Chapter 10

Simple Harmonic Motion and Elasticity

continued

Spring constants & oscillations

Hooke's Law $\boxed{F_A = kx}$ Displacement proportional to applied force

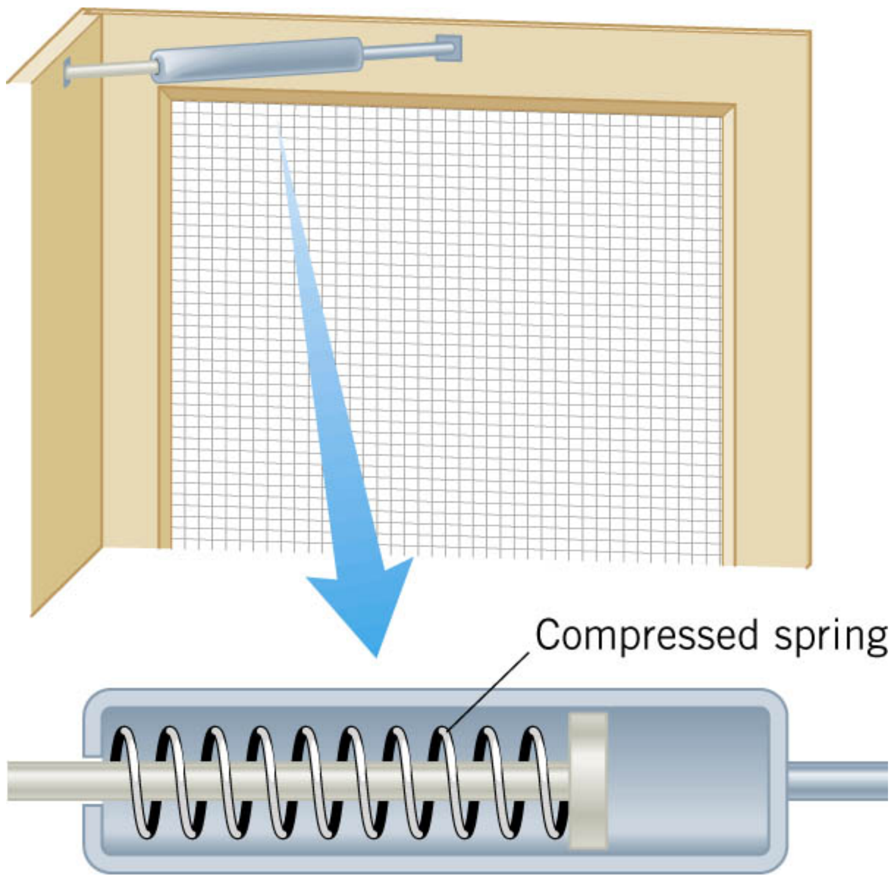
Oscillations $\boxed{\omega = \sqrt{\frac{k}{m}}}$ Angular frequency $(\omega = 2\pi f = 2\pi/T)$

position: $\boxed{x = A\cos(\omega t)}$

velocity: $\boxed{v_x = -\underbrace{A\omega}_{v_{\max}}\sin(\omega t)}$

acceleration: $\boxed{a_x = -\underbrace{A\omega^2}_{a_{\max}}\cos\omega t}$

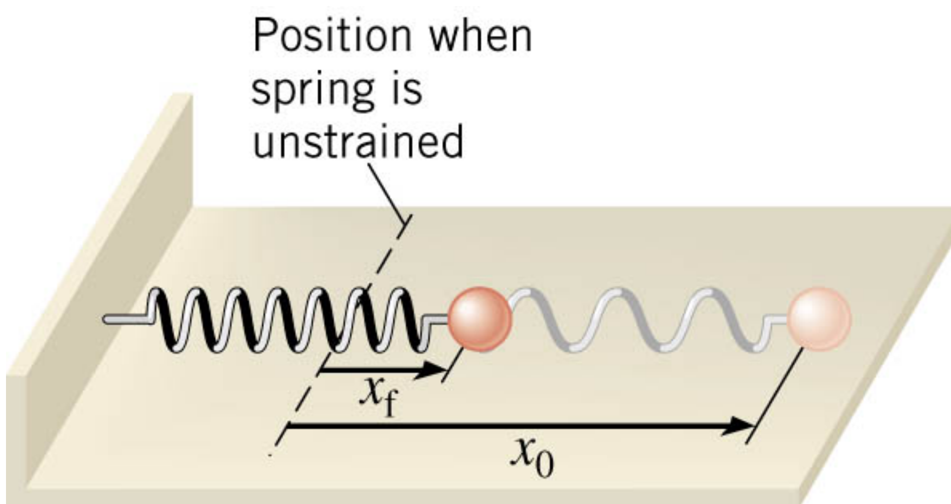
A compressed spring can do work.



A compressed spring can do work.

$$W_{\text{elastic}} = \left(\bar{F}\cos\theta\right)s = \tfrac{1}{2}\left(kx_o + kx_f\right)\cos 0^\circ\left(x_o - x_f\right)$$

$$W_{\text{elastic}} = \tfrac{1}{2}kx_o^2 - \tfrac{1}{2}kx_f^2$$

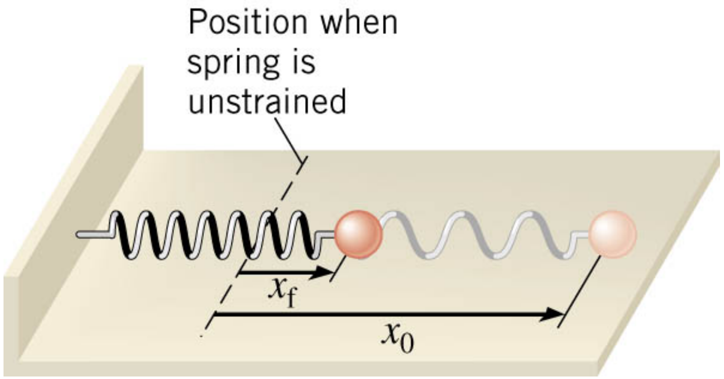


DEFINITION OF ELASTIC POTENTIAL ENERGY

The elastic potential energy is the energy that a spring has by virtue of being stretched or compressed. For an ideal spring, the elastic potential energy is

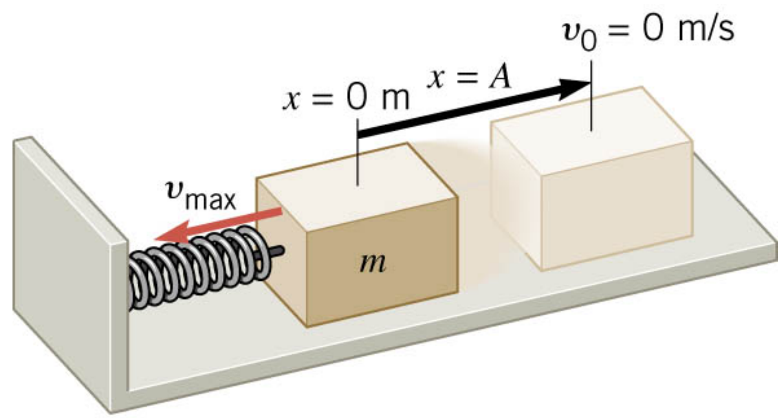
$$\mathrm{PE}_{\text{elastic}} = \tfrac{1}{2}kx^2$$

SI Unit of Elastic Potential Energy: joule (J)

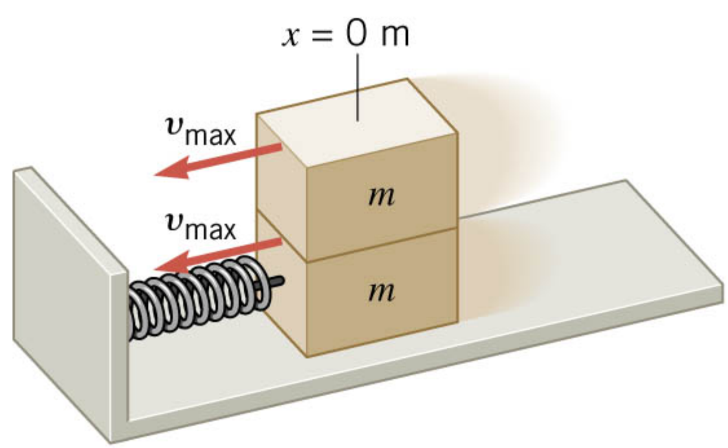


Conceptual Example 8 Changing the Mass of a Simple Harmonic Oscilator

The box rests on a horizontal, frictionless surface. The spring is stretched to x=A and released. When the box is passing through x=0, a second box of the same mass is attached to it. Discuss what happens to the (a) maximum speed (b) amplitude (c) angular frequency.



(a)



(b)

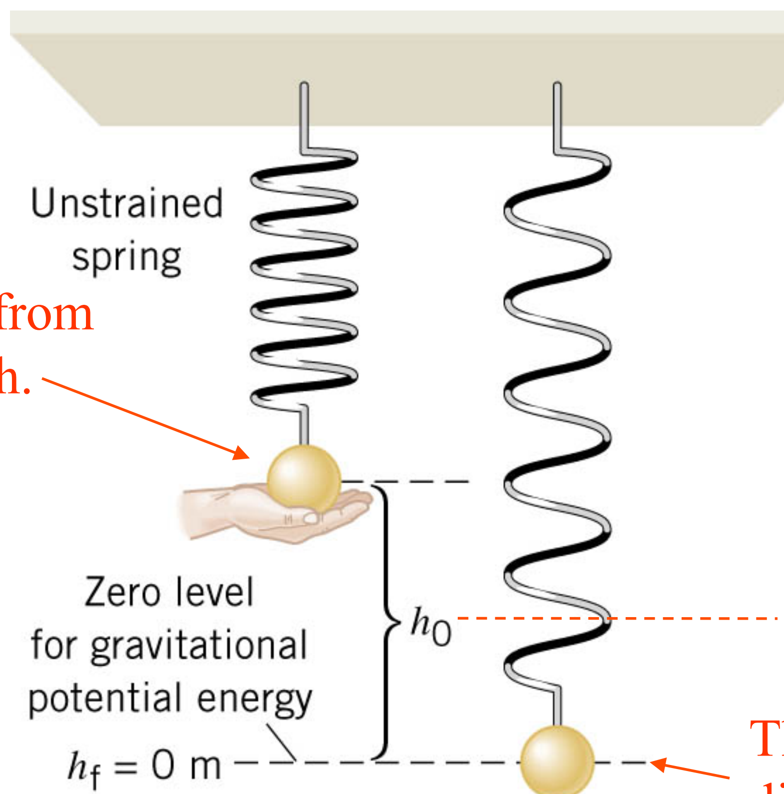
a) When 1st box reaches maximum velocity, second box added at the same velocity

In homework, the mass is added when mass reaches maximum displacement, and velocity is zero.

Example 8 Changing the Mass of a Simple Harmonic Oscillator

A 0.20-kg ball is attached to a vertical spring. The spring constant is 28 N/m. When released from rest, how far does the ball fall before being brought to a momentary stop by the spring?

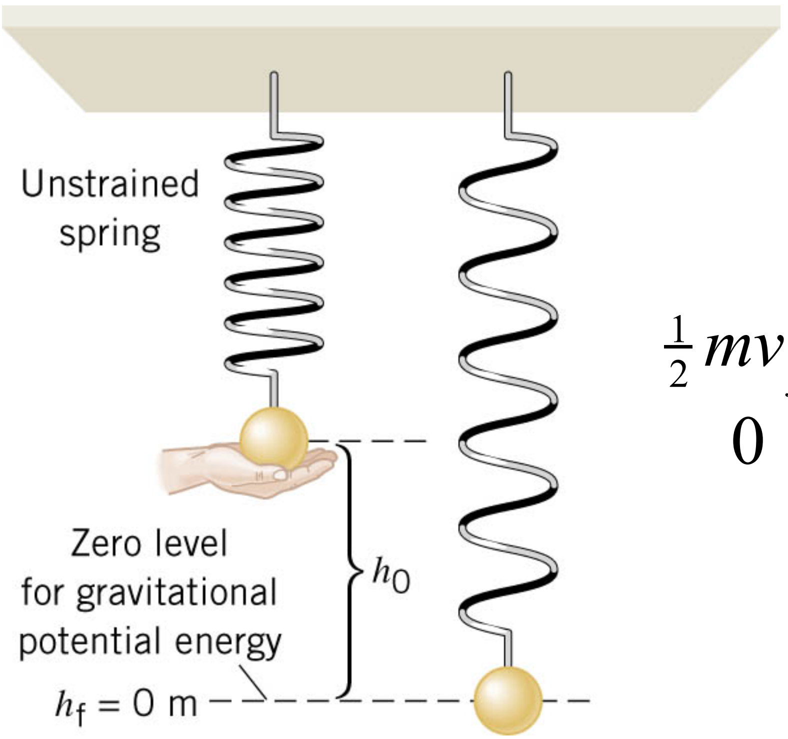
The ball is releasted from the unstretched length.



The ball reaches maximum speed at the equilibrium point.

The ball reaches maximum displacement when the velocity reaches zero.

After release, only conservative forces act.



Energy Conservation

$$E_f = E_o$$

$$\underbrace{\tfrac{1}{2}mv_f^2}_{0} + \underbrace{mgh_f}_{0} + \tfrac{1}{2}ky_f^2 = \underbrace{\tfrac{1}{2}mv_o^2}_{0} + mgh_o + \underbrace{\tfrac{1}{2}ky_o^2}_{0}$$

$$\tfrac{1}{2}kh_o^2 = mgh_o$$

CYU: Gravitational potential energy converted to elastic potential energy

$$h_o = \frac{2mg}{k} = \frac{2(0.20\text{ kg})(9.8\text{ m/s}^2)}{28\text{ N/m}} = 0.14\text{ m}$$

Clicker Question 10.1

A short spring with a spring constant of 980 N/m is compressed by 0.1 m. How high above the starting point will a 0.2 kg mass rise if fired vertically by this spring?

$$PE_S = \tfrac{1}{2}kx^2,\ PE_G = mgh$$

a) 20 m

b) 1.5 m

c) 2.5 m

d) 5.0 m

e) 100 m

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$$PE_S = \tfrac{1}{2}kx^2, \; PE_G = mgh$$

$$E_f = E_0$$

$$KE_f + PE_f = KE_0 + PE_0$$

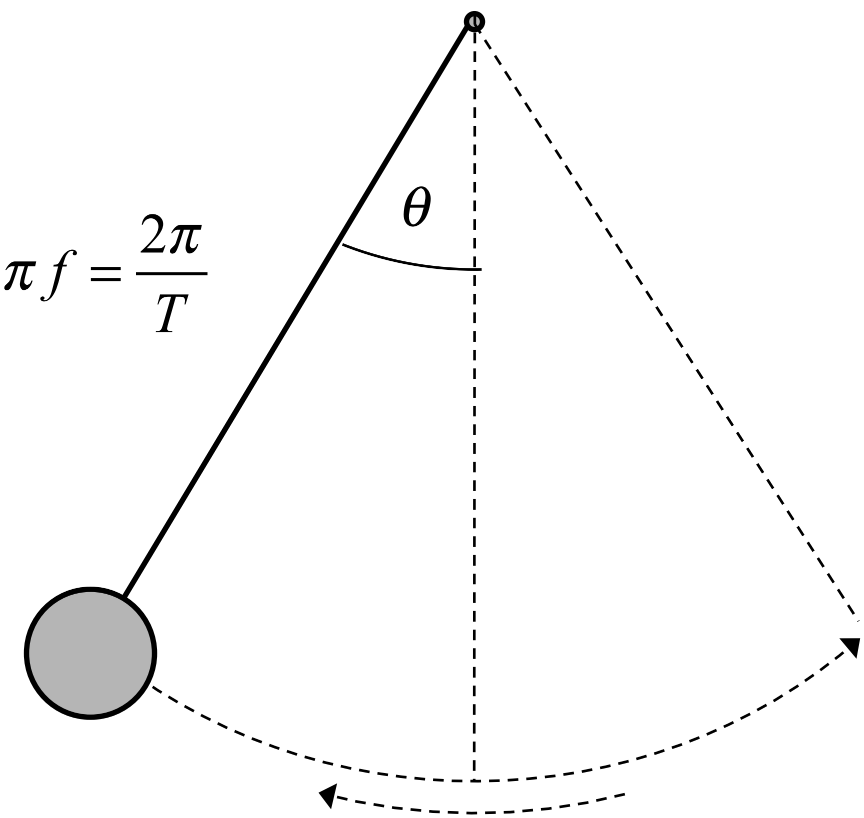
$$0 \; + \; mgh \; = \; 0 \; + \; \tfrac{1}{2}kx^2$$

$$h = \frac{kx^2}{2mg} = \frac{(980\ \mathrm{N/m^2})(0.1\mathrm{m})^2}{2(0.2\ \mathrm{kg})(9.8\ \mathrm{m/s^2})}$$

$$= 2.5\ \mathrm{m}$$

A ***simple pendulum*** consists of a particle attached to a frictionless pivot by a cable of negligible mass.

$$\omega = 2\pi f = \frac{2\pi}{T}$$



Angular frequency

$$\omega = \sqrt{\frac{g}{L}} \qquad \text{(small angles only)}$$

$$I = mL^2$$

$$\frac{1}{L} = \frac{mL}{I}$$

$$\omega = \sqrt{\frac{mgL}{I}} \qquad \text{(small angles only)}$$

Clicker Question 10.2

At the surface of Mars, the acceleration due to gravity is $3.71\ \mathrm{m/s^2}$. What is the length of a pendulum on Mars that oscillates with a period of one second?

a) 0.0940 m

b) 0.143 m

c) 0.248 m

d) 0.296 m

e) 0.655 m

$$\omega_{\text{pendulum}} = \frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

Clicker Question 10.1

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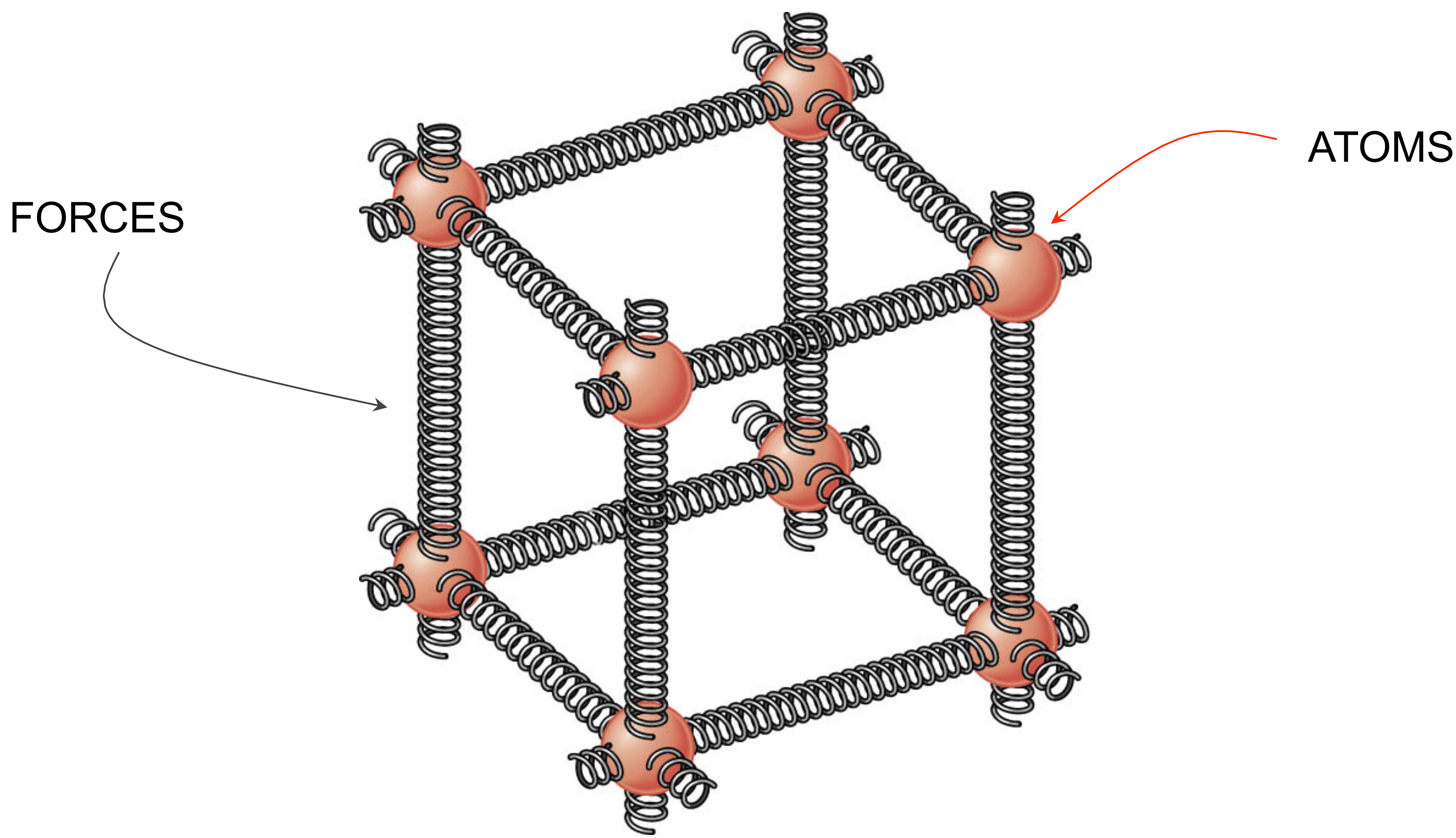
$$\omega_{\text{pendulum}} = \frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

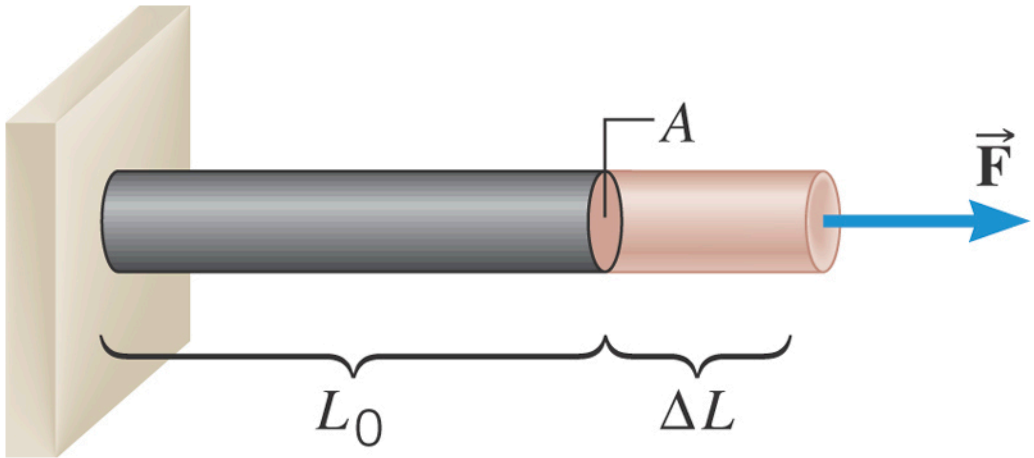
$$\boxed{\begin{aligned} \frac{(2\pi)^2}{T^2} &= \frac{g_{\text{Mars}}}{L} \\ L &= \frac{g_{\text{Mars}} T^2}{(2\pi)^2} = \frac{\left(3.71\ \text{m/s}^2\right)(1\ \text{s})^2}{(2\pi)^2} \\ &= 0.094\ \text{m} \end{aligned}}$$

10.7 Elastic Deformation

Because of these atomic-level “springs”, a material tends to return to its initial shape once forces have been removed.



STRETCHING, COMPRESSION, AND YOUNG'S MODULUS



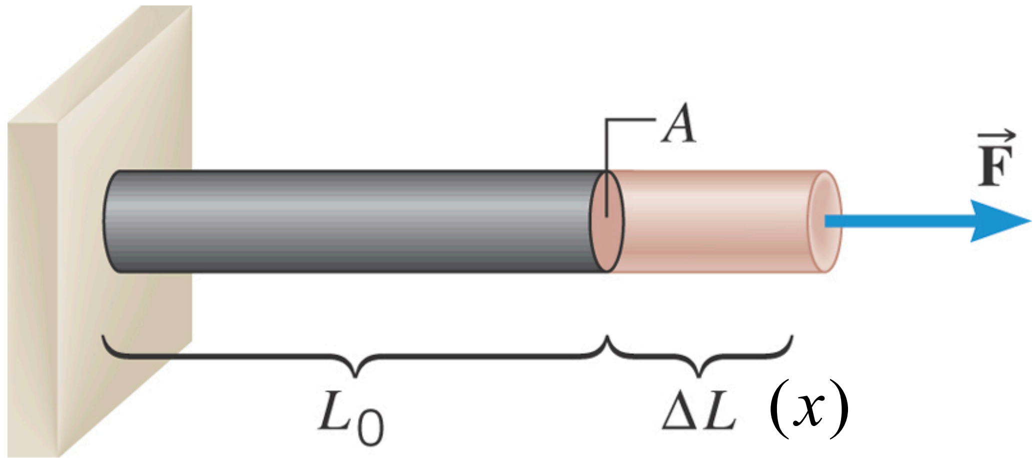
$$F = Y\left(\frac{\Delta L}{L_o}\right)A$$

Young's modulus has the units of pressure: N/m^2

Young's modulus is a characteristic of the material (see table 10.1)

$$Y_{\text{Steel}} = 2.0\times 10^{11}\ \text{N/m}^2$$

Spring Constants and Young's Modulus



Y : Young's Modulus

A, L_0 : Area and length of rod

ΔL : Change in rod length (x)

$$\boxed{\begin{aligned} F &= Y\left(\frac{\Delta L}{L_o}\right)A = \left(\frac{YA}{L_0}\right)\Delta L; \quad \text{let } \Delta L = x \\ &= kx, \quad k = \left(\frac{YA}{L_0}\right) \end{aligned}}$$

Table 10.1 Values for the Young's Modulus of Solid Materials

Material	Young's Modulus Y (N/m^2)
Aluminum	6.9×10^{10}
Bone	
Compression	9.4×10^{9}
Tension	1.6×10^{10}
Brass	9.0×10^{10}
Brick	1.4×10^{10}
Copper	1.1×10^{11}
Mohair	2.9×10^{9}
Nylon	3.7×10^{9}
Pyrex glass	6.2×10^{10}
Steel	2.0×10^{11}
Teflon	3.7×10^{8}
Titanium	1.2×10^{11}
Tungsten	3.6×10^{11}

Note: 1 Pascal (Pa) = 1 N/m^2

$$1 \text{ GPa} = 1\times10^{9} \text{ N/m}^2$$

In general the quantity *F/A* is called the **Stress**.

The change in the quantity divided by that quantity is called the **Strain**:

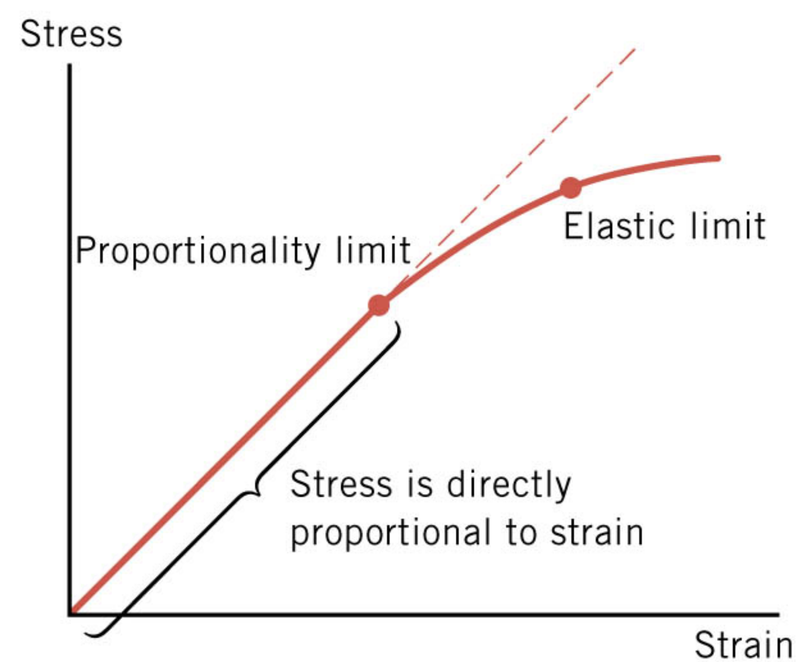
$$\Delta V/V_o \quad \Delta L/L_o \quad \Delta x/L_o$$

HOOKE'S LAW FOR STRESS AND STRAIN

Stress is directly proportional to strain.

Strain is a unitless quantitiy.

SI Unit of Stress: N/m^2



Example 12 Bone Compression

In a circus act, a performer supports the combined weight (1080 N) of a number of colleagues. Each thighbone of this performer has a length of 0.55 m and an effective cross sectional area of 7.7×10^{-4} m^2. Determine the amount that each thighbone compresses under the extra weight.

$$F = Y\left(\frac{\Delta L}{L_o}\right)A$$

$$\text{each leg} = \frac{1080\ \text{n}}{2}$$

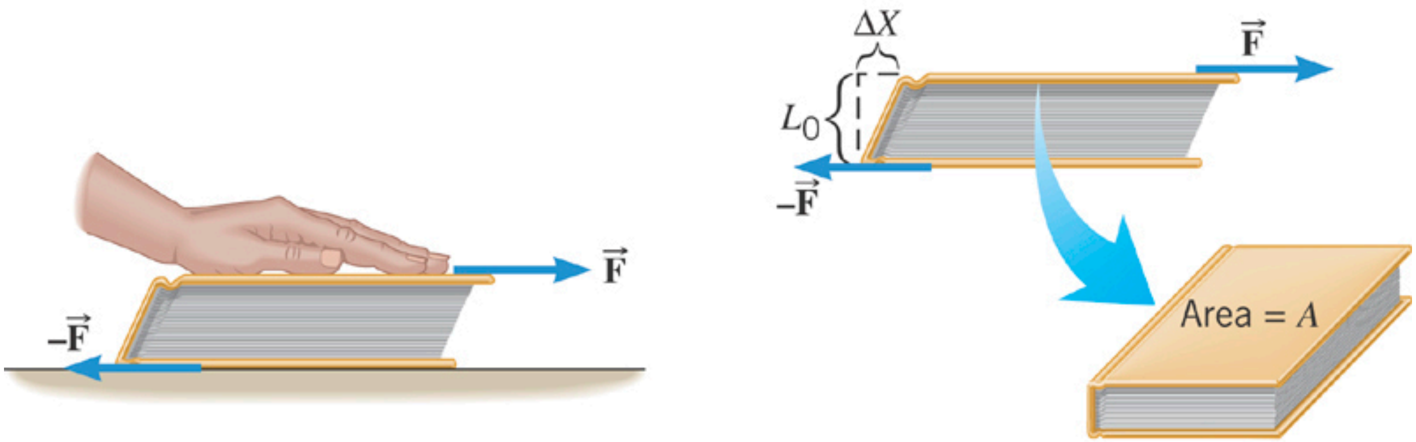
$$\Delta L = \frac{FL_o}{YA}$$

$$= \frac{(540\ \text{N})(0.55\ \text{m})}{(9.4\times10^{9}\ \text{N/m}^2)(7.7\times10^{-4}\ \text{m}^2)}$$

$$= 4.1\times10^{-5}\,\text{m} = 0.041\text{mm}$$

10.7 *Elastic Deformation*

SHEAR DEFORMATION AND THE SHEAR MODULUS

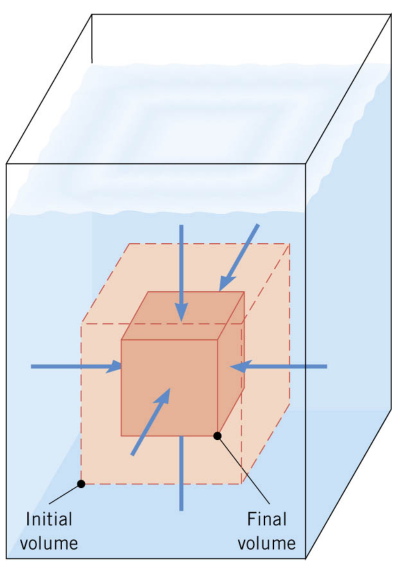


(a) (b)

$$F = S\left(\frac{\Delta x}{L_o}\right)A$$

S: Shear modulus
Table 10.2

VOLUME DEFORMATION AND THE BULK MODULUS



Pressure Change

$$\Delta P = -B\left(\frac{\Delta V}{V_o}\right)$$

B: Bulk modulus
Table 10.3

Chapter 11

Fluids

DEFINITION OF MASS DENSITY

The mass density of a substance is the mass of a substance divided by its volume:

$$\rho = \frac{m}{V}$$

SI Unit of Mass Density: kg/m^3

Table 11.1 Mass Densities[a] of Common Substances

Substance	Mass Density ρ (kg/m^3)
Solids	
Aluminum	2700
Brass	8470
Concrete	2200
Copper	8890
Diamond	3520
Gold	19 300
Ice	917
Iron (steel)	7860
Lead	11 300
Quartz	2660
Silver	10 500
Wood (yellow pine)	550
Liquids	
Blood (whole, 37 °C)	1060
Ethyl alcohol	806
Mercury	13 600
Oil (hydraulic)	800
Water (4 °C)	1.000×10^3
Gases	
Air	1.29
Carbon dioxide	1.98
Helium	0.179
Hydrogen	0.0899
Nitrogen	1.25
Oxygen	1.43

[a] Unless otherwise noted, densities are given at 0 °C and 1 atm pressure.

Example 1 Blood as a Fraction of Body Weight

The body of a man whose weight is 690 N contains about $5.2\text{x}10^{-3}$ m^3 of blood.

(a) Find the blood's weight and (b) express it as a percentage of the body weight.

$$m = \rho V$$

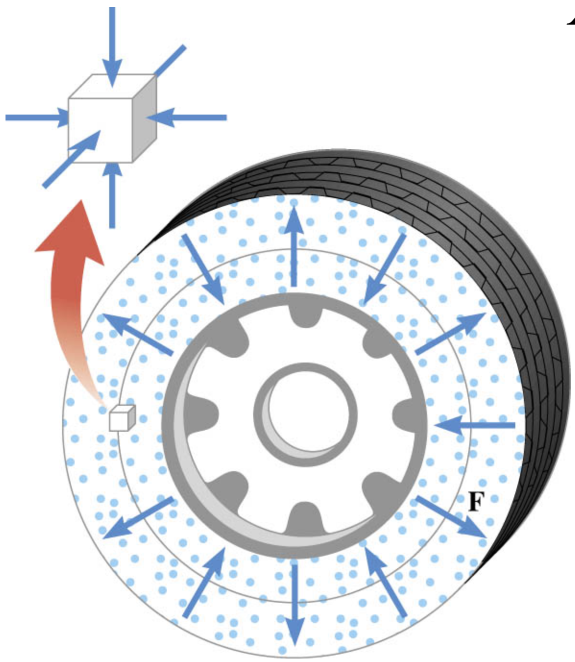
$$\text{(a) } W = mg = \rho V g = \left(1060 \text{ kg/m}^3\right)\left(5.2\times10^{-3} \text{ m}^3\right)\left(9.80\,\text{m/s}^2\right) = 54 \text{ N}$$

$$\text{(b) Percentage} = \frac{54 \text{ N}}{690 \text{ N}}\times 100\% = 7.8\%$$

11.2 Pressure

$$P = \frac{F}{A}$$

Pressure = Force per unit Area



The same pressure acts inward in every direction on a small volume.

SI Unit of Pressure: 1 N/m^2 = 1Pa

Pascal