Chapter 1

Introduction and Mathematical Concepts

Continued

Clicker Question 1.1: Using the dimensions given for the variables in the table, determine which one of the following expressions is correct.

A) $f = \dfrac{k}{m}$

B) $f = \sqrt{mk}$

C) $f = \dfrac{1}{\sqrt{mk}}$

D) $2\pi f = \sqrt{k/m}$

E) $f = \dfrac{1}{2\pi}\sqrt{m/k}$

variable	dimension
f	$\dfrac{1}{[\mathrm{T}]}$
m	$[\mathrm{M}]$
k	$\dfrac{[\mathrm{M}]}{[\mathrm{T}]^2}$

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m	$[\mathrm{M}]$
k	$\frac{[\mathrm{M}]}{[\mathrm{T}]^2}$

Left side Right side

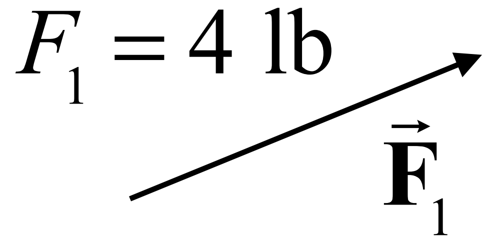
$$\frac{1}{[\mathrm{T}]} = \sqrt{\left(\frac{[\mathrm{M}]}{[\mathrm{T}]^2}\right)\Big/[\mathrm{M}]} = \sqrt{\frac{1}{[\mathrm{T}]^2}} = \frac{1}{[\mathrm{T}]}$$

1.5 Scalars and Vectors

Directions of vectors $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ appear to be the same.

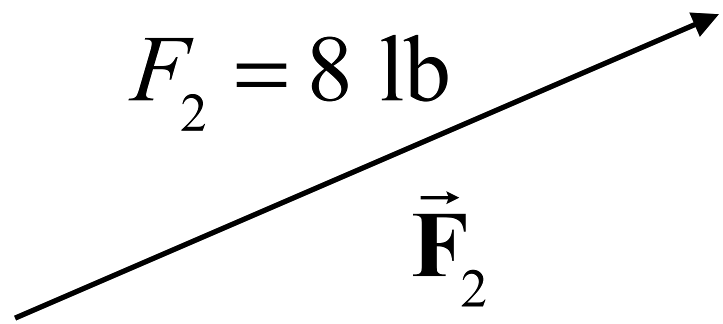
Vector $\vec{\mathbf{F}}_1$, (bold + arrow over it)

has 2 parts: $\begin{cases} \text{magnitude} = F_1 \ \textit{(italics)} \\ \text{direction} = \text{up \& to the right} \end{cases}$



Vector $\vec{\mathbf{F}}_2$, (bold + arrow over it)

has 2 parts: $\begin{cases} \text{magnitude} = F_2 \ \textit{(italics)} \\ \text{direction} = \text{up \& to the right} \end{cases}$



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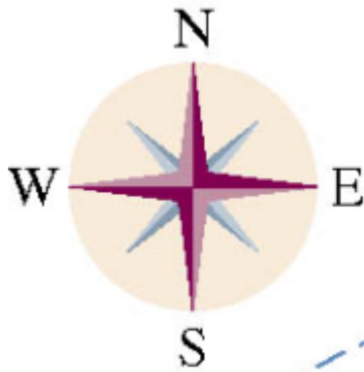
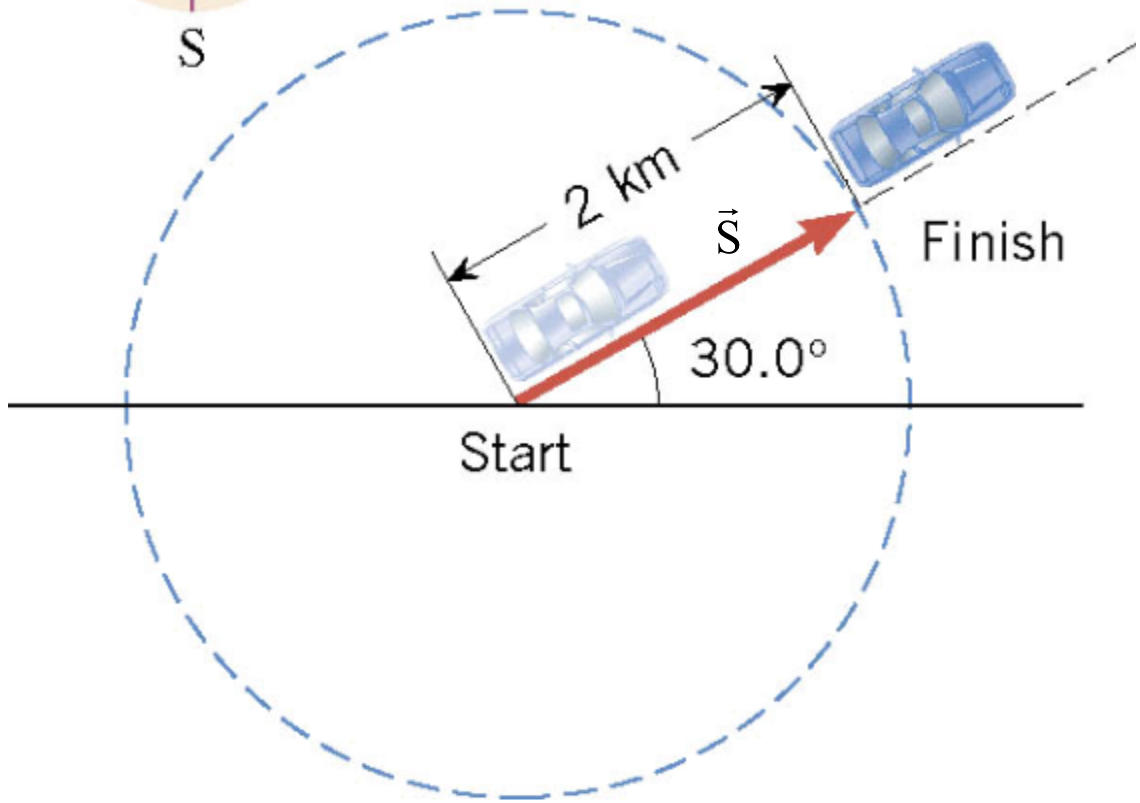
1.5 Scalars and Vectors



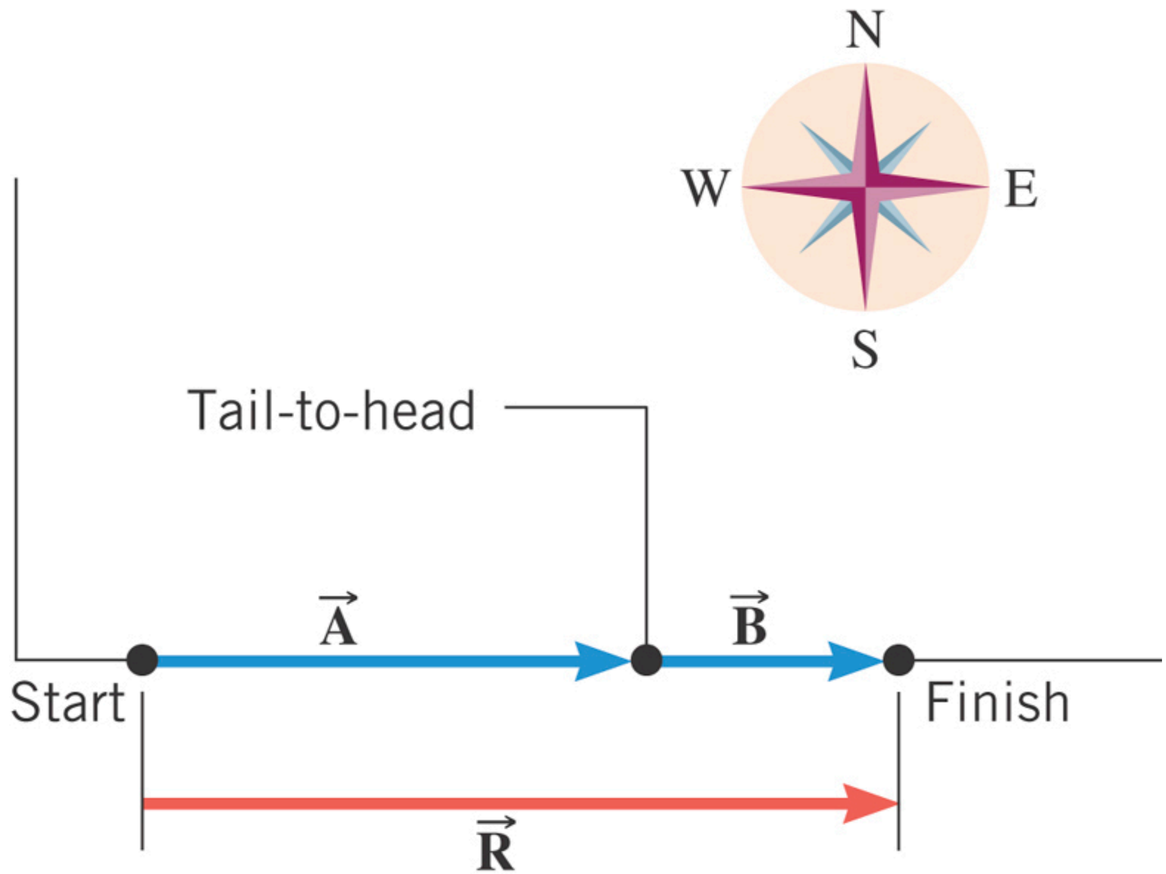
Displacement vector $\vec{S}$

has 2 parts: $\begin{cases} S = 2 \text{ km (magnitude) , and} \\ 30° \text{ North of East (direction)} \end{cases}$

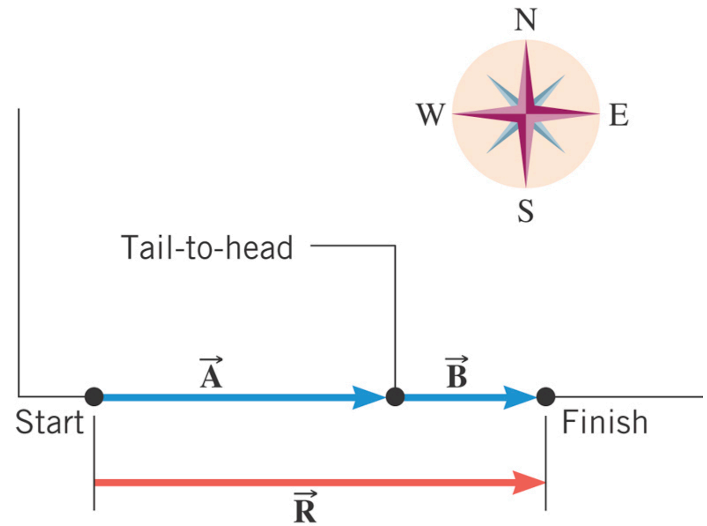


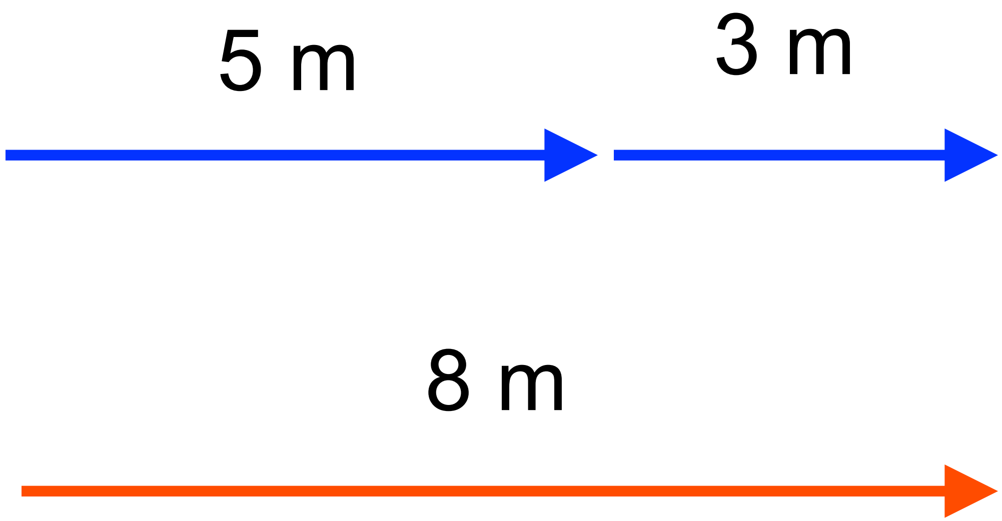
1.6 Vector Addition and Subtraction

Often it is necessary to add one vector to another.



1.6 Vector Addition and Subtraction



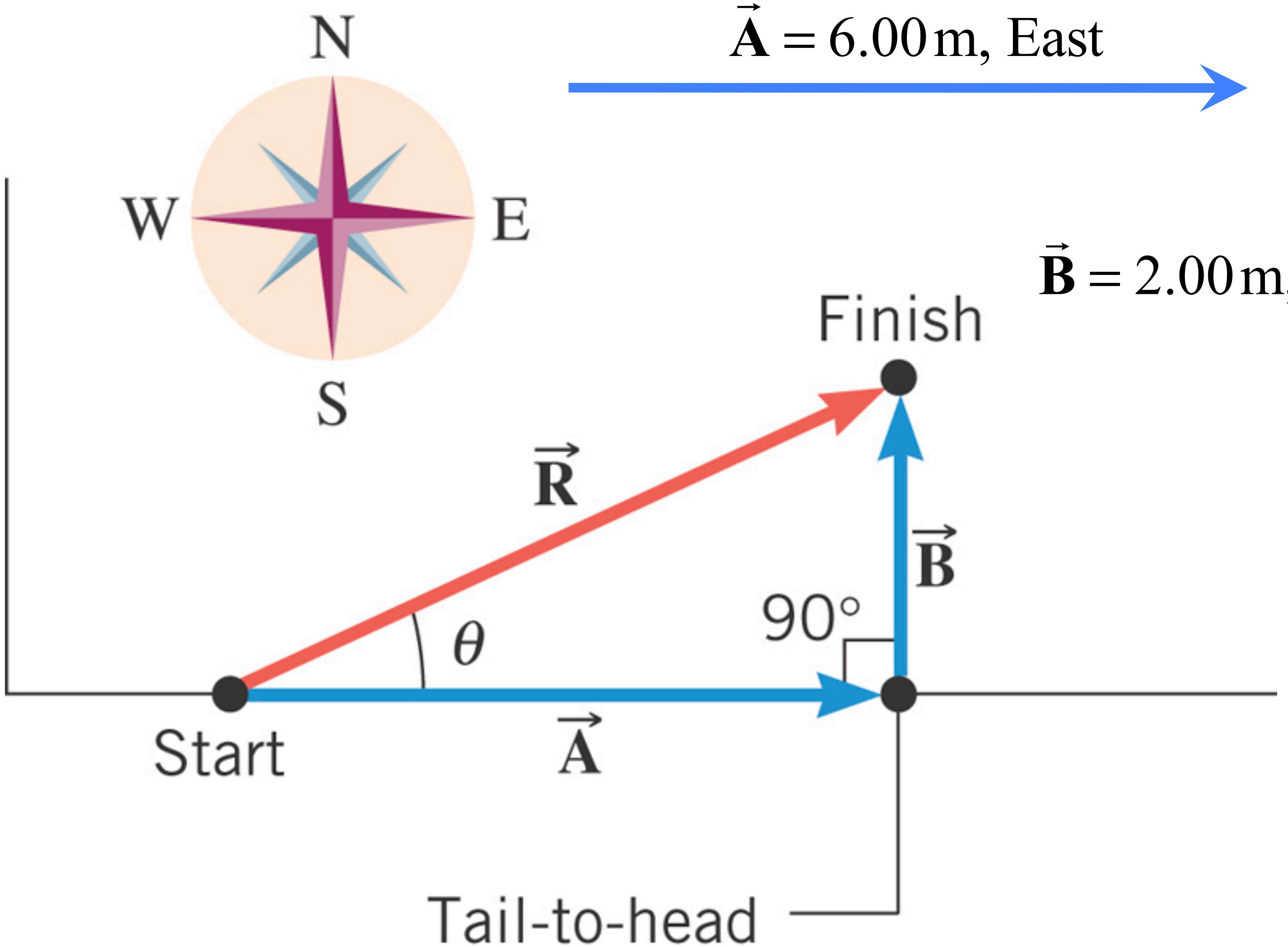


1.6 Vector Addition and Subtraction

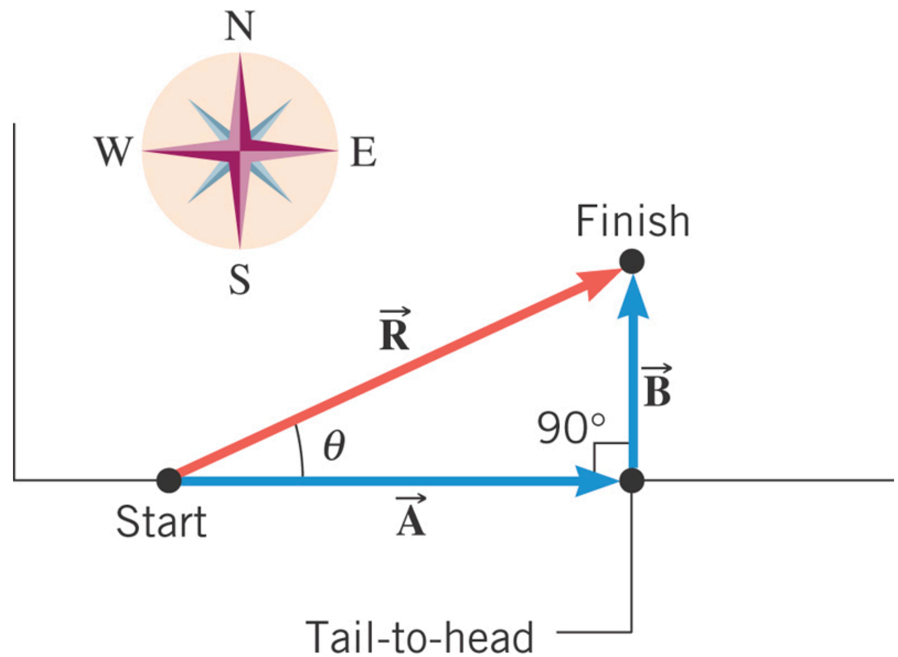
Two displacement vectors

$\vec{\mathbf{A}} = 6.00\,\text{m, East}$

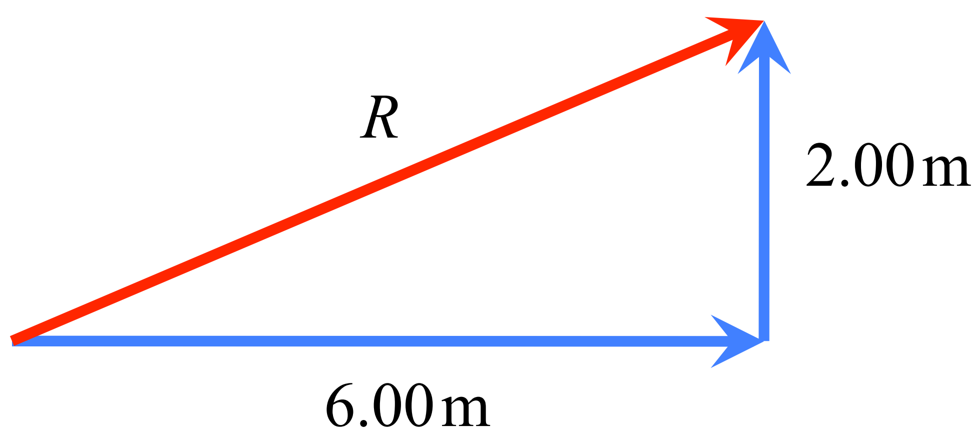
$\vec{\mathbf{B}} = 2.00\,\text{m, North}$



1.6 Vector Addition and Subtraction



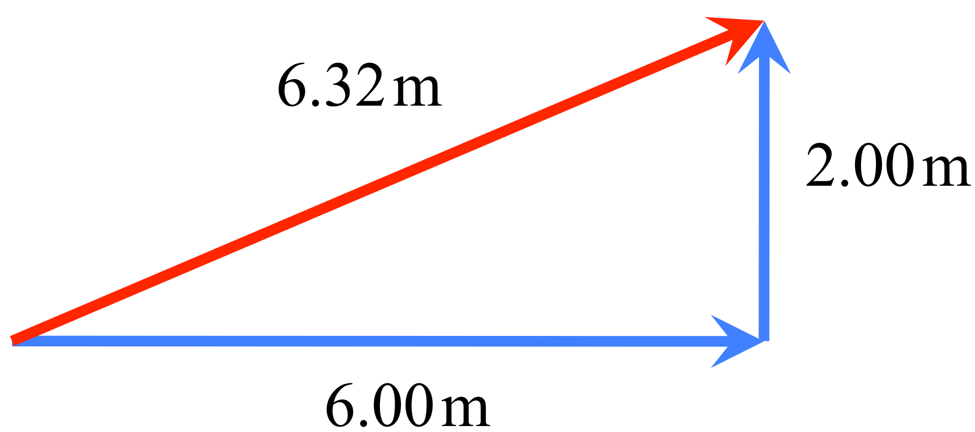
Shown here are the magnitudes of the displacement vectors



1.6 Vector Addition and Subtraction

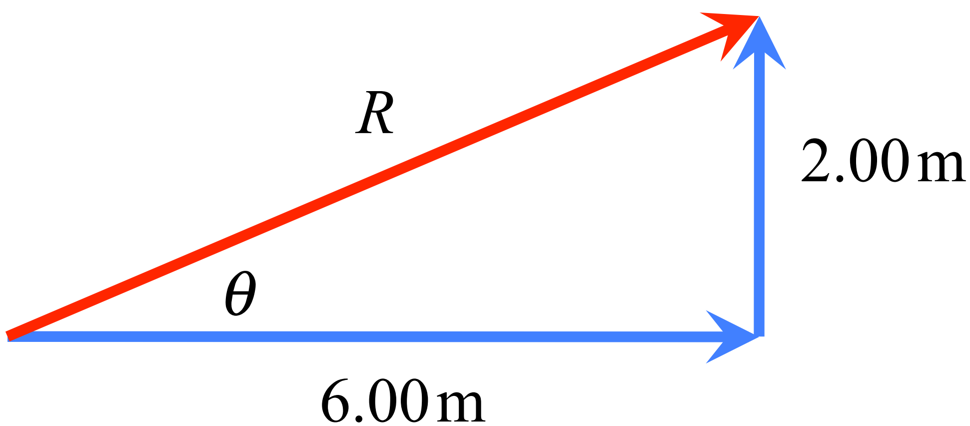
$$R^2 = (2.00\ \text{m})^2 + (6.00\ \text{m})^2$$

$$R = \sqrt{(2.00\ \text{m})^2 + (6.00\ \text{m})^2} = 6.32\text{m}$$



$$\tan\theta = 2.00/6.00$$

$$\theta = \tan^{-1}\left(2.00/6.00\right) = 18.4^{\circ}$$



Clicker Question 1.2

Given three vectors: $\vec{\mathbf{A}}, \vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$, what choice below is always equal to $\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}$?

A) $\vec{\mathbf{C}}+\vec{\mathbf{B}}+\vec{\mathbf{A}}$

B) $\vec{\mathbf{B}}+\vec{\mathbf{A}}+\vec{\mathbf{C}}$

C) $\vec{\mathbf{C}}+\vec{\mathbf{A}}+\vec{\mathbf{B}}$

D) All of the above

E) None of the above

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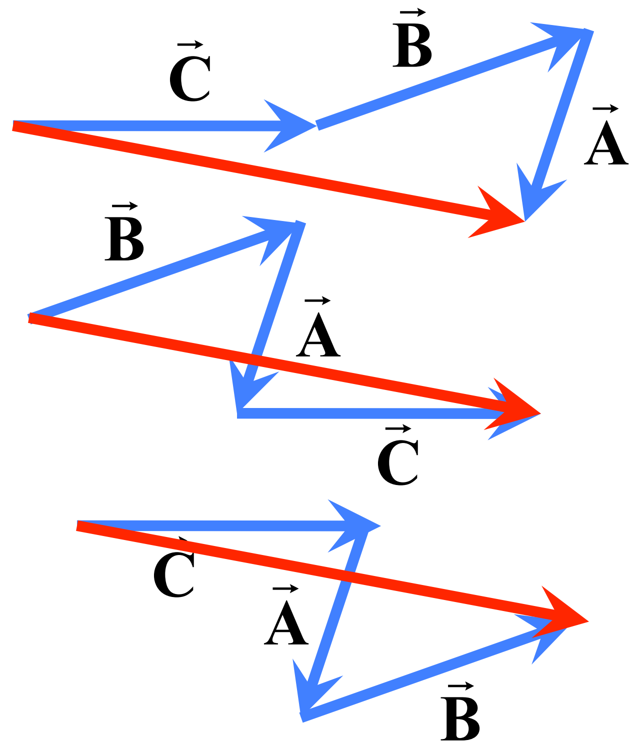
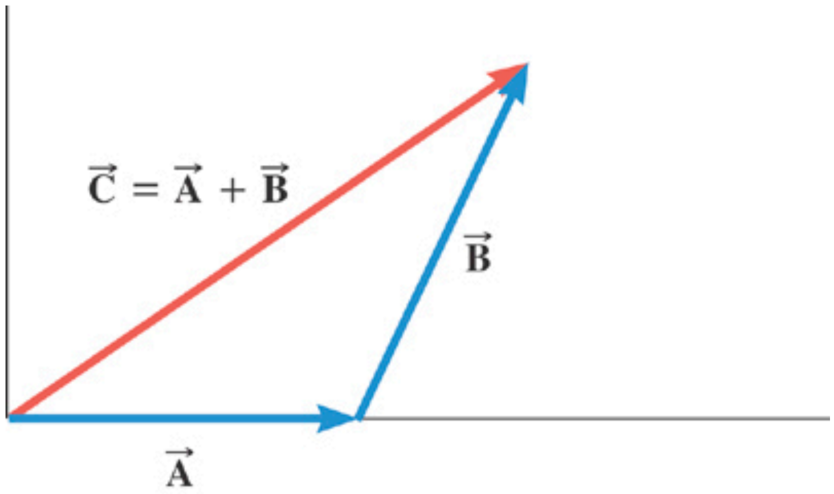
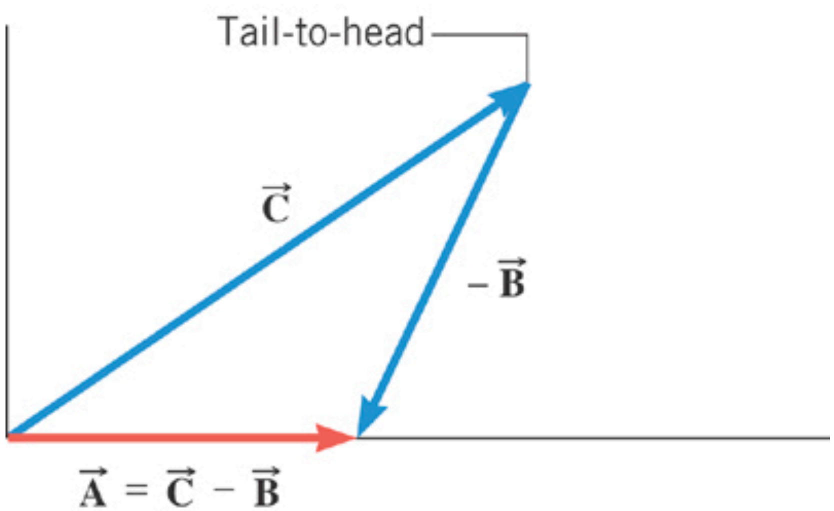


1.6 Vector Addition and Subtraction



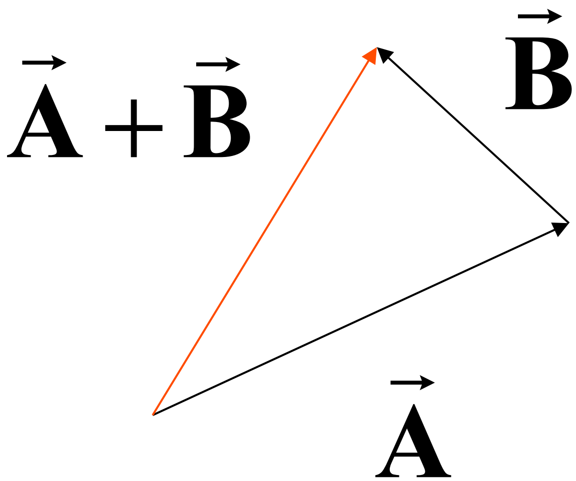
(a)

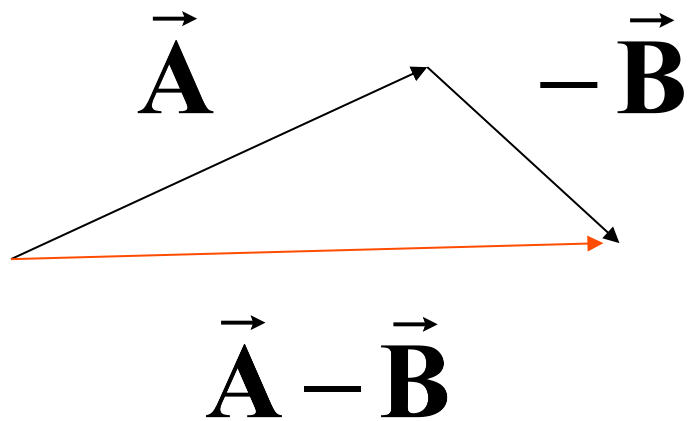


(b)

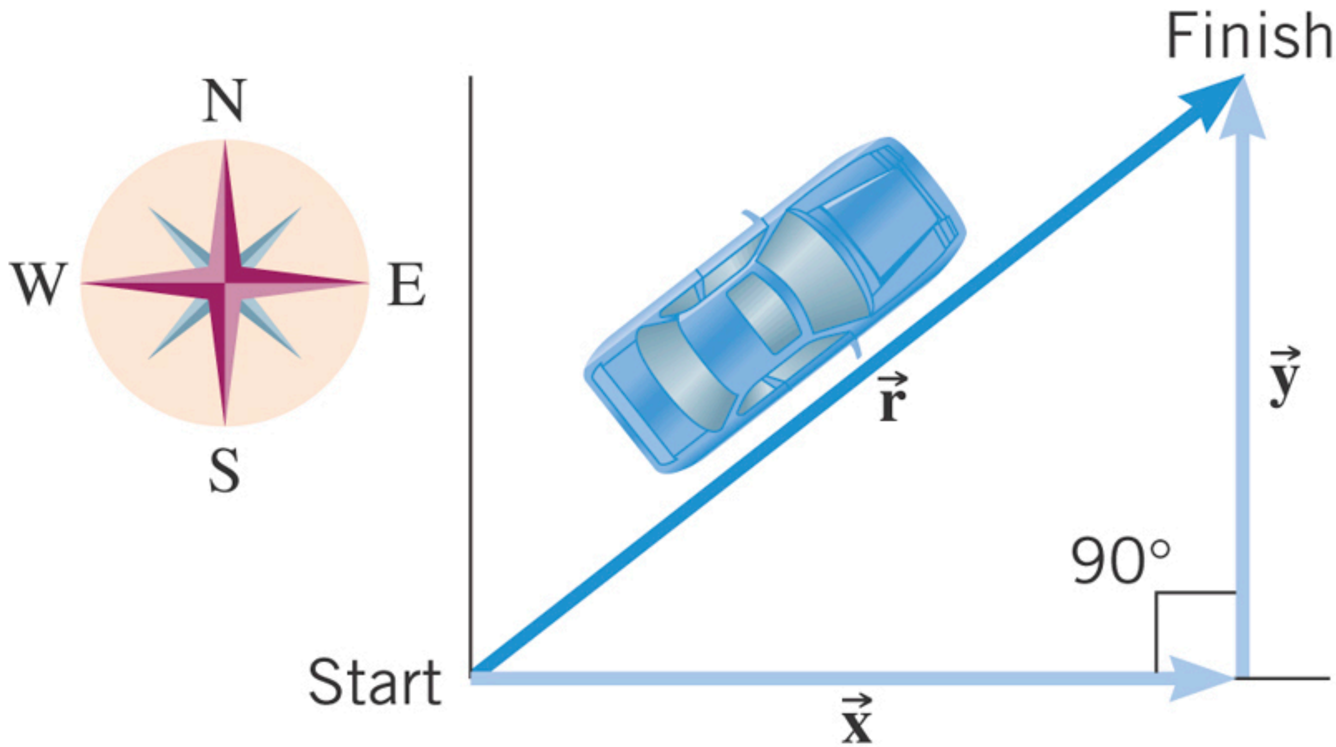
When a vector is multiplied by -1, the magnitude of the vector remains the same, but the direction of the vector is reversed.

1.6 Vector Addition and Subtraction



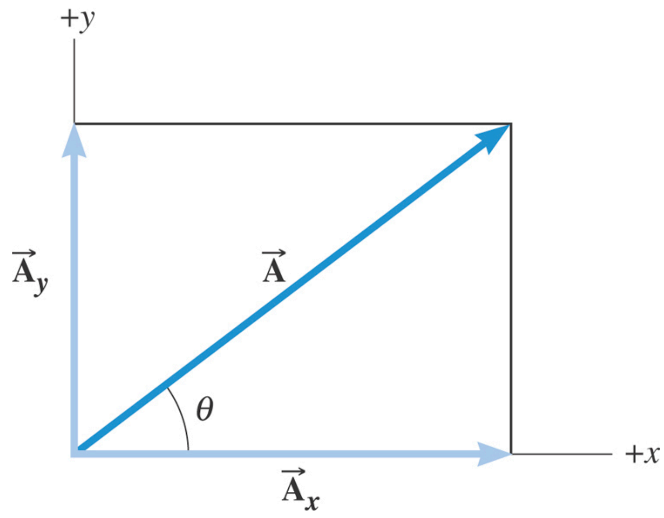


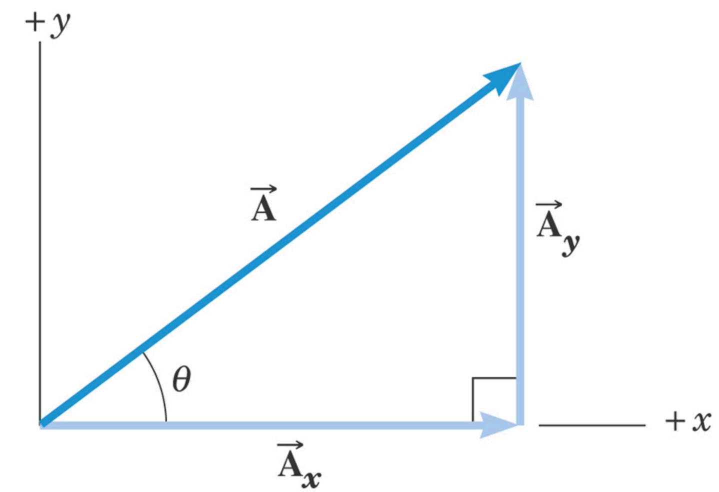
1.7 The Components of a Vector



$\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$ are called the x vector component and the y vector component of $\vec{\mathbf{r}}$.

1.7 The Components of a Vector

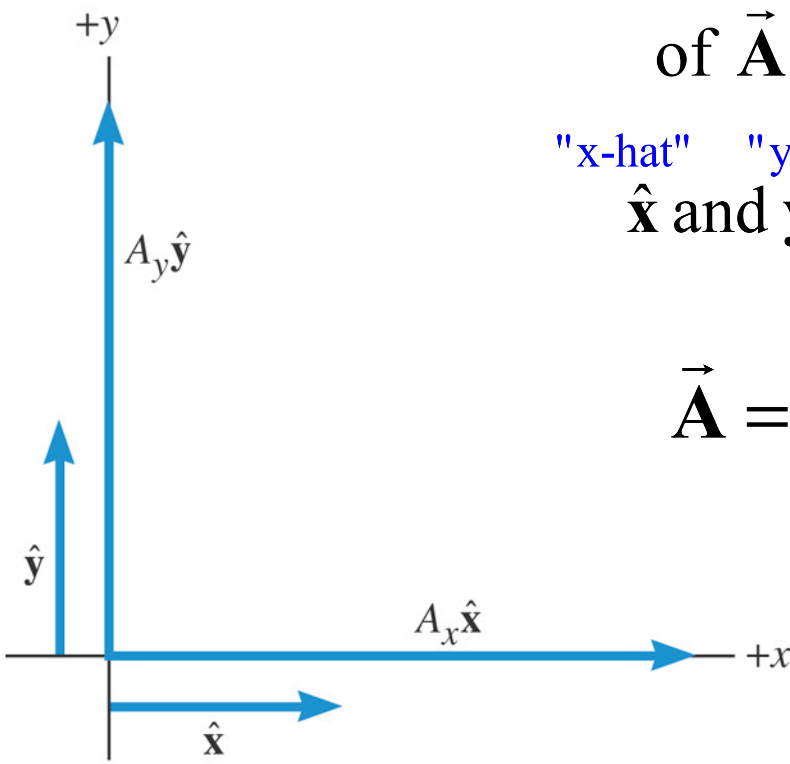




The vector components of $\vec{\mathbf{A}}$ are two perpendicular vectors $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$ that are parallel to the x and y axes, and add together vectorially so that $\vec{\mathbf{A}} = \vec{\mathbf{A}}_\mathbf{x} + \vec{\mathbf{A}}_\mathbf{y}$.

1.7 *The Components of a Vector*

It is often easier to work with the **scalar components** rather than the vector components.



A_x and A_y are the scalar components of $\vec{\mathbf{A}}$.

"x-hat" "y-hat"

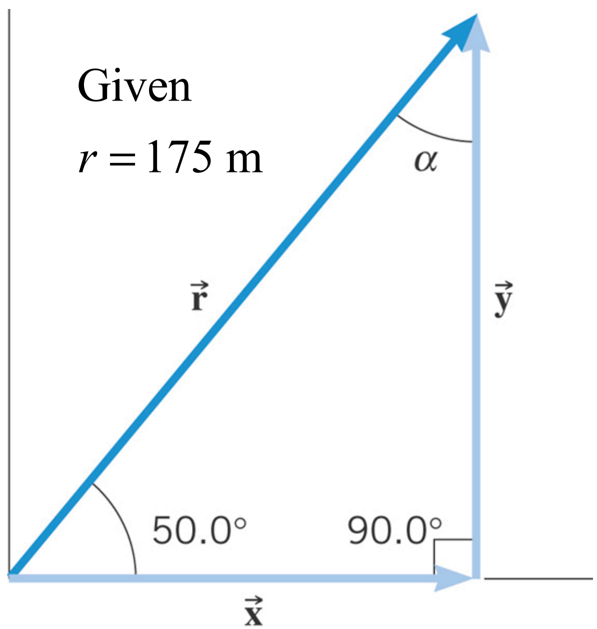
$\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are unit vectors with magnitude 1.

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}}$$

1.7 *The Components of a Vector*

Example

A displacement vector has a magnitude of 175 m and points at an angle of 50.0 degrees relative to the *x* axis. Find the *x* and *y* components of this vector.



vector $\vec{\mathbf{x}}$ has magnitude x

$$\sin\theta = y/r$$

y-component of the vector $\vec{\mathbf{r}}$

$$y = r\sin\theta = (175\text{ m})(\sin 50.0^\circ) = 134\text{ m}$$

$$\cos\theta = x/r$$

x-component of the vector $\vec{\mathbf{r}}$

$$x = r\cos\theta = (175\text{ m})(\cos 50.0^\circ) = 112\text{ m}$$

$$\vec{\mathbf{r}} = (112\text{ m})\hat{\mathbf{x}} + (134\text{ m})\hat{\mathbf{y}}$$

Clicker Question 1.3 $\vec{\mathbf{A}} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}}$, where $A_x = 3\text{m}$, and $A_y = 4\text{m}$.

What is the magnitude of the vector $\vec{\mathbf{A}}$?

A) $7\,\text{m}$

B) $5\,\text{m}$

C) $\sqrt{7}\,\text{m}$

D) $6\,\text{m}$

E) $25\,\text{m}$

Clicker Question 1.3 $\vec{\mathbf{A}} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}}$, where $A_x = 3\text{m}$, and $A_y = 4\text{m}$.

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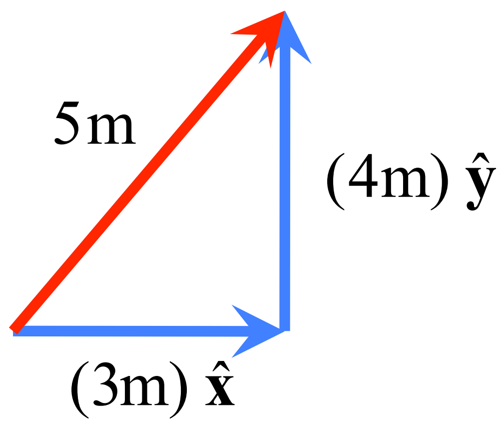
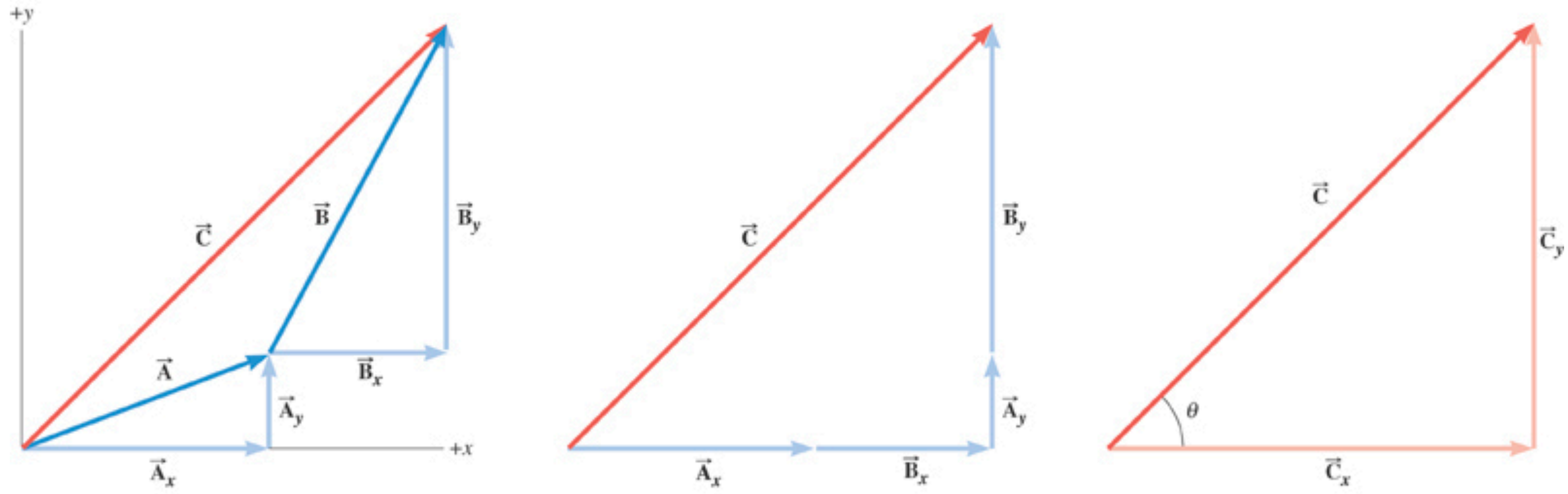


$$\vec{\mathbf{A}} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}}$$

magnitude of $\vec{\mathbf{A}}$, $A = \sqrt{A_x^2 + A_y^2} = \sqrt{(3\text{m})^2 + (4\text{m})^2}$

$$= \left(\sqrt{9+16}\right)\text{m} = \sqrt{25}\ \text{m} = 5\text{m}$$

1.8 Addition of Vectors by Means of Components

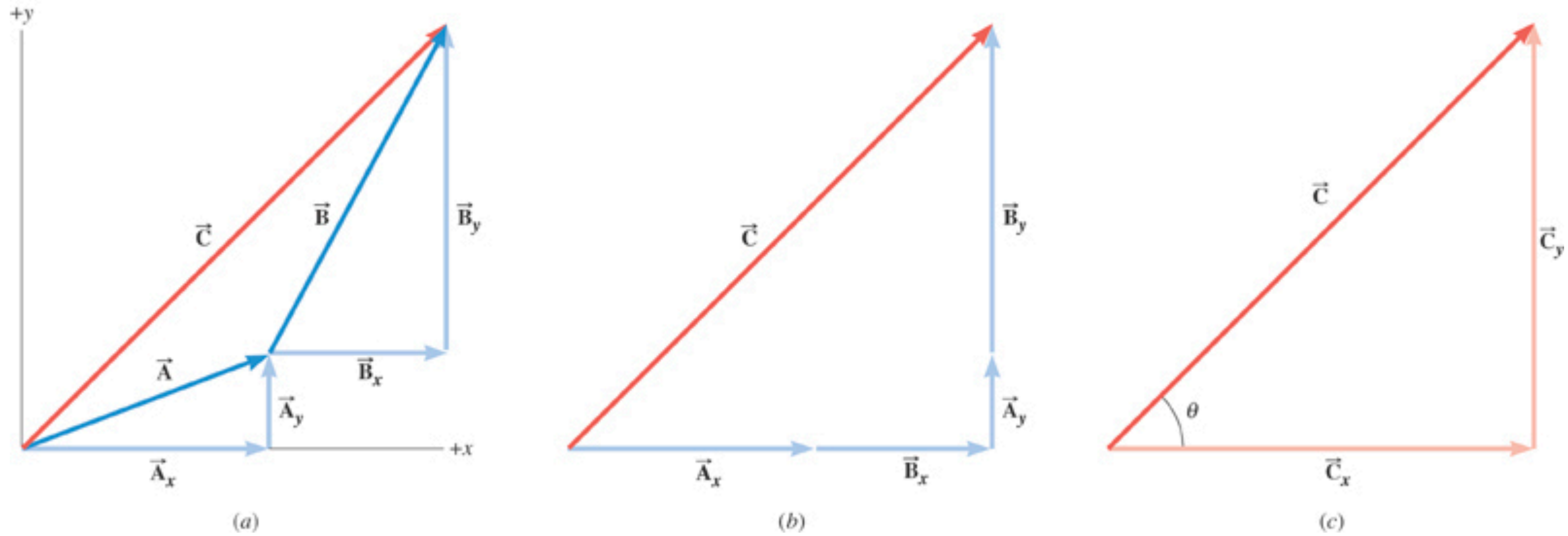


(a) (b) (c)

$$\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$

$$\vec{\mathbf{A}} = A_x \hat{\mathbf{x}} + A_y \hat{\mathbf{y}} \qquad \vec{\mathbf{B}} = B_x \hat{\mathbf{x}} + B_y \hat{\mathbf{y}}$$

1.8 Addition of Vectors by Means of Components



$$\vec{\mathbf{C}} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}}$$

$$= \left(A_x + B_x\right)\hat{\mathbf{x}} + \left(A_y + B_y\right)\hat{\mathbf{y}}$$

$$C_x = A_x + B_x \qquad\qquad C_y = A_y + B_y$$