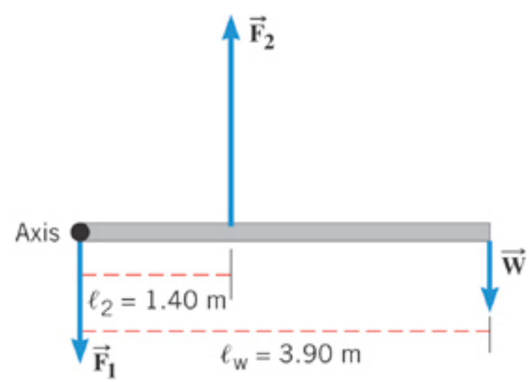


Counter-clockwise torque is positive

A woman whose weight is 530 N is poised at the right end of a diving board with length 3.90 m. The board has negligible weight and is supported by a fulcrum 1.40 m away from the left end. Find the forces that the bolt and the fulcrum exert on the board.



(a)



(b) Free-body diagram of the diving board

F_1 acts on rotation axis - produces no torque.

$$\sum \tau = 0 = F_2 \ell_2 - W \ell_W$$
$$F_2 = W(\ell_W / \ell_2) = 530\text{N}(3.9/1.4) = 1480\ \text{N}$$

$$\sum F_y = 0 = -F_1 + F_2 - W$$
$$F_1 = F_2 - W = (1480 - 530)\text{N} = 950\ \text{N}$$

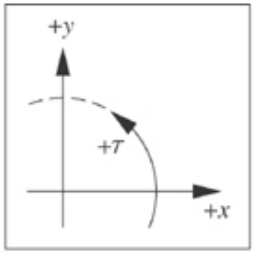
9.2 Rigid Objects in Equilibrium

Choice of pivot is arbitary (most convenient)

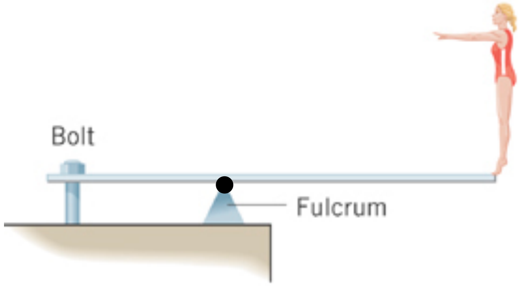
Pivot at fulcum: F_2 produces no torque.

$$\sum \tau = 0 = F_1 \ell_2 - W(\ell_W - \ell_2)$$
$$F_1 = W(\ell_W / \ell_2 - 1) = (530\text{N})(1.8) = 950\,\text{N}$$

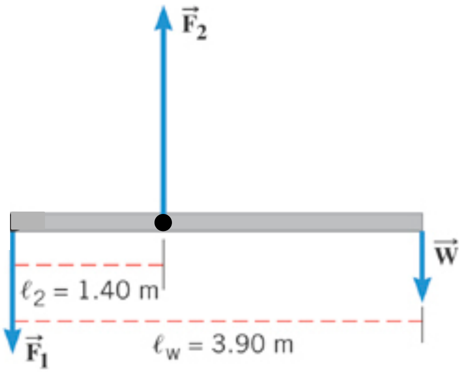
$$\sum F_y = 0 = -F_1 + F_2 - W$$
$$F_2 = F_1 + W = (950 + 530)\text{N} = 1480\ \text{N}$$



Counter-clockwise torque is positive



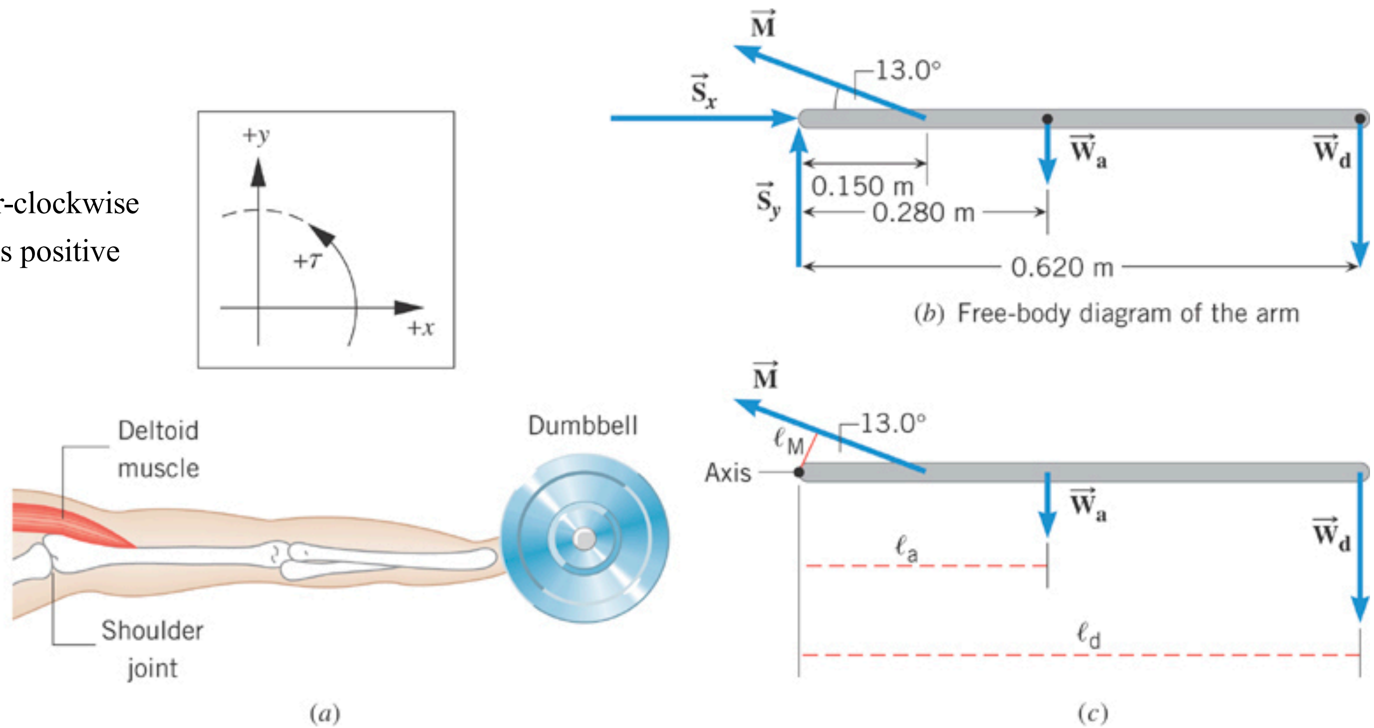
(a)



(b) Free-body diagram of the diving board

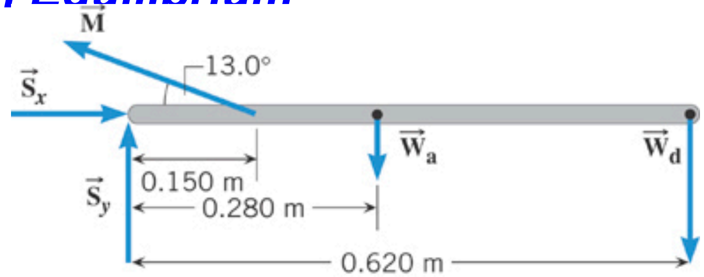
9.2 Rigid Objects in Equilibrium

Counter-clockwise torque is positive

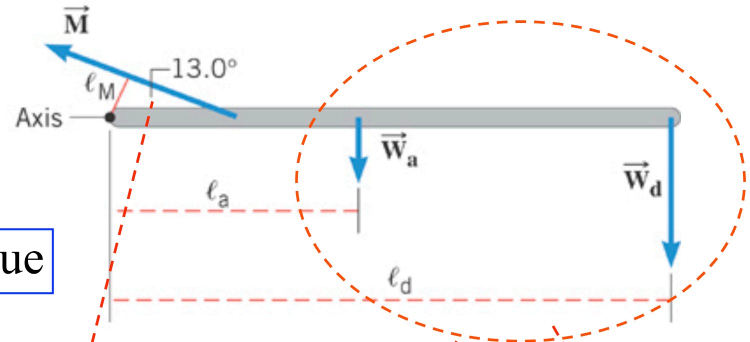


Example 5 Bodybuilding

The arm is horizontal and weighs 31.0 N. The deltoid muscle can supply 1840 N of force. What is the weight of the heaviest dumbell he can hold?



(*b*) Free-body diagram of the arm



(*c*)

positive torque

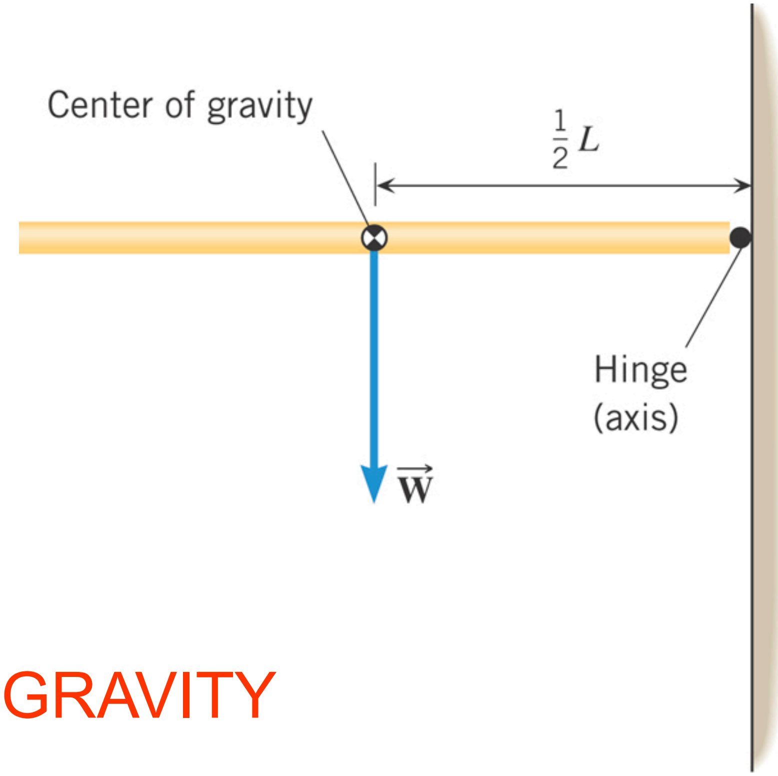
negative torques

$$\sum \tau = M\left(\sin 13°\right)L_M - W_a L_a - W_d L_d = 0$$

$$W_d = \left[+M\left(\sin 13°\right)L_M - W_a L_a\right]/L_d$$

$$= \left[1840\text{N}(.225)(0.15\text{m}) - 31\text{N}(0.28\text{m})\right]/0.62\text{m}$$

$$= 86.1\text{N}$$

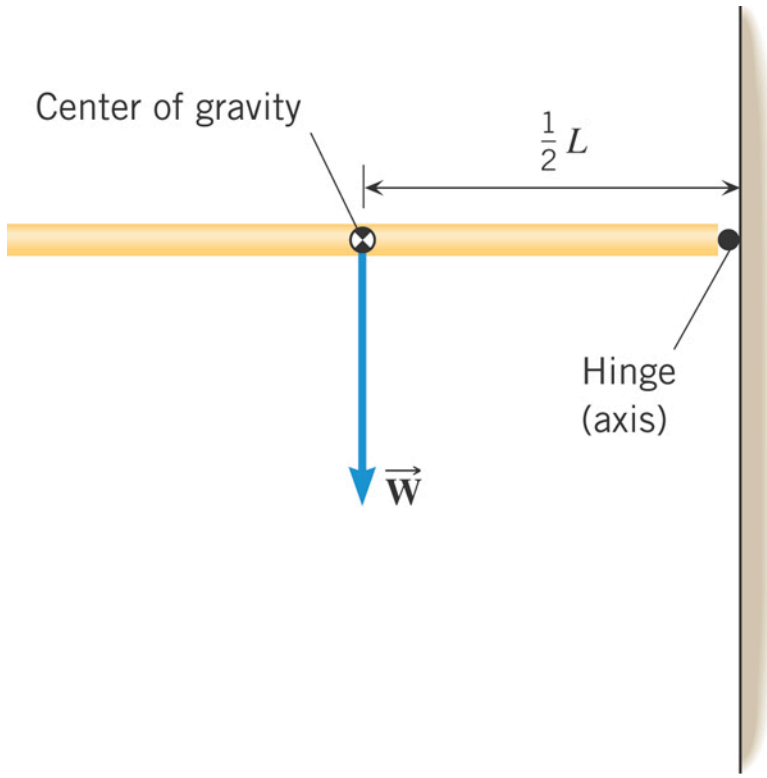


DEFINITION OF CENTER OF GRAVITY

The center of gravity of a rigid body is the point at which its weight can be considered to act when the torque due to the weight is being calculated.

9.3 Center of Gravity

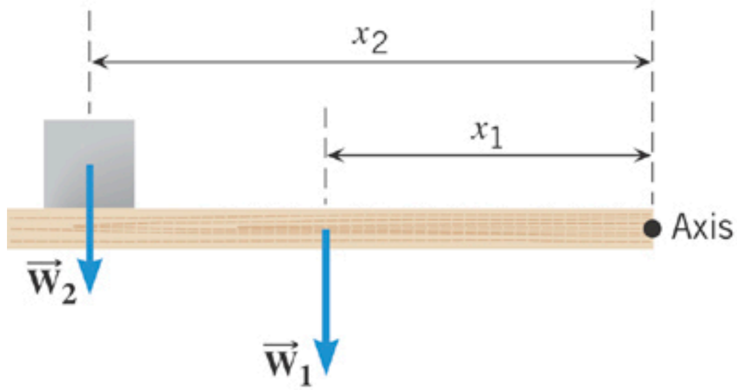
When an object has a symmetrical shape and its weight is distributed uniformly, the center of gravity lies at its geometrical center.



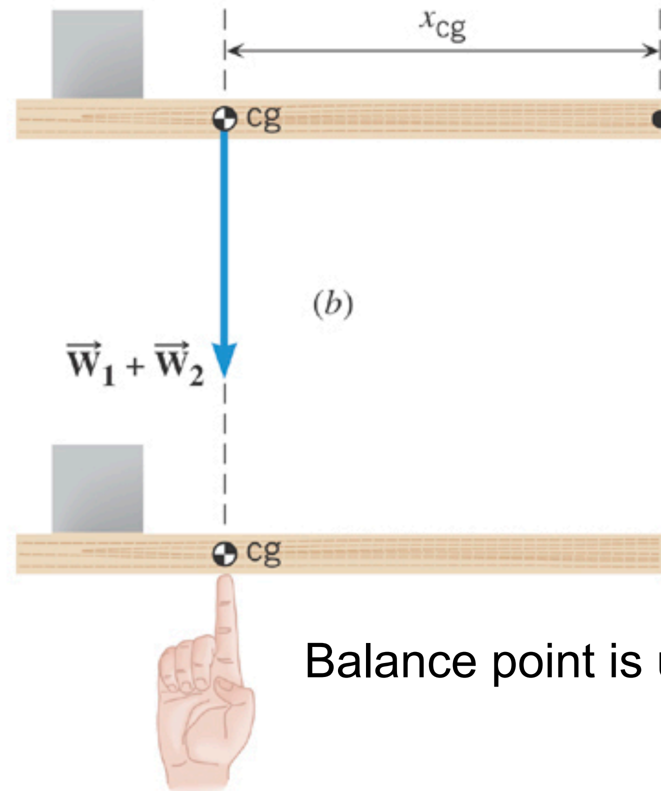
9.3 Center of Gravity

General Form of x_{cg}

$$x_{cg} = \frac{W_1 x_1 + W_2 x_2 + \cdots}{W_1 + W_2 \ + \cdots}$$



(a)



Balance point is under x_{cg}

(c)

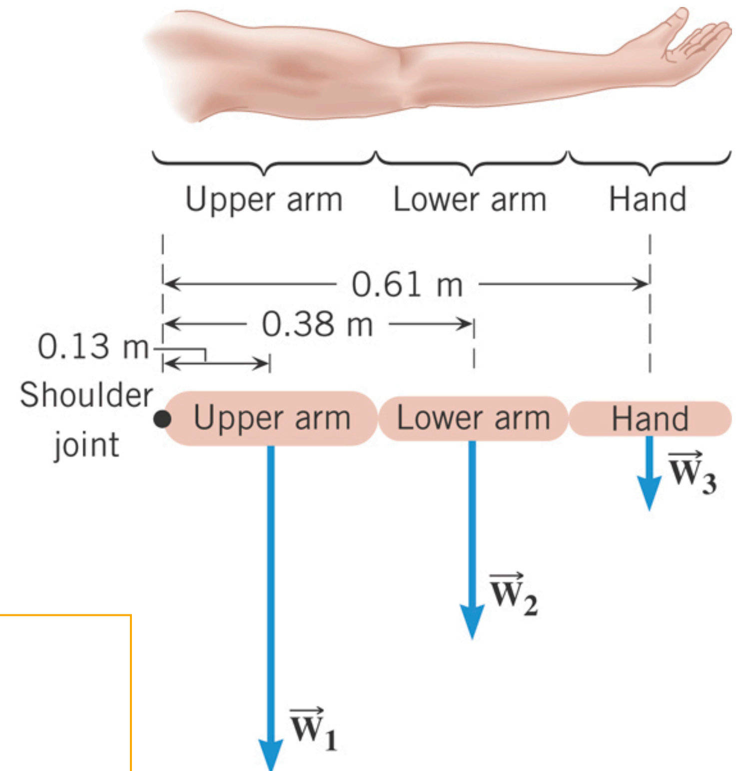
Center of Gravity, x_{cg} , for 2 masses

$$x_{cg} = \frac{W_1 x_1 + W_2 x_2}{W_1 + W_2}$$

Example 6 The Center of Gravity of an Arm

The horizontal arm is composed of three parts: the upper arm (17 N), the lower arm (11 N), and the hand (4.2 N).

Find the center of gravity of the arm relative to the shoulder joint.

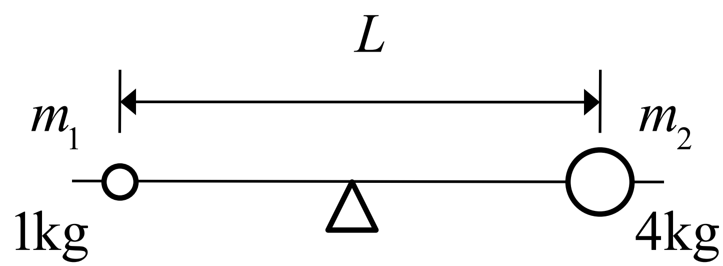


$$x_{cg} = \frac{W_1 x_1 + W_2 x_2 + W_3 x_3}{W_1 + W_2 + W_3}$$

$$= \frac{\left[17(0.13) + 11(0.38) + 4.2(0.61)\right]\mathrm{N \cdot m}}{(17 + 11 + 4.2)\ \mathrm{N}} = 0.28\ \mathrm{m}$$

Clicker Question 9.2 Torque and Equilibrium

A 4-kg ball and a 1-kg ball are positioned a distance L apart on a bar of negligible mass. How far from the 4-kg mass should the fulcum be placed to balance the bar?



a) $\frac{1}{2}L$

b) $\frac{1}{3}L$

c) $\frac{1}{4}L$

d) $\frac{1}{5}L$

e) $\frac{1}{6}L$

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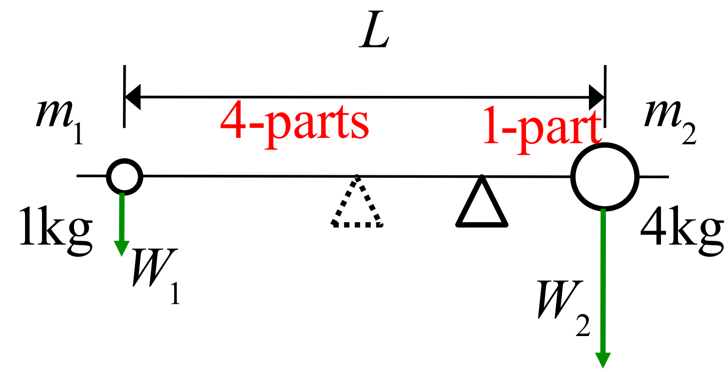

For equilibrium the sum of the torques must be zero

Need to separate length into 4 parts on 1-kg mass side and 1 part on the 4-kg mass side. Total is 5 parts.

Fulcum must be 1/5 of the total length from the 4-kg mass.

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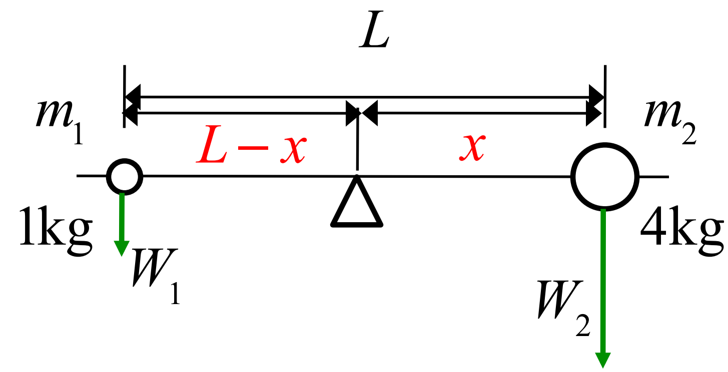


For equilibrium the sum of the torques must be zero

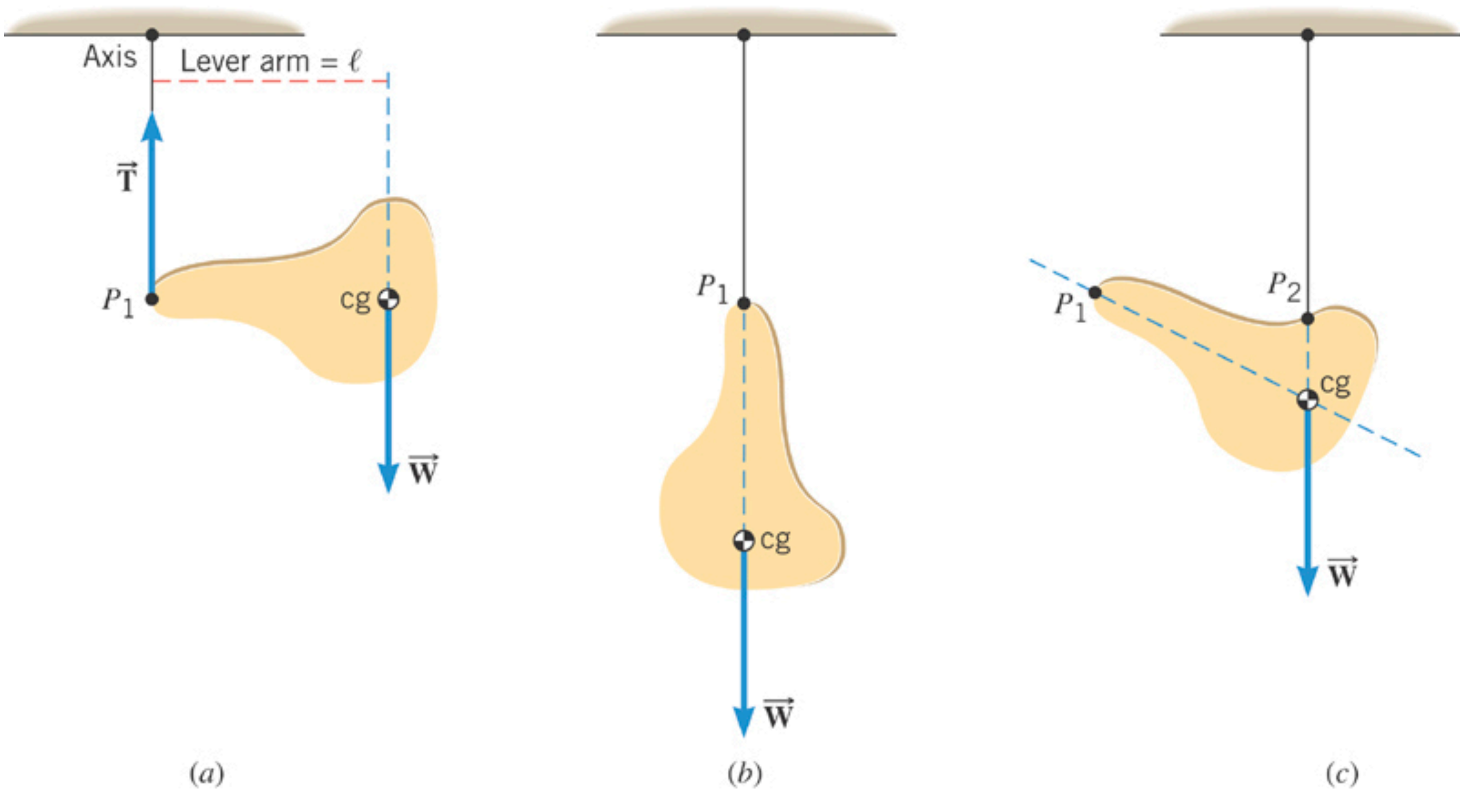
Let x be the distance of fulcum from 4-kg mass.

$$\sum \tau = 0 = m_1 g(L-x) + (-m_2 g x)$$

$$(m_1 + m_2)x = m_1 L$$

$$x = \frac{m_1}{(m_1+m_2)}L = \frac{1}{(1+4)}L = \frac{1}{5}L$$

9.3 Center of Gravity



Finding the center of gravity of an irregular shape.

9.4 Newton's Second Law for Rotational Motion About a Fixed Axis

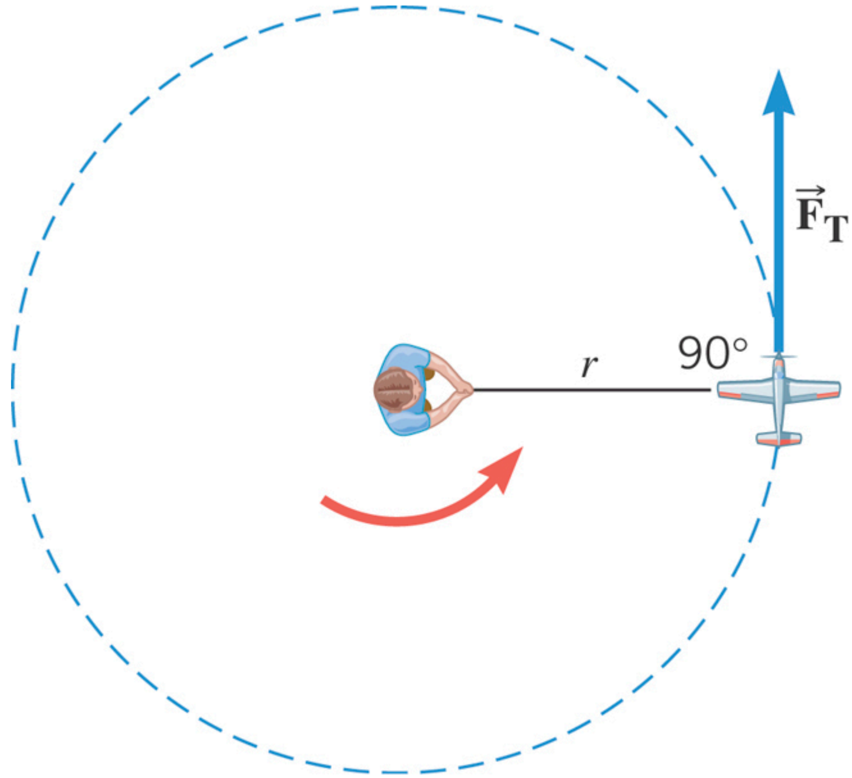
$$\tau = F_T r$$
$$= m a_T r$$
$$= m \alpha r^2$$
$$= \left(m r^2\right)\alpha$$
$$= I\alpha$$

$$\boxed{F_T = m a_T}$$

$$\boxed{a_T = r\alpha}$$

$$\boxed{\text{Let } I = m r^2}$$

Moment of Inertia

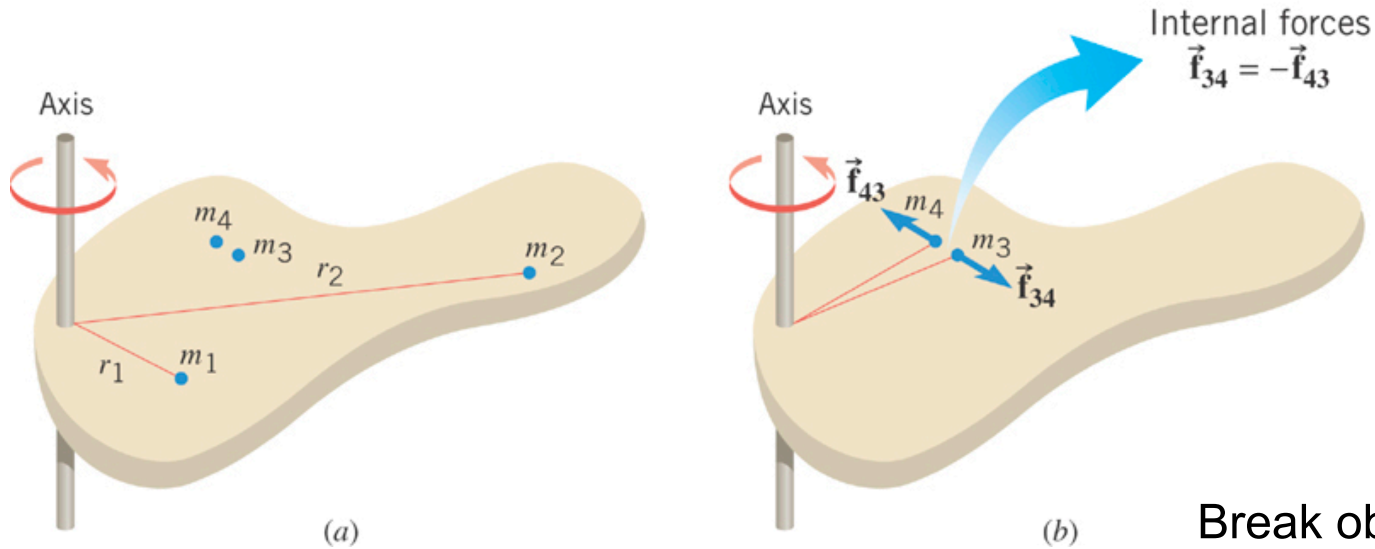


Moment of Inertia, $I = mr^2$, for a point-mass, m, at the end of a massless arm of length, r.

$$\boxed{\tau = I\alpha}$$

Newton's 2nd Law for rotations

9.4 Newton's Second Law for Rotational Motion About a Fixed Axis



Break object into N individual masses

$$\sum \tau = \sum \left(mr^2 \right) \alpha$$

Net external torque

Moment of inertia

$$\tau_1 = \left(m_1 r_1^2 \right) \alpha$$

$$\tau_2 = \left(m_2 r_2^2 \right) \alpha$$

$$\vdots$$

$$\tau_N = \left(m_N r_N^2 \right) \alpha$$

9.4 Newton's Second Law for Rotational Motion About a Fixed Axis

ROTATIONAL ANALOG OF NEWTON'S SECOND LAW FOR A RIGID BODY ROTATING ABOUT A FIXED AXIS

$$\text{Net external torque} = \begin{pmatrix} \text{Moment of} \\ \text{inertia} \end{pmatrix} \times \begin{pmatrix} \text{Angular} \\ \text{acceleration} \end{pmatrix}$$

$$\sum \tau = I\,\alpha$$

$$\boxed{I = \sum \left(m r^2 \right)}$$

Requirement: Angular acceleration must be expressed in radians/s^2.