

Chapter 9

Rotational Dynamics ***continued***

9.4 Newton's Second Law for Rotational Motion About a Fixed Axis

ROTATIONAL ANALOG OF NEWTON'S SECOND LAW FOR A RIGID BODY ROTATING ABOUT A FIXED AXIS

$$\text{Net external torque} = \left(\begin{array}{c} \text{Moment of} \\ \text{inertia} \end{array} \right) \times \left(\begin{array}{c} \text{Angular} \\ \text{acceleration} \end{array} \right)$$

$$\sum \tau = I \alpha$$

$$I = \sum (mr^2)$$

Point masses

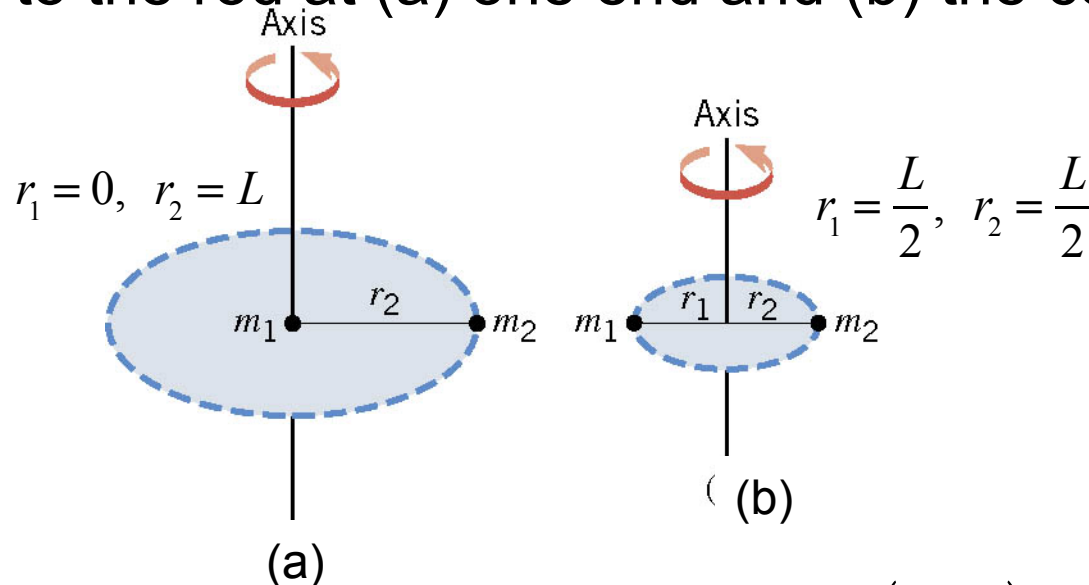
Requirement: Angular acceleration must be expressed in radians/s².

9.4 Newton's Second Law for Rotational Motion About a Fixed Axis

Moment of Inertia depends on axis of rotation.

Two particles each with mass, m , and are fixed at the ends of a thin rigid rod. The length of the rod is L . Find the moment of inertia when this object rotates relative to an axis that is perpendicular to the rod at (a) one end and (b) the center.

$$m_1 = m_2 = m$$



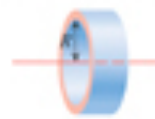
$$\begin{aligned} \text{(a)} \quad I &= \sum (mr^2) = m_1 r_1^2 + m_2 r_2^2 \\ &= m(0)^2 + m(L)^2 \\ &= mL^2 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad I &= \sum (mr^2) = m_1 r_1^2 + m_2 r_2^2 \\ &= m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 \\ &= \frac{1}{2} mL^2 \end{aligned}$$

9.4 Newton's Second Law for Rotational Motion About a Fixed Axis

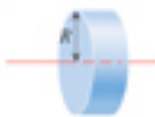
Table 9.1 Moments of Inertia I for Various Rigid Objects of Mass M

Thin-walled hollow cylinder or hoop



$$I = MR^2$$

Solid cylinder or disk



$$I = \frac{1}{2}MR^2$$

Thin rod, axis perpendicular to rod and passing through center



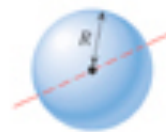
$$I = \frac{1}{12}ML^2$$

Thin rod, axis perpendicular to rod and passing through one end



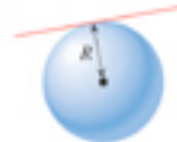
$$I = \frac{1}{3}ML^2$$

Solid sphere, axis through center



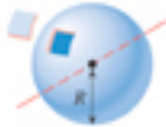
$$I = \frac{2}{5}MR^2$$

Solid sphere, axis tangent to surface



$$I = \frac{7}{5}MR^2$$

Thin-walled spherical shell, axis through center



$$I = \frac{2}{3}MR^2$$

Thin rectangular sheet, axis parallel to one edge and passing through center of other edge



$$I = \frac{1}{12}ML^2$$

Thin rectangular sheet, axis along one edge



$$I = \frac{1}{3}ML^2$$

$$I = MR^2$$

$$I = \frac{1}{2}MR^2$$

$$I = \frac{1}{12}MR^2$$

$$I = \frac{1}{3}MR^2$$

$$I = \frac{2}{5}MR^2$$

$$I = \frac{7}{5}MR^2$$

$$I = \frac{2}{3}MR^2$$

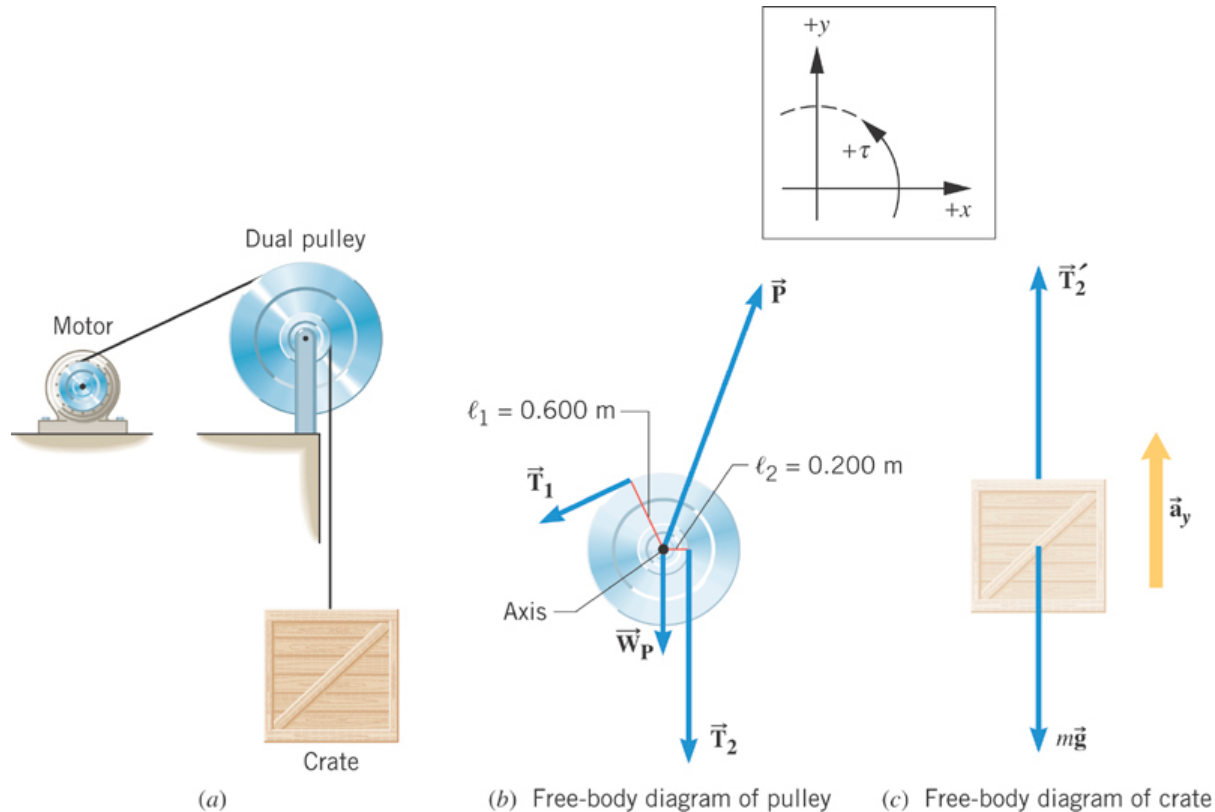
$$I = \frac{1}{12}MR^2$$

$$I = \frac{1}{3}MR^2$$

9.4 Newton's Second Law for Rotational Motion About a Fixed Axis

Example 12 Hoisting a Crate

The combined moment of inertia of the dual pulley is $50.0 \text{ kg}\cdot\text{m}^2$. The crate weighs 4420 N . A tension of 2150 N is maintained in the cable attached to the motor. Find the angular acceleration of the dual pulley.



9.4 Newton's Second Law for Rotational Motion About a Fixed Axis

2nd law for linear motion of crate

$$\sum F_y = T_2 - mg = ma_y \quad a_y = \ell_2 \alpha$$

2nd law for rotation of the pulley

$$\sum \tau = T_1 \ell_1 - T_2 \ell_2 = I \alpha$$

$$I = 46 \text{ kg} \cdot \text{m}^2$$

$$mg = 4420 \text{ N}$$

$$T_1 = 2150 \text{ N}$$

$$T_2 - mg = m \ell_2 \alpha$$

$$T_1 \ell_1 - T_2 \ell_2 = I \alpha$$

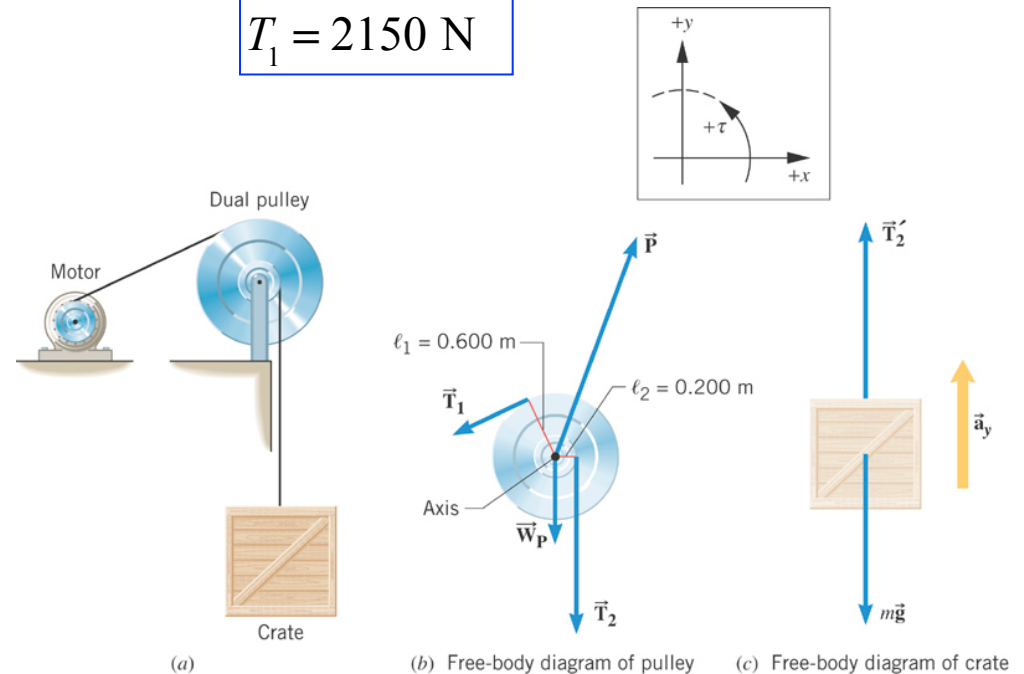
$$T_2 = m \ell_2 \alpha + mg$$

$$T_2 = \frac{T_1 \ell_1 - I \alpha}{\ell_2}$$

$$m \ell_2^2 \alpha + I \alpha = T_1 \ell_1 - mg \ell_2$$

$$\alpha = \frac{T_1 \ell_1 - mg \ell_2}{m \ell_2^2 + I} = 6.3 \text{ rad/s}^2$$

$$T_2 = m \ell_2 \alpha + mg = (451(.6)(6.3) + 4420) \text{ N} = 6125 \text{ N}$$



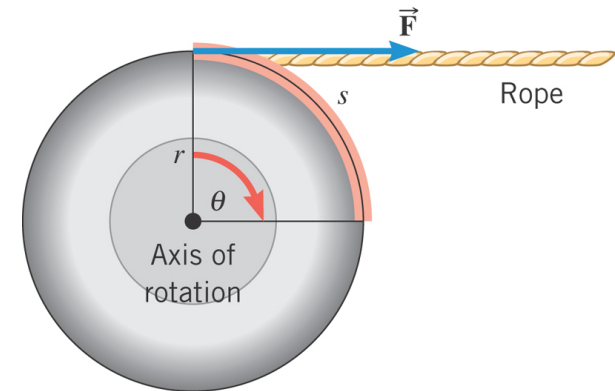
$$\vec{T}_2' = +T_2$$

$$\vec{T}_2 = -T_2$$

9.5 Rotational Work and Energy

Work
rotating
a mass:

$$\begin{aligned} W &= Fs & s &= r\theta \\ &= Fr\theta & Fr &= \tau \\ &= \tau\theta \end{aligned}$$



DEFINITION OF ROTATIONAL WORK

The rotational work done by a constant torque in turning an object through an angle is

$$W_R = \tau\theta$$

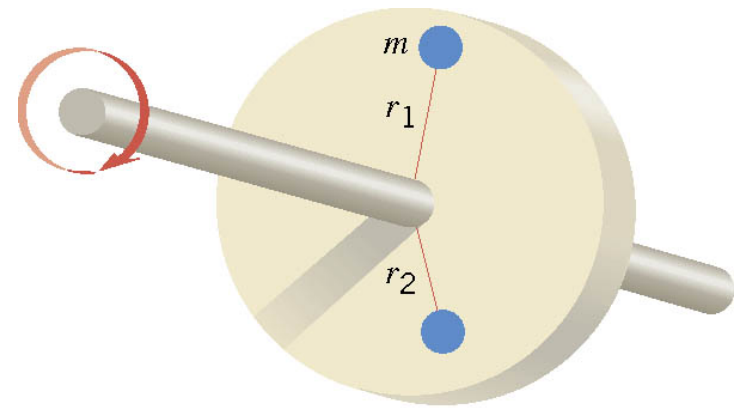
Requirement: The angle must be expressed in radians.

SI Unit of Rotational Work: joule (J)

9.5 Rotational Work and Energy

Kinetic Energy of a rotating **one** point mass

$$\begin{aligned} KE &= \frac{1}{2} m v_T^2 & v_T &= r\omega \\ &= \frac{1}{2} m r^2 \omega^2 \end{aligned}$$



Kinetic Energy of many rotating point masses

$$KE = \sum \left(\frac{1}{2} m r^2 \omega^2 \right) = \frac{1}{2} \left(\sum m r^2 \right) \omega^2 = \frac{1}{2} I \omega^2$$

DEFINITION OF ROTATIONAL KINETIC ENERGY

The rotational kinetic energy of a rigid rotating object is

$$KE_R = \frac{1}{2} I \omega^2$$

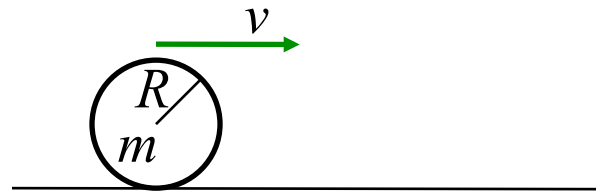
Requirement: The angular speed must be expressed in rad/s.

SI Unit of Rotational Kinetic Energy: joule (J)

Clicker Question 9.3 Rotational Energy

A bowling ball is rolling without slipping at constant speed toward the pins on a lane. What percentage of the ball's total kinetic energy is translational kinetic energy?

- a) 50%
- b) 71%
- c) 46%
- d) 29%
- e) 33%



Hint: Express both KE types with $\omega^2 R^2$

$$KE = \frac{1}{2}mv^2, \text{ with } v = \omega R$$

$$KE_{rot} = \frac{1}{2}I\omega^2, \text{ with } I = \frac{2}{5}mR^2$$

$$\text{fraction} = \frac{KE}{KE + KE_{rot}}$$

Clicker Question 9.3 Rotational Energy

A bowling ball is rolling without slipping at constant speed toward the pins on a lane. What percentage of the ball's total kinetic energy is translational kinetic energy?

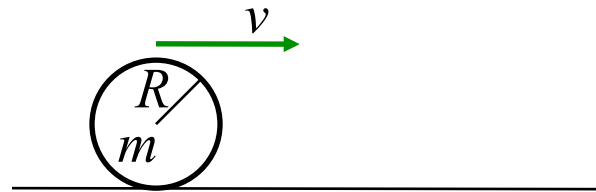
a) 50%

b) 71%

c) 46%

d) 29%

e) 33%



$$KE = \frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 R^2, \text{ because } v = \omega R$$

$$KE_{rot} = \frac{1}{2} I\omega^2 = \frac{1}{5} m\omega^2 R^2, \text{ because } I = \frac{2}{5} mR^2$$

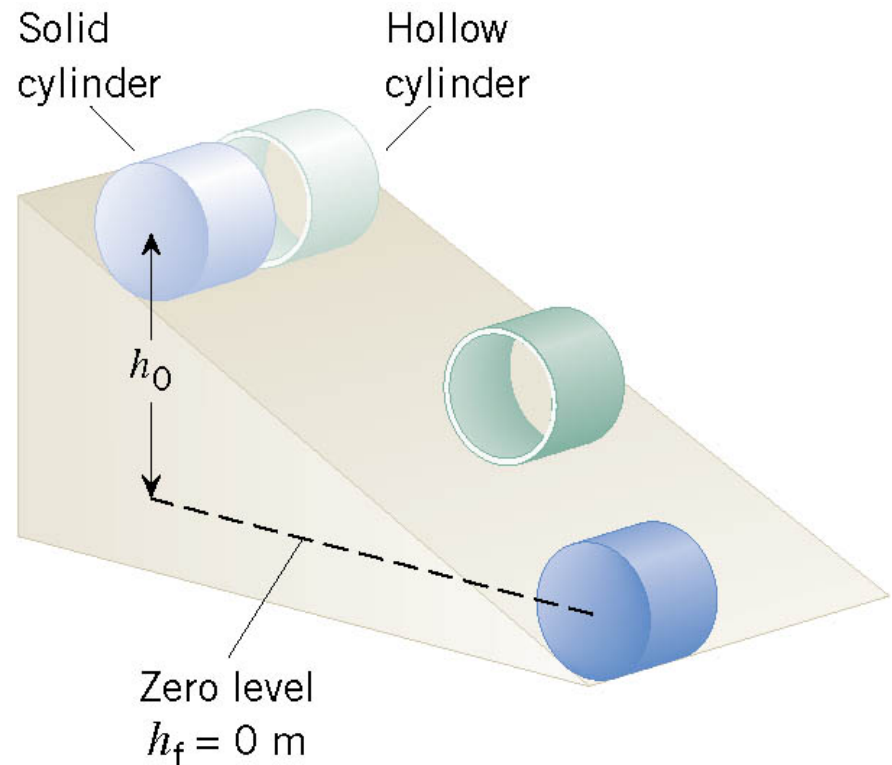
$$\frac{KE}{KE + KE_{rot}} = \frac{\frac{1}{2}}{\frac{1}{5} + \frac{1}{2}} = \frac{\frac{1}{2}}{\frac{2}{10} + \frac{5}{10}} = \frac{5}{7} = 71\%$$

9.5 Rotational Work and Energy

Example 13 Rolling Cylinders

A thin-walled hollow cylinder (mass = m_h , radius = r_h) and a solid cylinder (mass = m_s , radius = r_s) start from rest at the top of an incline.

Determine which cylinder has the greatest translational speed upon reaching the bottom.



9.5 Rotational Work and Energy

Total Energy = Kinetic Energy + Rotational Energy + Potential Energy

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh$$

ENERGY CONSERVATION $E_f = E_0$

$$\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + mgh_f = \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2 + mgh_0$$

$$\omega_f = v_f / r$$

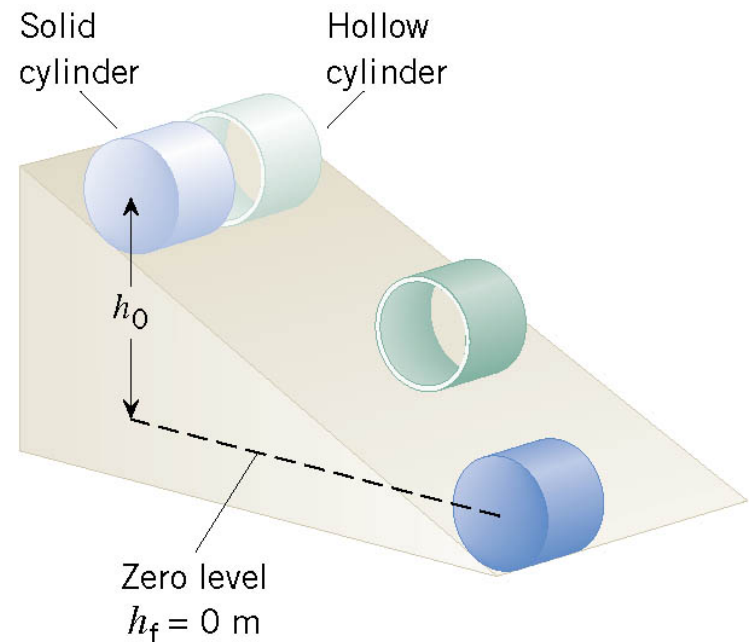
$$\frac{1}{2}mv_f^2 + \frac{1}{2}Iv_f^2 / r^2 = mgh_0$$

$$v_f^2(m + I/r^2) = 2mgh_0$$

$$v_f = \sqrt{\frac{2mgh_0}{m + I/r^2}}$$

The cylinder with the **smaller** moment of inertia will have a **greater** final translational speed.

Same mass for cylinder and hoop



9.6 *Angular Momentum*

DEFINITION OF ANGULAR MOMENTUM

The angular momentum L of a body rotating about a fixed axis is the product of the body's moment of inertia and its angular velocity with respect to that axis:

$$L = I\omega$$

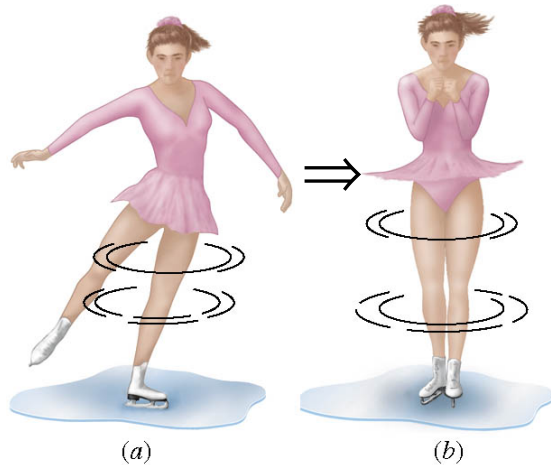
Requirement: The angular speed must be expressed in rad/s.

SI Unit of Angular Momentum: $\text{kg}\cdot\text{m}^2/\text{s}$

9.6 Angular Momentum

PRINCIPLE OF CONSERVATION OF ANGULAR MOMENTUM

The angular momentum of a system remains constant (is conserved) if the net external torque acting on the system is zero.



Moment of Inertia
decreases

$$I = \sum mr^2, \quad r_f < r_i$$
$$I_f < I_i$$
$$\frac{I_i}{I_f} > 1$$

Angular momentum, L

$$L_i = I_i \omega_i; \quad L_f = I_f \omega_f$$

No external torque

$\Rightarrow$ Angular momentum conserved

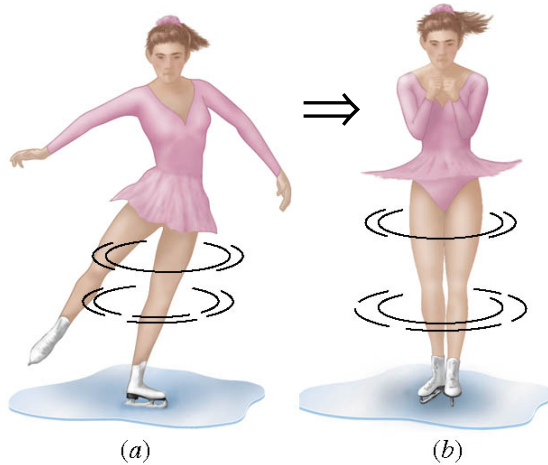
$$L_f = L_i$$

$$I_f \omega_f = I_i \omega_i$$

$$\omega_f = \frac{I_i}{I_f} \omega_i; \quad \frac{I_i}{I_f} > 1$$

$\omega_f > \omega_i$ (angular speed increases)

9.6 Angular Momentum



From Angular Momentum Conservation

$$\omega_f = \left(I_i / I_f \right) \omega_i$$

because $I_i / I_f > 1$

Angular velocity increases

Is Energy conserved?

$$\begin{aligned} KE_f &= \frac{1}{2} I_f \omega_f^2 \\ &= \frac{1}{2} I_f \left(I_i / I_f \right)^2 \omega_i^2 \\ &= \left(I_i / I_f \right) \left(\frac{1}{2} I_i \omega_i^2 \right) \quad KE_i = \frac{1}{2} I_i \omega_i^2; \\ &= \left(I_i / I_f \right) KE_i \Rightarrow \text{Kinetic Energy increases} \end{aligned}$$

Energy is **not conserved** because pulling in the arms does
(NC) work on their mass and **increases the kinetic energy** of rotation

Rotational Dynamics Summary

linear rotational

x

θ

v

ω

a

α

m

$I = mr^2$ (point m)

F

$\tau = Fr \sin \theta$

p

$L = I\omega$

linear

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

$$W = Fs \cos \theta$$

$$KE = \frac{1}{2}mv^2$$

$$W \Rightarrow \Delta KE$$

$$\vec{\mathbf{F}}\Delta t \Rightarrow \Delta \vec{\mathbf{p}}$$

rotational

$$\vec{\tau} = I\vec{\alpha}$$

$$\vec{L} = I\vec{\omega}$$

$$W_{rot} = \tau\theta$$

$$KE_{rot} = \frac{1}{2}I\omega^2$$

$$W_{rot} \Rightarrow \Delta KE_{rot}$$

$$\vec{\tau}\Delta t \Rightarrow \Delta \vec{L}$$

Gravitational

$$PE_G = mgh$$

Conserved: $KE + PE$

Conservation laws

If $W_{NC} = 0$,

$KE + PE$

If $\mathbf{F}_{ext} = 0$,

$\mathbf{P} = \sum \mathbf{p}$

If $\tau_{ext} = 0$,

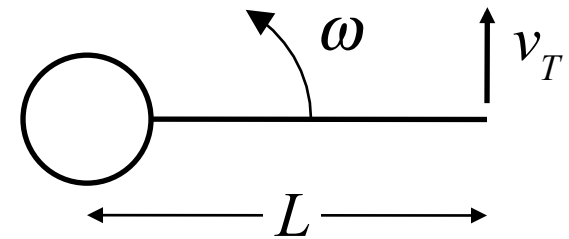
$L = I\omega$

8.37 Length of nylon rotates around one end. Rotation angular speed, 47 rev/s. Tip has tangential speed of 54 m/s . What is length of nylon string?

$$\omega = 47 \text{ rev/s} = 2\pi(47) \text{ rad/s} = 295 \text{ rad/s}$$

$$v_T = 54 \text{ m/s}, v_T = \omega r = \omega L$$

$$L = v_T / \omega = (54 \text{ m/s}) / (295 \text{ rad/s}) = 0.18 \text{ m}$$



8.51 Sun moves in circular orbit, radius r , around the center of the galaxy at an angular speed, ω . a) tangential speed of sun? b) centripetal force?

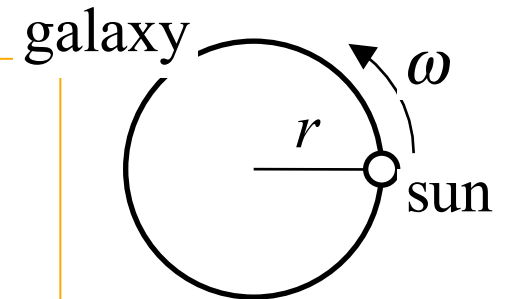
$$r = (2.3 \times 10^4 \text{ l-yr}) 9.5 \times 10^{15} \text{ m/l-yr} = 2.2 \times 10^{20} \text{ m}$$

$$\omega = 1.1 \times 10^{-15} \text{ rad/s}, m = 2 \times 10^{30} \text{ kg}$$

$$\text{a) } v_T = \omega r = 2.4 \times 10^5 \text{ m/s}$$

$$\text{b) } F = ma_c = m(v_T^2 / r) = m\omega^2 r$$

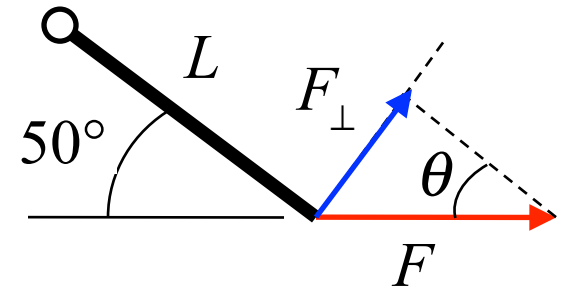
$$= (2 \times 10^{30}) (1.1 \times 10^{-15})^2 (2.2 \times 10^{20} \text{ m}) = 5.6 \times 10^{20} \text{ N}$$



9.3 Wrench length L . Hand force F , to get torque 45 Nm?

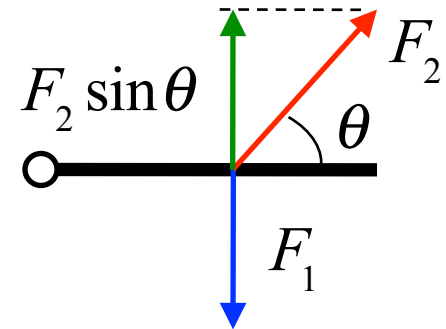
$$\tau = F_{\perp} L = (F \sin \theta) L$$

$$F = \tau / L \sin \theta = 45 \text{ Nm} / (0.28 \text{ m}) \sin 50^{\circ} \\ = 210 \text{ N}$$



9.9 $F_1 = 38 \text{ N}$ perpendicular, $F_2 = 55 \text{ N}$ at angle θ .
What is angle of F_2 for net torque = zero?

$$\tau_{\text{Net}} = (F_2 \sin \theta) L - F_1 L = 0 \\ \sin \theta = F_1 / F_2 = 38 / 55 = 0.69 \\ \theta = 43.7^{\circ}$$



9.23 Board against wall, coefficient of friction of ground 0.650. What is smallest angle without slipping? (See *Example 4*)

Forces:

$$G_y = W$$

$$P = G_x = \mu G_y = \mu W$$

Torque:

$$\tau_P = +P_{\perp} L = (P \sin \theta) L$$

$$\tau_W = -W_{\perp} (L/2) = -(W \cos \theta) (L/2)$$

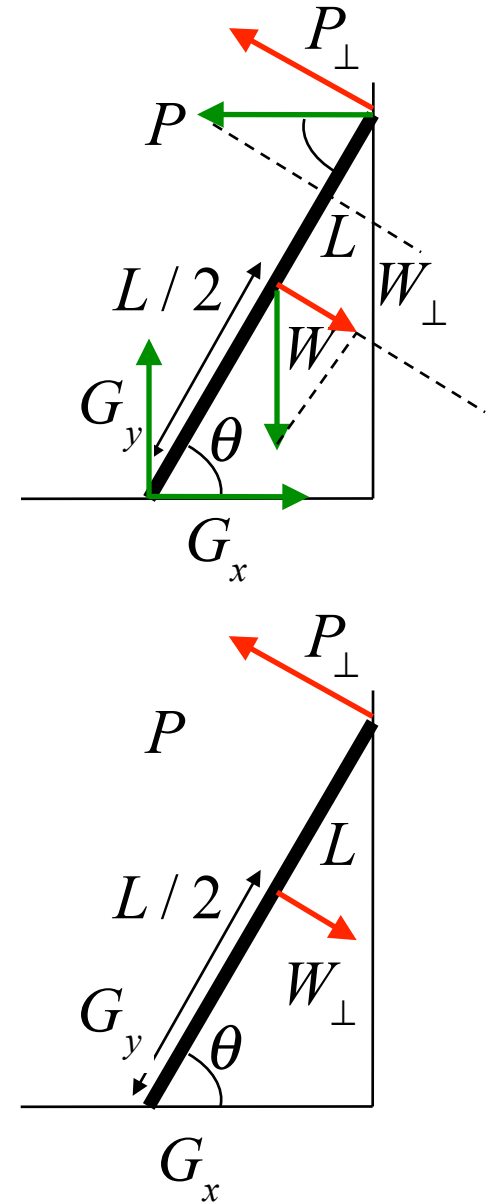
$$\tau_W + \tau_P = 0 \Rightarrow (P \sin \theta) = (W \cos \theta) / 2$$

$$W = 2P \sin \theta / \cos \theta$$

$$W = 2P \sin \theta / \cos \theta = 2\mu W \tan \theta$$

$$\tan \theta = 1/(2\mu) = 1/(1.3) = 0.77$$

$$\theta = 37.6^\circ$$



9.51 Energy $1.2 \times 10^9 \text{ J}$. Flywheel $M = 13 \text{ kg}$, $R = 0.3\text{m}$.

$$\begin{aligned} KE &= \frac{1}{2} I \omega^2; \quad I_{\text{disk}} = \frac{1}{2} MR^2 \\ &= \frac{1}{2} \left(\frac{1}{2} MR^2 \right) \omega^2 \\ \omega^2 &= \frac{4KE}{MR^2} = \frac{4.8 \times 10^9 \text{ J}}{(13\text{kg})(0.3\text{m})^2} \\ \omega &= \sqrt{4.1 \times 10^9} = 6.4 \times 10^4 \text{ rad/s} \\ &= 6.4 \times 10^4 \text{ rad/s} \left(\text{rev}/(2\pi) \text{ rad} \right) \\ &= 1.02 \times 10^4 \text{ rev/s} \left(60 \text{ s/min} \right) = 6.1 \times 10^5 \text{ rpm} \end{aligned}$$