

Chapter 14

The Ideal Gas Law and Kinetic Theory

14.1 Molecular Mass, the Mole, and Avogadro's Number

The **atomic number** of an element is the # of protons in its nucleus.

Isotopes of an element have different # of neutrons in its nucleus.

The **atomic mass unit** (symbol u) is used to compare the mass of elements.

The reference is the most abundant isotope of carbon, which is called carbon-12.

H 1 1.00794	Atomic number Atomic mass
Li 3 6.941	Be 4 9.01218
Na 11 22.9898	Mg 12 24.305

$$1 \text{ u} = 1.6605 \times 10^{-24} \text{ g} = 1.6605 \times 10^{-27} \text{ kg}$$

The atomic mass is given in atomic mass units. For example, a Li atom has a mass of 6.941u.

One **mole** (mol) of a substance (element or molecule) contains as many particles as there are atoms in 12 grams of the isotope carbon-12. The number of atoms in 12 grams of carbon-12 is known as Avogadro's number, N_A .

Avogadro's number

$$N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$$

14.1 Molecular Mass, the Mole, and Avogadro's Number

The mass per mole (in g/mol) of a substance has the same numerical value as the atomic or molecular mass of the substance (in atomic mass units).

For example Hydrogen has an atomic mass of 1.00794 g/mol, while the mass of a single hydrogen atom is 1.00794 u.

N : # of atoms or molecules,

n : # of moles of element or molecule

m_p : atomic mass (amu) \Rightarrow also grams/mole

$$N = nN_A$$

$$m = nm_p$$

14.1 Molecular Mass, the Mole, and Avogadro's Number

Example 1 Hope Diamond & Rosser Reeves Ruby

The Hope diamond (44.5 carats) is almost pure carbon. The Rosser Reeves ruby (138 carats) is primarily aluminum oxide (Al_2O_3). One carat is equivalent to a mass of 0.200 g. Determine (a) the number of carbon atoms in the Hope diamond and (b) the number of Al_2O_3 molecules in the ruby.

$$\left[2(26.98) + 3(15.99) \right] \text{g/mole}$$

$$(a) \quad n = \frac{m}{\text{Mass per mole}} = \frac{(44.5 \text{ carats}) \left[(0.200 \text{ g}) / (1 \text{ carat}) \right]}{12.011 \text{ g/mol}} = 0.741 \text{ mol}$$

$$N = nN_A = (0.741 \text{ mol}) (6.022 \times 10^{23} \text{ mol}^{-1}) = 4.46 \times 10^{23} \text{ atoms}$$

$$(b) \quad n = \frac{m}{\text{Mass per mole}} = \frac{(138 \text{ carats}) \left[(0.200 \text{ g}) / (1 \text{ carat}) \right]}{101.96 \text{ g/mol}} = 0.271 \text{ mol}$$

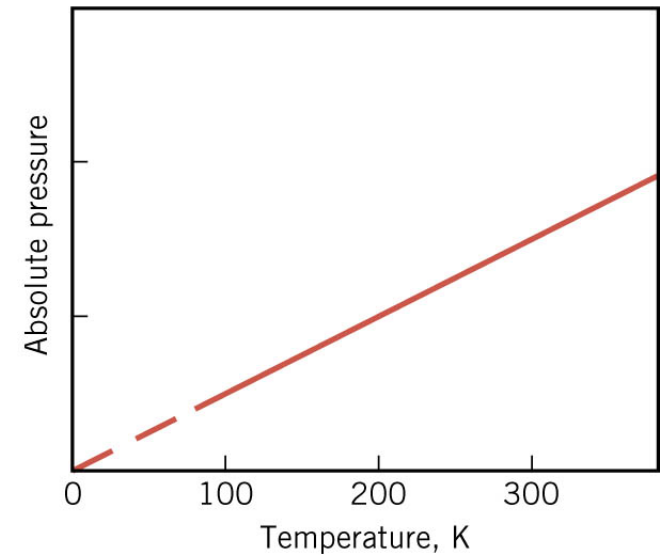
$$N = nN_A = (0.271 \text{ mol}) (6.022 \times 10^{23} \text{ mol}^{-1}) = 1.63 \times 10^{23} \text{ atoms}$$

14.2 The Ideal Gas Law

An **ideal gas** is an idealized model for real gases that have sufficiently low densities, **interacting only by elastic collisions**.

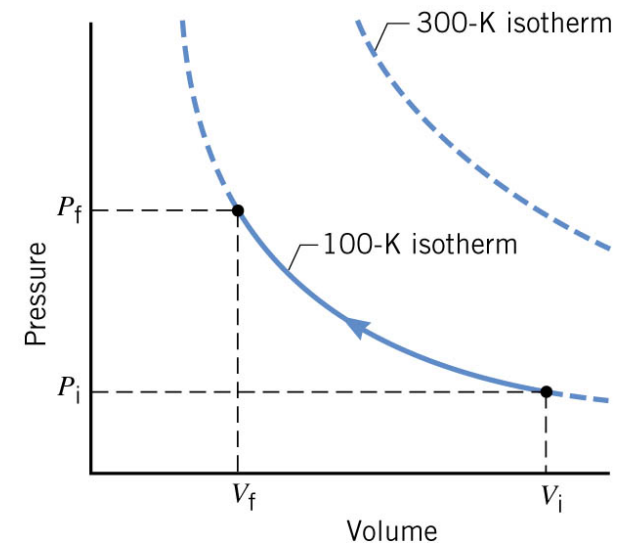
At constant volume the pressure is proportional to the temperature.

$$P \propto T$$



At constant temperature, the pressure is inversely proportional to the volume.

$$P \propto 1/V$$



The pressure is also proportional to the amount of gas.

$$P \propto n$$

Clicker Question 14.1

Under which of the following circumstances does a real gas behave like an ideal gas?

- a) The gas particles move very slowly.
- b) The gas particles do not collide with each other very often.
- c) The gas particles just bounce off each other.
- d) The interaction between the gas particles and the walls of the container is negligible.
- e) There are only one kind of particles in the container.

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14.2 The Ideal Gas Law

THE IDEAL GAS LAW

The absolute pressure of an ideal gas is directly proportional to the Kelvin temperature and the number of moles (n) of the gas and is inversely proportional to the volume of the gas.

$$P = \frac{nRT}{V}$$

$$PV = nRT$$

$$R = 8.31 \text{ J}/(\text{mol} \cdot \text{K})$$

Another form for the Ideal Gas Law using the number of atoms (N)

$$PV = nRT$$

$$= N \left(\frac{R}{N_A} \right) T$$

$$N = nN_A$$

$$PV = NkT$$

$$k = \frac{R}{N_A} = \frac{8.31 \text{ J}/(\text{mol} \cdot \text{K})}{6.022 \times 10^{23} \text{ mol}^{-1}} = 1.38 \times 10^{-23} \text{ J/K}$$

14.2 The Ideal Gas Law

Example 2 Oxygen in the Lungs

In the lungs, the respiratory membrane separates tiny sacs of air (pressure $1.00 \times 10^5 \text{ Pa}$) from the blood in the capillaries. These sacs are called alveoli. The average radius of the alveoli is 0.125 mm , and the air inside contains 14% oxygen. Assuming that the air behaves as an ideal gas at 310 K , find the number of oxygen molecules in one of these sacs.

$$PV = NkT$$

$$N_{tot} = \frac{PV}{kT} = \frac{(1.00 \times 10^5 \text{ Pa}) \left[\frac{4}{3} \pi (0.125 \times 10^{-3} \text{ m})^3 \right]}{(1.38 \times 10^{-23} \text{ J/K})(310 \text{ K})}$$
$$= 1.9 \times 10^{14}$$

$$N_{\text{Oxy}} = (1.9 \times 10^{14}) \times (0.14) = 2.7 \times 10^{13}$$

Clicker Question 14.2

An ideal gas is enclosed within a container by a moveable piston. If the final temperature is two times the initial temperature and the volume is reduced to one-fourth of its initial value, what will the final pressure of the gas be relative to its initial pressure, P_1 ?

- a) $8P_1$
- b) $4P_1$
- c) $2P_1$
- d) $P_1/2$
- e) $P_1/4$

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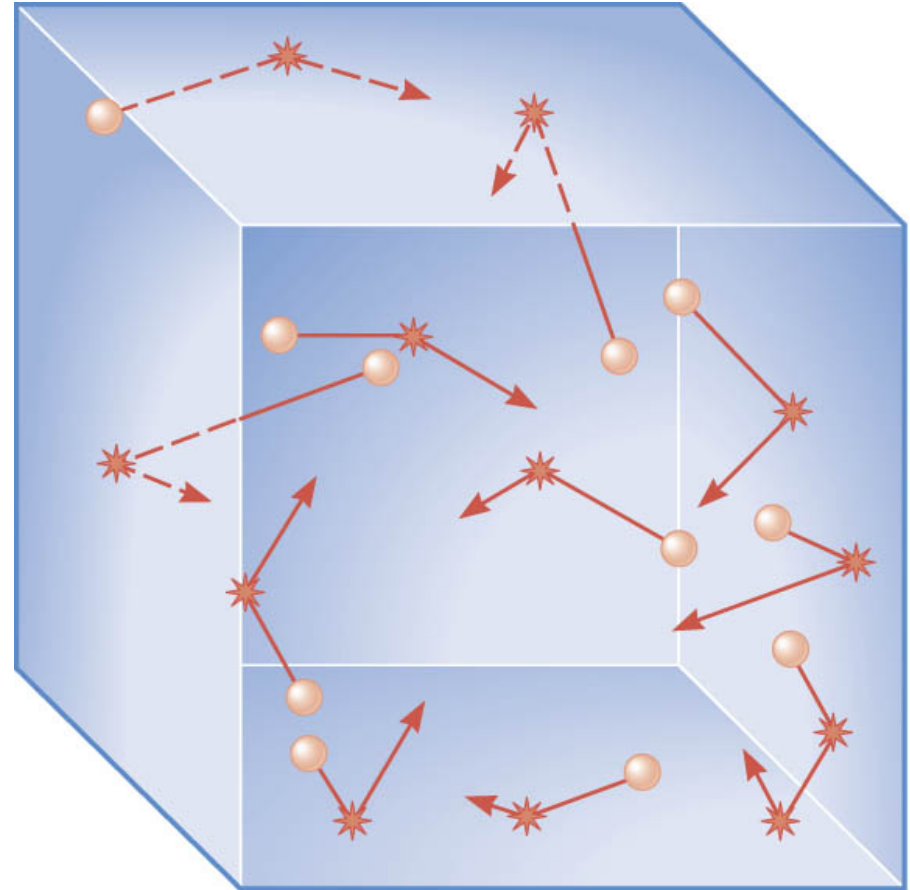
$$P_1 V_1 = nRT_1; \quad V_2 = V_1/4; \quad T_2 = 2T_1$$
$$P_2 = \frac{nRT_2}{V_2} = \frac{nR(2T_1)}{V_1/4} = 8 \frac{nRT_1}{V_1} = 8P_1$$

14.3 *Kinetic Theory of Gases*

The particles are in constant, random motion, colliding with each other and with the walls of the container.

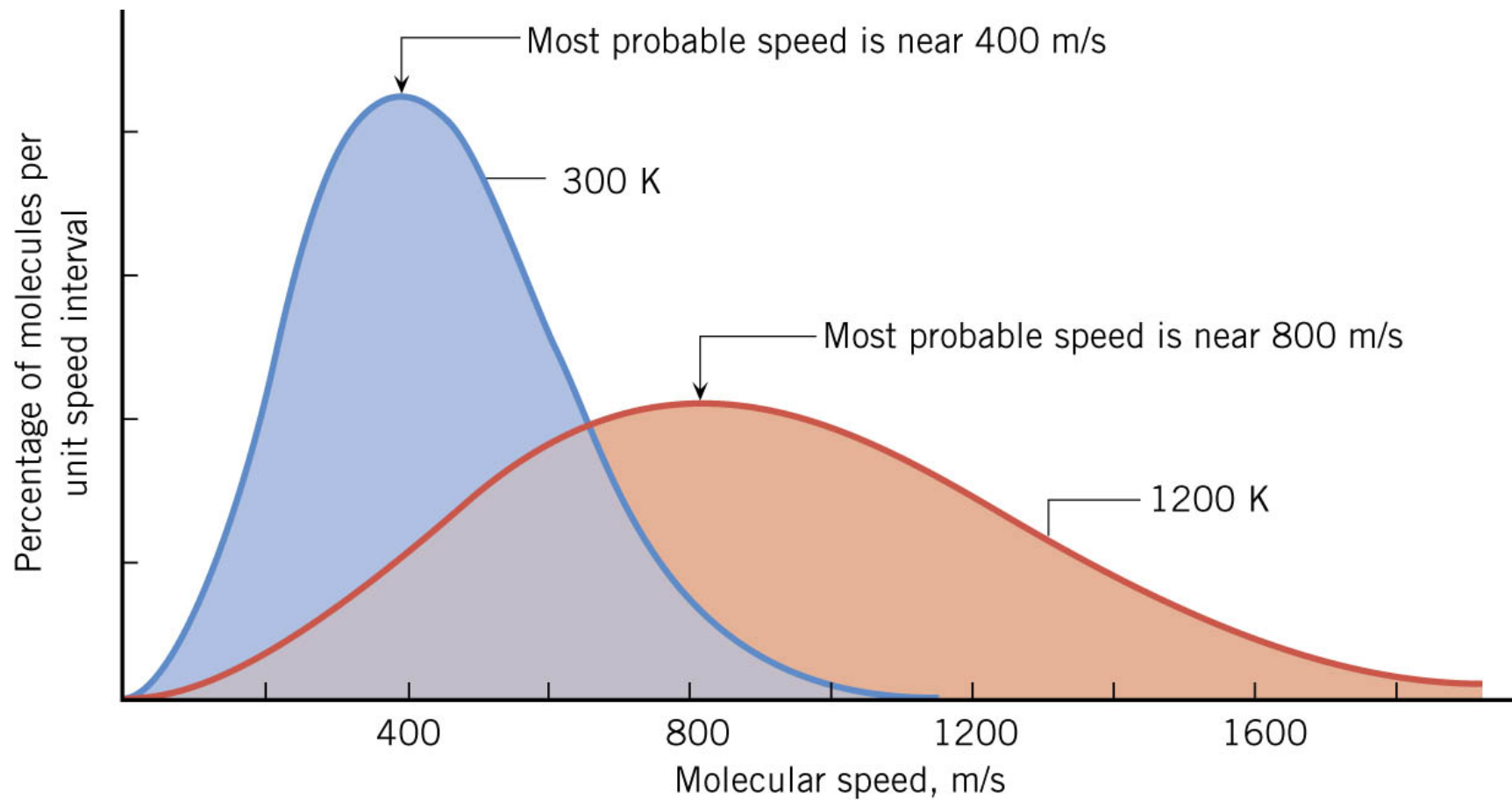
Each collision changes the particle's speed.

As a result, the atoms and molecules have different speeds.



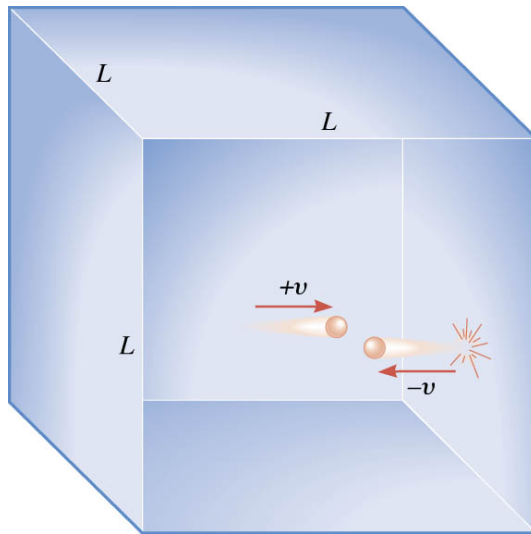
14.3 Kinetic Theory of Gases

THE DISTRIBUTION OF MOLECULAR SPEEDS



14.3 Kinetic Theory of Gases

KINETIC THEORY



$$\sum F = ma = m \frac{\Delta v}{\Delta t} = \frac{\Delta(mv)}{\Delta t}$$

$$\begin{aligned} \text{Average force on each gas molecule when hitting the wall} &= \frac{\text{Final momentum} - \text{Initial momentum}}{\text{Time between successive collisions}} \\ &= \frac{(-mv) - (+mv)}{2L/v} = \frac{-mv^2}{L} \end{aligned}$$

Average force
on the wall

$$\bar{F} = \left(\frac{N}{3} \right) \left(\frac{mv^2}{L} \right) \Rightarrow P = \frac{\bar{F}}{A} = \frac{\bar{F}}{L^2} = \left(\frac{N}{3} \right) \left(\frac{mv^2}{L^3} \right)$$

$$PV = \left(\frac{N}{3} \right) mv^2 = \frac{2}{3} N \left(\frac{1}{2} mv^2 \right)$$

$$PV = NkT$$

$$\overline{KE} = \frac{1}{2} mv^2$$

$$v_{rms} = \sqrt{\overline{v^2}}$$

root mean
square speed

Temperature reflects the average
Kinetic Energy of the molecules

$$\frac{3}{2} kT = \frac{1}{2} mv_{rms}^2 = \overline{KE}$$

$$k = 1.38 \times 10^{-23} \text{ J/K}$$

14.3 Kinetic Theory of Gases

Example 6 The Speed of Molecules in Air

Air is primarily a mixture of nitrogen N_2 molecules (molecular mass 28.0u) and oxygen O_2 molecules (molecular mass 32.0u). Assume that each behaves as an ideal gas and determine the rms speeds of the nitrogen and oxygen molecules when the temperature of the air is 293K.

$$\frac{1}{2} m v_{rms}^2 = \frac{3}{2} kT$$

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

T must be in Kelvin
($\text{K} = \text{C}^\circ + 273$)

$$\begin{array}{l} \text{Nitrogen molecule} \\ m = \frac{28.0 \text{ g/mol}}{6.022 \times 10^{23} \text{ mol}^{-1}} \\ = 4.65 \times 10^{-26} \text{ kg} \end{array}$$

$$\begin{aligned} v_{rms} &= \sqrt{\frac{3kT}{m}} \\ &= \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})}{4.65 \times 10^{-26} \text{ kg}}} = 511 \text{ m/s} \end{aligned}$$

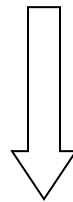
Molecules are moving really fast
but do not go very far before hitting
another molecule.

14.3 Kinetic Theory of Gases

THE INTERNAL ENERGY OF A MONATOMIC IDEAL GAS

$$\overline{\text{KE}} = \frac{1}{2} m v_{rms}^2 = \frac{3}{2} kT$$

Average KE per atom



multiply by the number of atoms

$$U = N \frac{3}{2} kT = \frac{3}{2} nRT$$

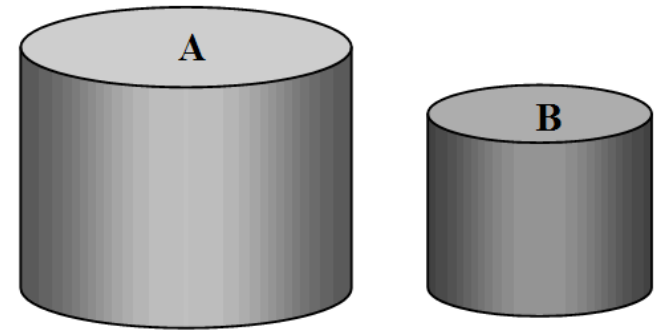
Total Internal Energy

THE INTERNAL ENERGY OF A MOLECULAR GAS
MUST INCLUDE MOLECULAR VIBRATIONS!

H₂, N₂, H₂O, SO₂, CO₂, ... (most gases except Nobel gases)

Clicker Question 14.3

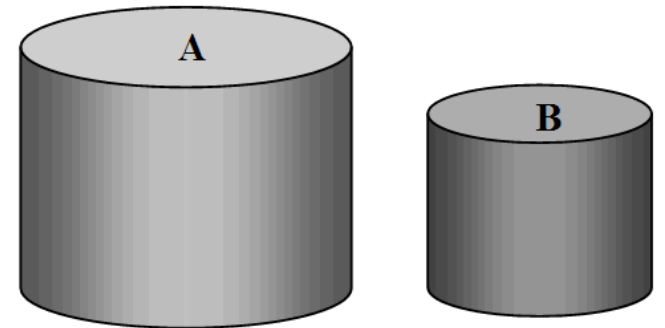
Two sealed containers, labeled A and B as shown, are at the same temperature and each contain the same number of moles of an ideal monatomic gas. Which one of the following statements concerning these containers is true?



- a) The rms speed of the atoms in the gas is greater in B than in A
- b) The frequency of collisions of the atoms with the walls of container B are greater than that for container A
- c) The kinetic energy of the atoms in the gas is greater in B than in A.
- d) The pressure within container B is less than the pressure inside container A.
- e) The force that the atoms exert on the walls of container B are greater than in for those in container A.

Clicker Question 14.3

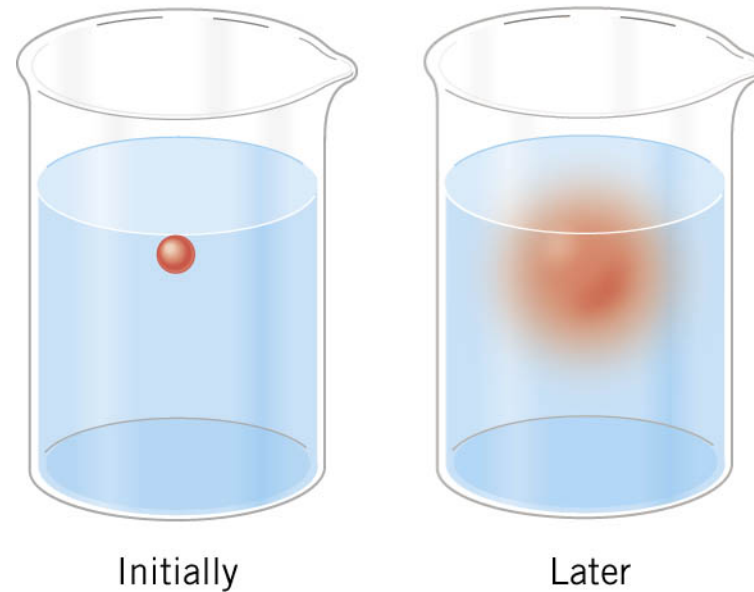
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14.4 Diffusion

The process in which molecules move from a region of higher concentration to one of lower concentration is called **diffusion**.



FICK'S LAW OF DIFFUSION

The mass m of solute that diffuses in a time t through a solvent contained in a channel of length L and cross sectional area A is

$$m = \frac{DA(\Delta C)t}{L}$$

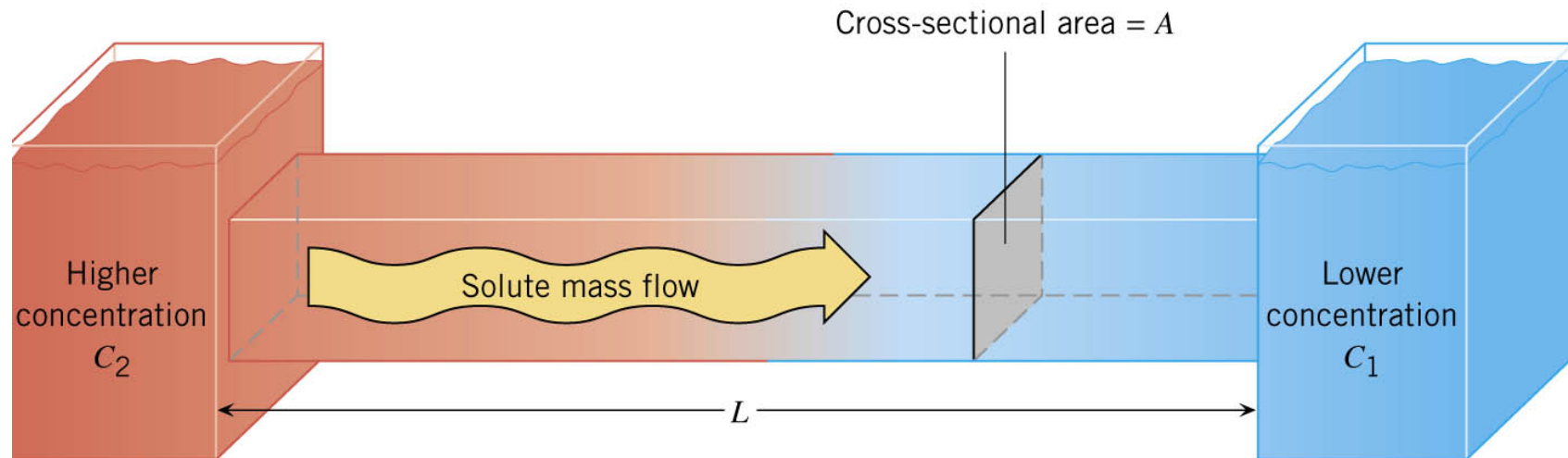
D Diffusion constant

ΔC Concentration gradient
between ends

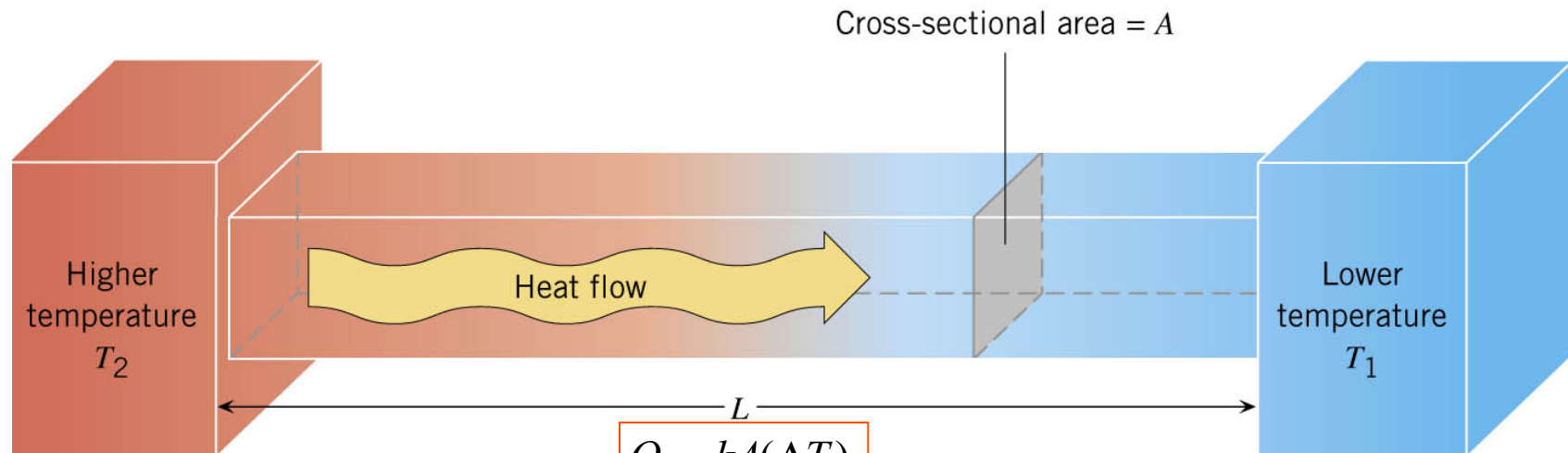
SI Units for the Diffusion Constant: m^2/s

14.4 Diffusion

Diffusion analogous to heat flow



$$\frac{m}{t} = \frac{DA(\Delta C)}{L}$$



$$\frac{Q}{t} = \frac{kA(\Delta T)}{L}$$

Chapter 15

Thermodynamics

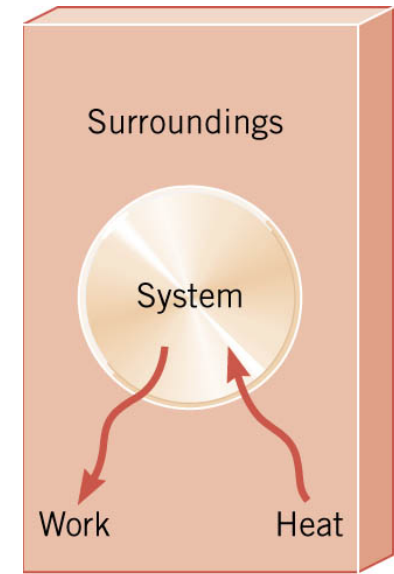
15.1 Thermodynamic Systems and Their Surroundings

Thermodynamics is the branch of physics that is built upon the fundamental laws that heat and work obey.

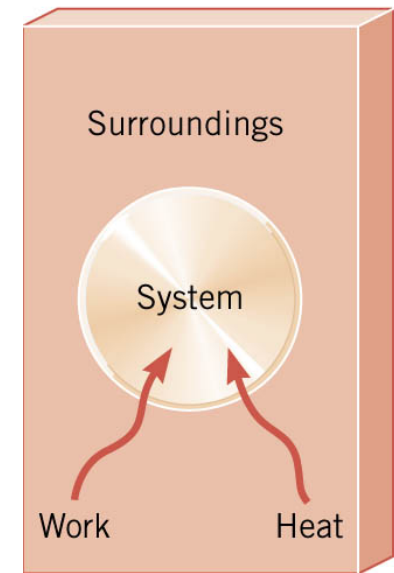
The collection of objects on which attention is being focused is called the **system**, while everything else in the environment is called the **surroundings**.

Walls that permit heat flow are called **diathermal walls**, while walls that do not permit heat flow are called **adiabatic (NO HEAT FLOW) walls**

To understand thermodynamics, it is necessary to describe the **state of a system**.



(a)

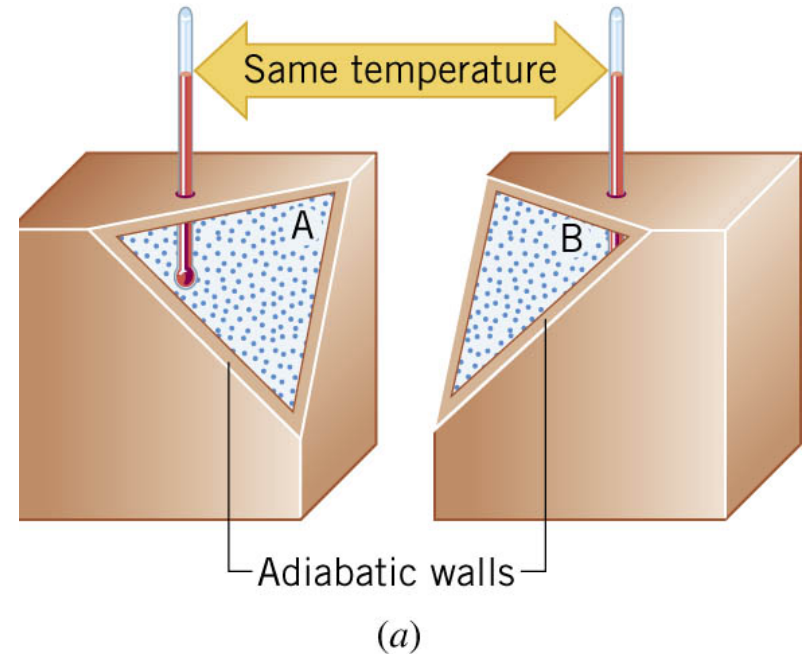


(b)

15.2 The Zeroth Law of Thermodynamics

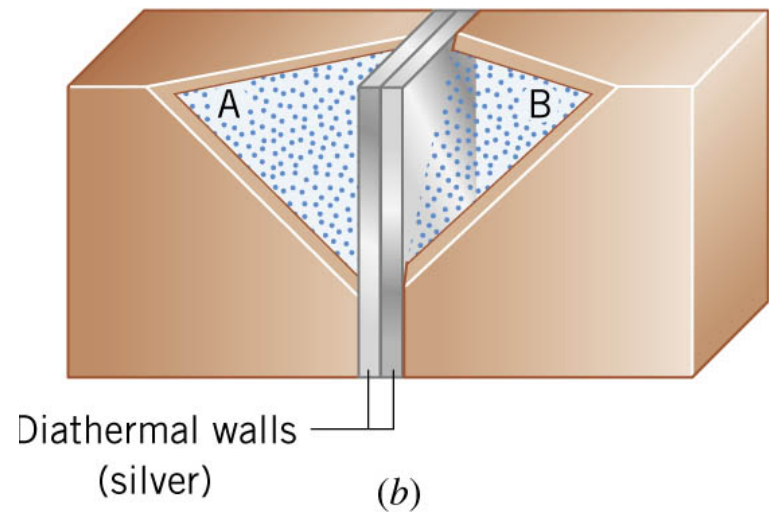
Two systems are said to be in **thermal equilibrium** if there is no heat flow between them when in contact.

Temperature: there is **no net flow of heat** between two systems in thermal contact that have the **same temperature**.



THE ZEROTH LAW OF THERMODYNAMICS

Two systems individually in thermal equilibrium with a third system are in thermal equilibrium with each other.



15.3 The First Law of Thermodynamics

- 1) Suppose that a system only gains heat (Q) and **nothing else changes**. Consistent with the law of conservation of energy, the internal energy (U) of the system changes:

$$\Delta U = U_f - U_i = Q \quad Q > 0 \text{ system gains heat}$$

- 2) If a system does work W on its surroundings but **there is no heat flow**, (**adiabatic**) conservation of energy indicates that the internal energy of the system will decrease:

$$\Delta U = U_f - U_i = -W \quad W > 0 \text{ if system does work}$$

THE FIRST LAW OF THERMODYNAMICS

The internal energy of a system changes due to heat and work:

$$\Delta U = U_f - U_i = Q - W \quad \begin{array}{l} Q > 0 \text{ system gains heat} \\ W > 0 \text{ if system does work} \end{array}$$

To determine ΔU , the values of both Q and W are required!

15.3 The First Law of Thermodynamics

Example 1 Positive and Negative Work

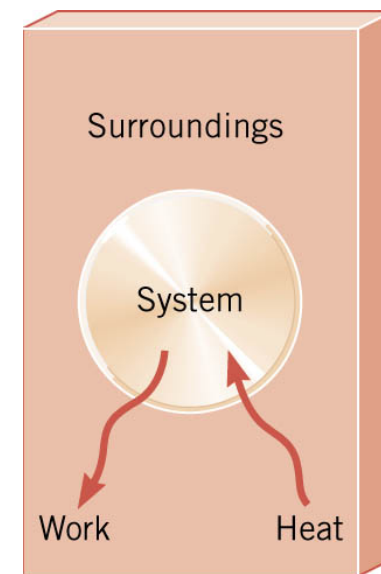
In part (a) of figure, the system gains 1500J of heat and 2200J of work is done **by the system** on its surroundings.

In part (b), the system also gains 1500J of heat, but 2200J of work is done **on the system**.

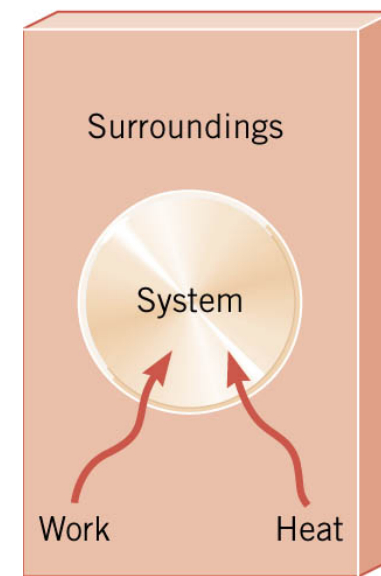
In each case, determine the change in internal energy of the system.

$$\begin{aligned} (a) \quad \Delta U &= Q - W \\ &= (+1500 \text{ J}) - (+2200 \text{ J}) = -700 \text{ J} \end{aligned}$$

$$\begin{aligned} (b) \quad \Delta U &= Q - W \\ &= (+1500 \text{ J}) - (-2200 \text{ J}) = +3700 \text{ J} \end{aligned}$$



(a)



(b)

15.3 The First Law of Thermodynamics

Example 2 An Ideal Gas

The temperature of three moles of a monatomic ideal gas is reduced from 540K to 350K as 5500J of heat flows into the gas.

Find (a) the change in internal energy and (b) the work done by the gas.

$$\Delta U = U_f - U_i = Q - W$$

$$U = \frac{3}{2} nRT$$

(a)

$$\begin{aligned}\Delta U &= \frac{3}{2} nR(T_f - T_i) \\ &= \frac{3}{2} (3.0 \text{ mol}) (8.31 \text{ J}/(\text{mol} \cdot \text{K})) (350 \text{ K} - 540 \text{ K}) \\ &= -7100 \text{ J}\end{aligned}$$

(b)

$$\begin{aligned}W &= Q - \Delta U \\ &= 5500 \text{ J} - (-7100 \text{ J}) \\ &= 12600 \text{ J}\end{aligned}$$

Clicker Question 15.1

The internal energy of a system increases during some time interval. Which one of the following statements concerning this situation must be true?

- a) The increase in internal energy indicates that work was done on the system.
- b) The increase in internal energy indicates that heat was added to the system.
- c) The increase in internal energy indicates that work was done by the system.
- d) The increase in internal energy indicates that heat was removed from the system.
- e) The information given is insufficient to indicate the reason for the increase.

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15.4 Thermal Processes

A **quasi-static** process is one that occurs slowly enough that a **uniform temperature and pressure** exist throughout all regions of the system at all times.

isobaric: constant pressure

$$W = F_s = P(A_s) = P\Delta V$$

isochoric: constant volume

$$W = P\Delta V = 0$$

isothermal: constant temperature

$$W = nRT \ln(V_f/V_i) \quad \text{ideal gas}$$

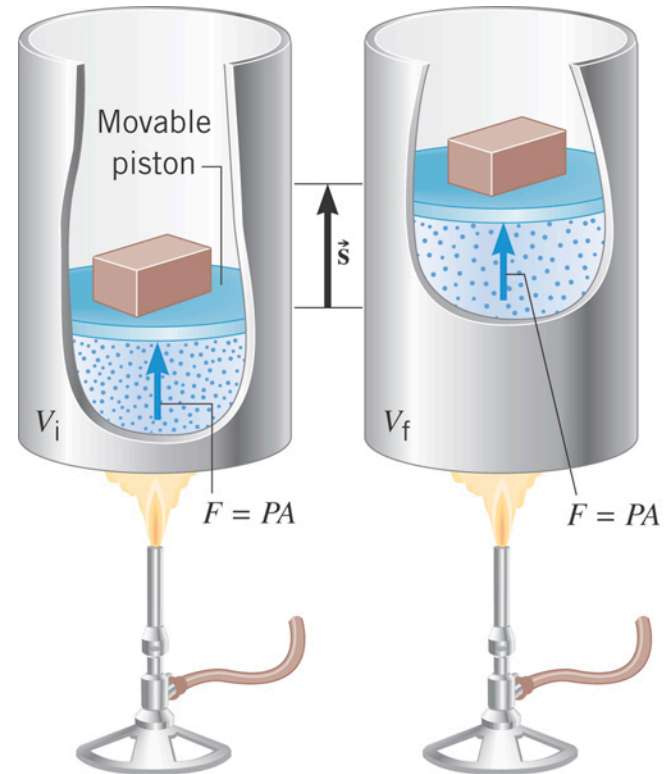
adiabatic: no transfer of heat

$$W = \frac{3}{2} nR(T_f - T_i) \quad \text{ideal gas}$$

15.4 Thermal Processes

An **isobaric** process is one that occurs at **constant pressure**.

$$W = Fs = P(\Delta s) = P\Delta V$$



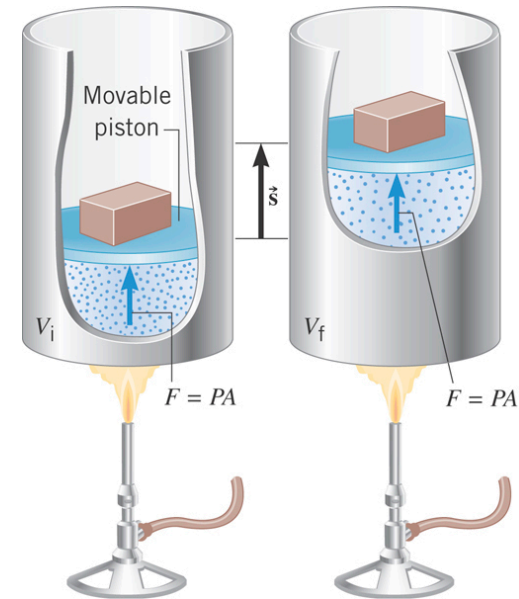
Isobaric process: $W = P\Delta V = P(V_f - V_i)$

15.4 Thermal Processes

Example 3 Isobaric Expansion of Water

One gram of water is placed in the cylinder and the pressure is maintained at $2.0 \times 10^5 \text{ Pa}$. The **temperature of the water is raised by 31°C** . The water is in the liquid phase and **expands by the small amount of $1.0 \times 10^{-8} \text{ m}^3$** .

Find the work done and the change in internal energy.



$$W = P\Delta V$$
$$= (2.0 \times 10^5 \text{ Pa})(1.0 \times 10^{-8} \text{ m}^3) = 0.0020 \text{ J}$$

$$\text{Liquid water } \Delta V \sim 0$$

$$Q = mc\Delta T$$
$$= (0.0010 \text{ kg})[4186 \text{ J}/(\text{kg} \cdot \text{C}^\circ)](31 \text{ C}^\circ) = 130 \text{ J}$$

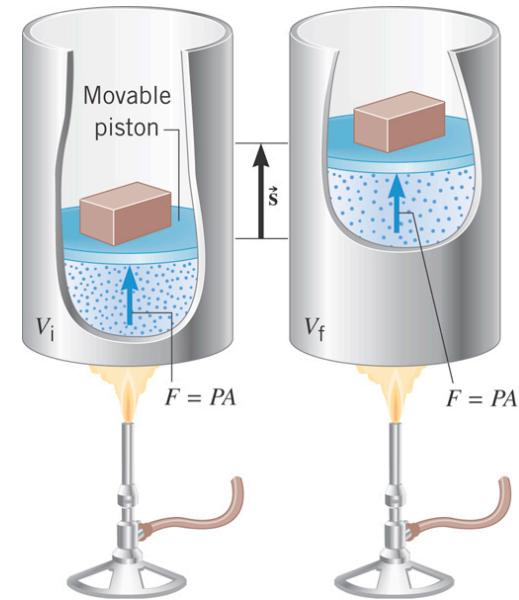
$$\Delta U = Q - W = 130 \text{ J} - 0.0020 \text{ J} = 130 \text{ J}$$

15.4 Thermal Processes

Example 3 Isobaric Expansion of **Water Vapor**

One gram of water vapor is placed in the cylinder and the pressure is maintained at $2.0 \times 10^5 \text{ Pa}$. The **temperature of the vapor is raised by 31°C** , and the gas expands by $7.1 \times 10^{-5} \text{ m}^3$. Heat capacity of the gas is $2020 \text{ J}/(\text{kg}\cdot^\circ\text{C})$.

Find the work done and the change in internal energy.



$$W = P\Delta V = (2.0 \times 10^5 \text{ Pa})(7.1 \times 10^{-5} \text{ m}^3) \\ = 14.2 \text{ J}$$

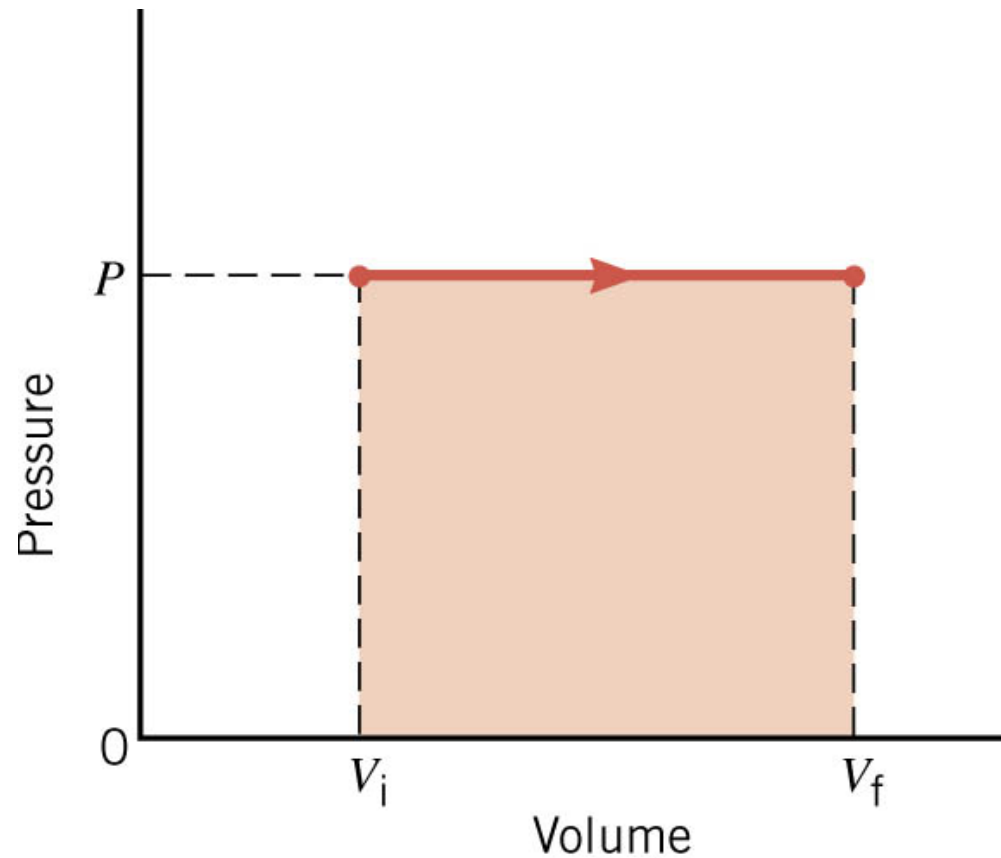
$$Q = mc\Delta T \\ = (0.0010 \text{ kg}) \left[2020 \text{ J}/(\text{kg} \cdot ^\circ\text{C}) \right] (31 ^\circ\text{C}) = 63 \text{ J}$$

$$\Delta U = Q - W = 63 \text{ J} - 14 \text{ J} = 49 \text{ J}$$

15.4 Thermal Processes

$$W = P\Delta V = P(V_f - V_i)$$

The work done at constant pressure the work done is the area under a P-V diagram.

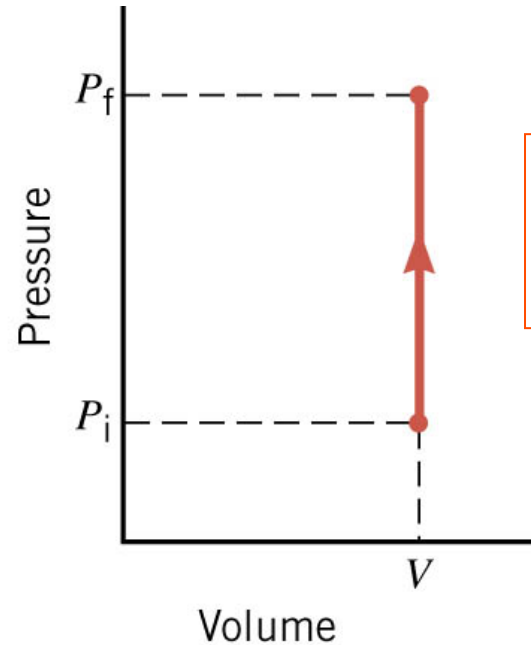


15.4 Thermal Processes

isochoric: constant volume



(a)



(b)

The work done at constant volume is the area under a P-V diagram. The area is **zero!**

$$W = 0$$

$$\Delta U = Q - W = Q$$

Change in internal energy is equal to the heat added.

15.4 Thermal Processes

Example 4 Work and the Area Under a Pressure-Volume Graph

Determine the work for the process in which the pressure, volume, and temperature of a gas are changed along the straight line in the figure.

The area under a pressure-volume graph is the work for any kind of process.

$$\begin{aligned} W &= 9(2.0 \times 10^5 \text{ Pa})(1.0 \times 10^{-4} \text{ m}^3) \\ &= +180 \text{ J} \end{aligned}$$

