Chapter 2

Kinematics in One Dimension

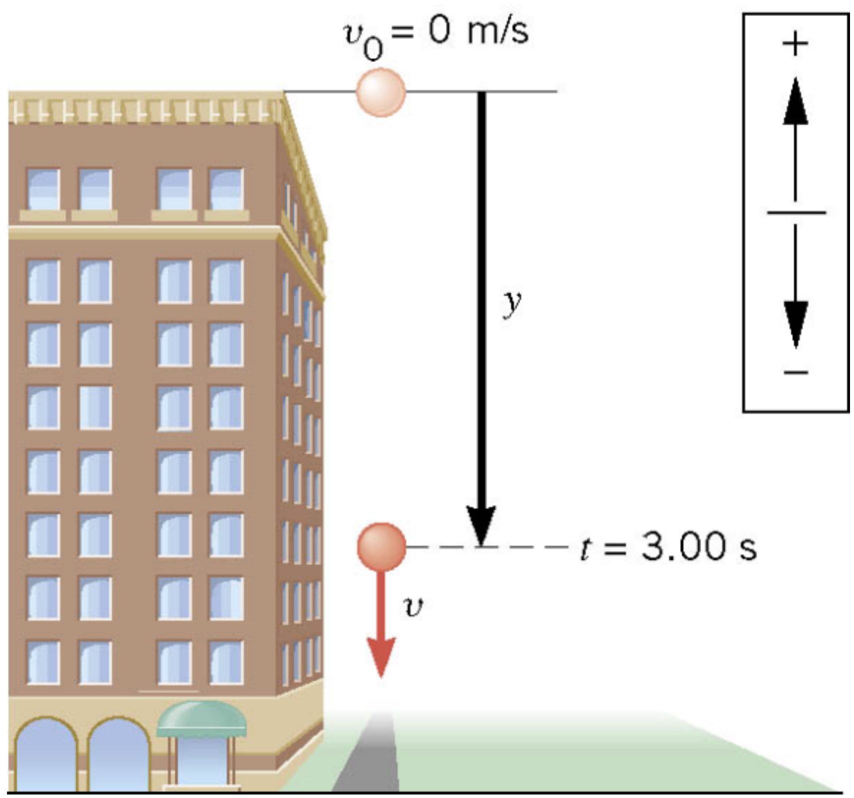
continued

C. Bromberg office hours changed:
11:00 am – 1:00 pm, Learning Center BPS1248
Also, see website: http://www.pa.msu.edu/courses/phy231
for TA hours.

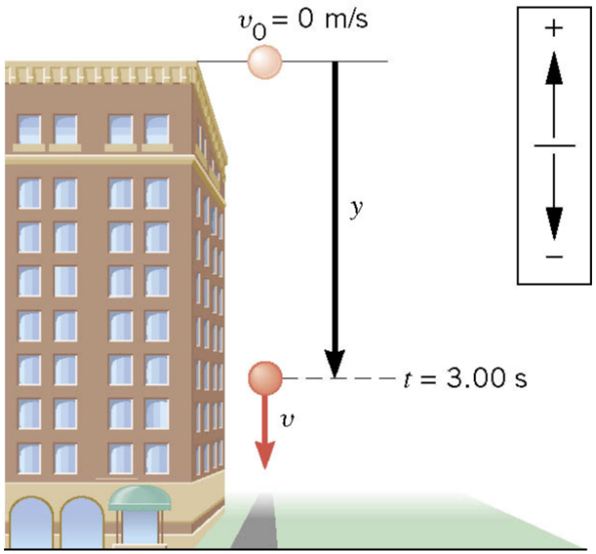
2.6 *Freely Falling Bodies*

Example 10 A Falling Stone

A stone is dropped from the top of a tall building. After 3.00s of free fall, what is the displacement *y* of the stone?



2.6 Freely Falling Bodies



y	a	v	v_o	t
?	-9.80 m/s^2		0 m/s	3.00 s

$$y = v_o t + \tfrac{1}{2} a t^2$$

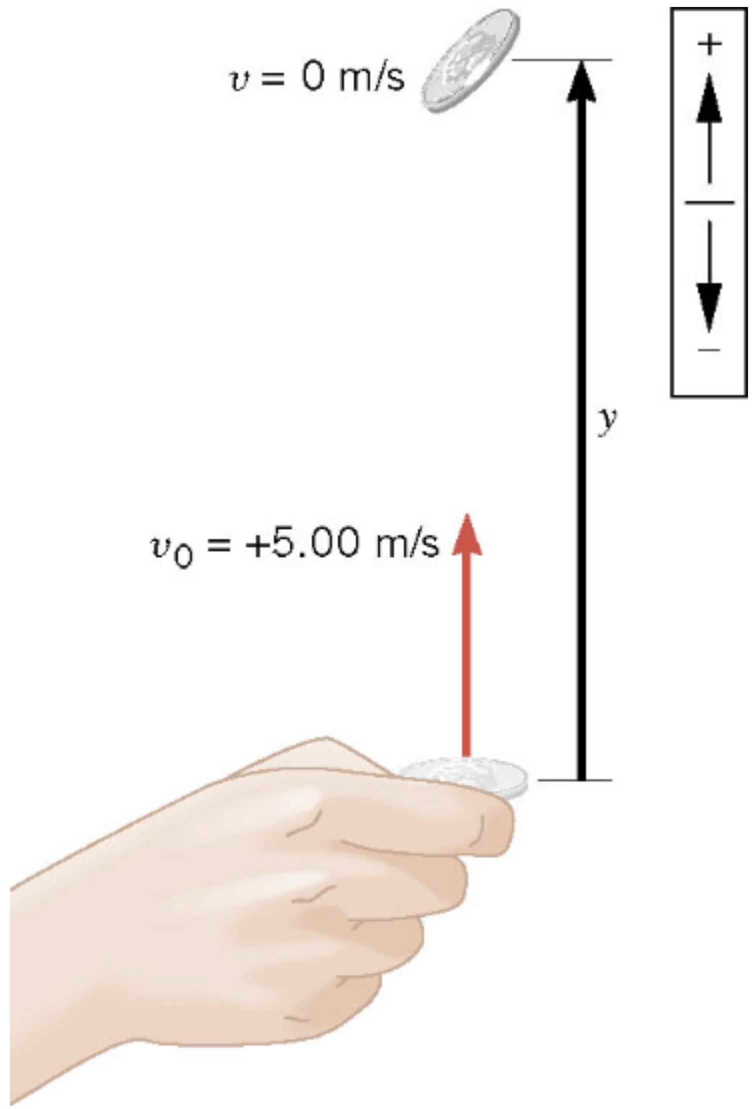
$$= \left(0\,\mathrm{m/s}\right)\left(3.00\ \mathrm{s}\right) + \tfrac{1}{2}\left(-9.80\,\mathrm{m/s^2}\right)\left(3.00\ \mathrm{s}\right)^2$$

$$= -44.1\ \mathrm{m}$$

2.6 *Freely Falling Bodies*

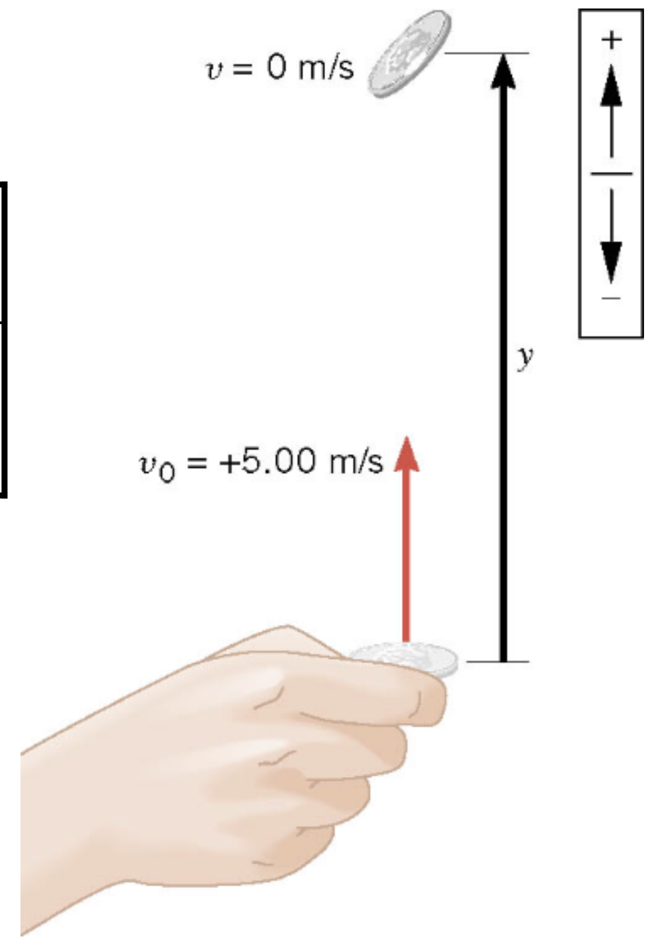
Example 12 How High Does it Go?

The referee tosses the coin up with an initial speed of 5.00m/s. In the absence if air resistance, how high does the coin go above its point of release?



2.6 Freely Falling Bodies

y	a	v	v_o	t
?	-9.80 m/s^2	0 m/s	+5.00 m/s	



$$v^2 = v_o^2 + 2ay \quad \Rightarrow \quad y = \frac{v^2 - v_o^2}{2a}$$

$$y = \frac{v^2 - v_o^2}{2a} = \frac{(0\,\mathrm{m/s})^2 - (5.00\,\mathrm{m/s})^2}{2\left(-9.80\,\mathrm{m/s^2}\right)} = 1.28\,\mathrm{m}$$

2.6 *Freely Falling Bodies*

Conceptual Example 14 Acceleration Versus Velocity

There are three parts to the motion of the coin.

1) On the way up, the coin has an upward-pointing velocity with a decreasing magnitude.
2) At that time the coin reaches the top of its path, the coin has an instantaneously zero velocity.
3) On the way down, the coin has a downward-pointing velocity with an increasing magnitude.

In the absence of air resistance, does the acceleration vector of the coin, like the velocity, change from one part to another?

Clicker Question 2.3

During the free flight (no air resistance) of a coin thrown upward, what are the values of the acceleration at these times during the motion ?

	$\vec{\mathbf{a}} =$ On the way up	$\vec{\mathbf{a}} =$ At the top	$\vec{\mathbf{a}} =$ On the way down
a)	$+9.80\mathrm{m/s}^2$	$+9.80\mathrm{m/s}^2$	$+9.80\mathrm{m/s}^2$
b)	$+9.80\mathrm{m/s}^2$	$0.0\ \mathrm{m/s}^2$	$+9.80\mathrm{m/s}^2$
c)	$+9.80\mathrm{m/s}^2$	$0.0\ \mathrm{m/s}^2$	$-9.80\mathrm{m/s}^2$
d)	$-9.80\mathrm{m/s}^2$	$0.0\ \mathrm{m/s}^2$	$-9.80\mathrm{m/s}^2$
e)	$-9.80\mathrm{m/s}^2$	$-9.80\mathrm{m/s}^2$	$-9.80\mathrm{m/s}^2$

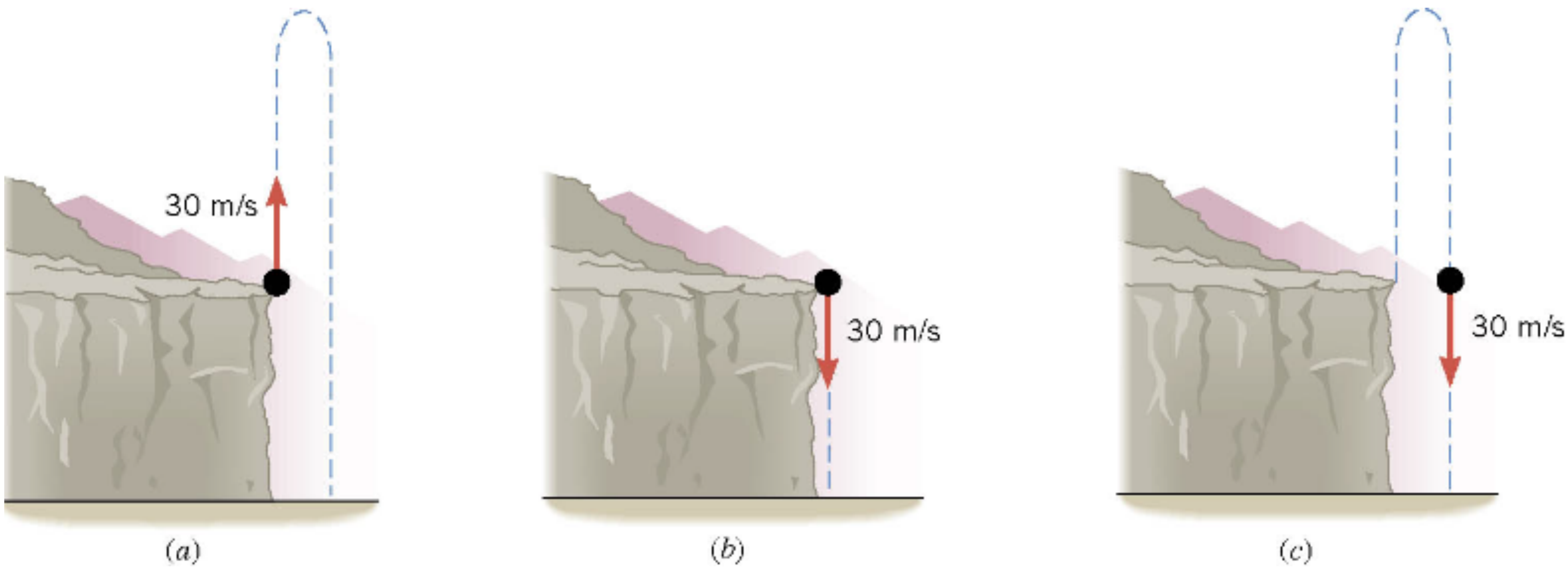
Clicker Question 2.3

During the free flight (no air resistance) of a coin thrown upward, what are the values of the acceleration at these times during the motion ?

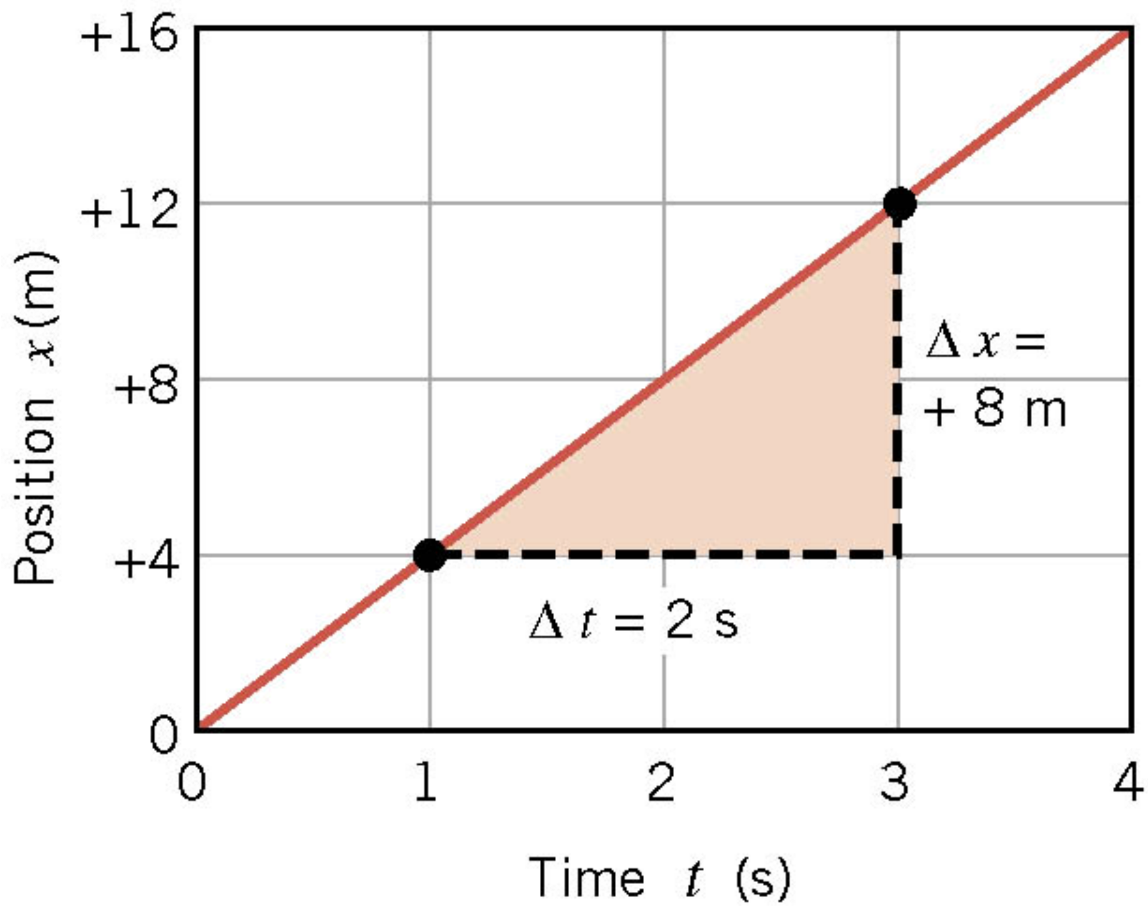
	$\vec{\mathbf{a}} =$ On the way up	$\vec{\mathbf{a}} =$ At the top	$\vec{\mathbf{a}} =$ On the way down
a)	$+9.80\text{m/s}^2$	$+9.80\text{m/s}^2$	$+9.80\text{m/s}^2$
b)	$+9.80\text{m/s}^2$	$0.0\ \text{m/s}^2$	$+9.80\text{m/s}^2$
c)	$+9.80\text{m/s}^2$	$0.0\ \text{m/s}^2$	-9.80m/s^2
d)	-9.80m/s^2	$0.0\ \text{m/s}^2$	-9.80m/s^2
e)	-9.80m/s^2	-9.80m/s^2	-9.80m/s^2

Conceptual Example 15 Taking Advantage of Symmetry

Does the pellet in part *b* strike the ground beneath the cliff with a smaller, greater, or the same speed as the pellet in part *a*?

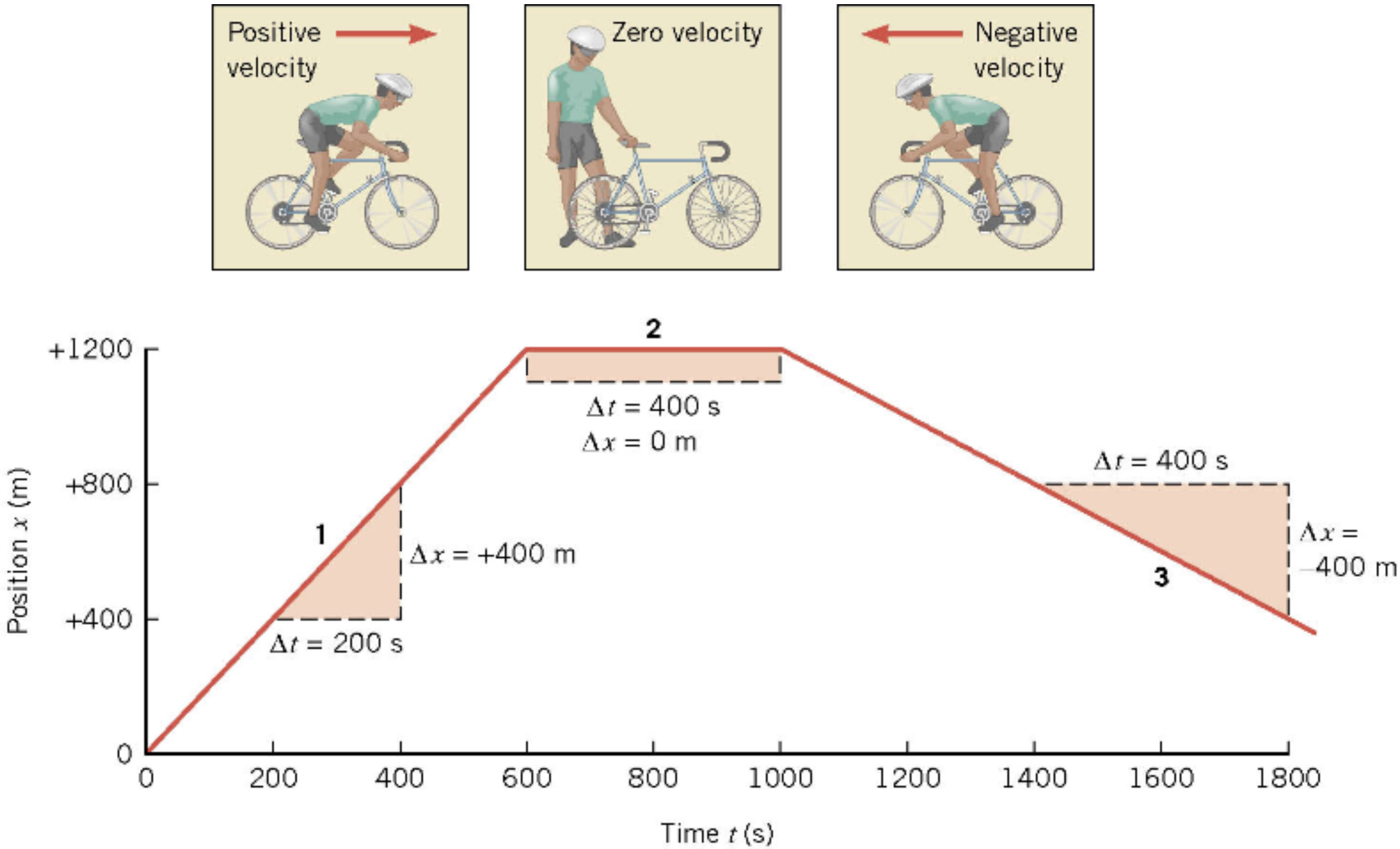


2.7 Graphical Analysis of Velocity and Acceleration

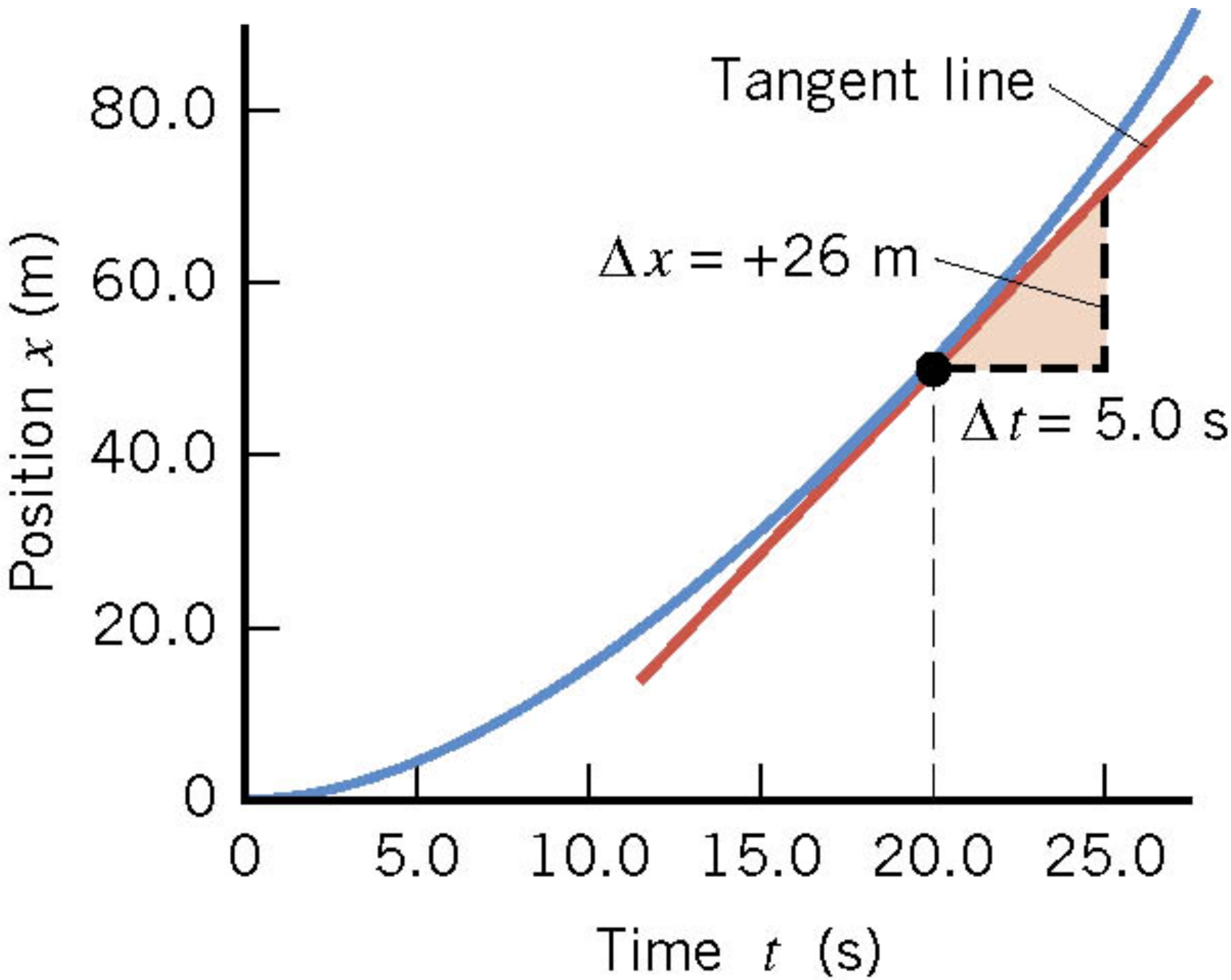


$$\text{Slope} = \frac{\Delta x}{\Delta t} = \frac{+8\,\text{m}}{2\,\text{s}} = +4\,\text{m/s}$$

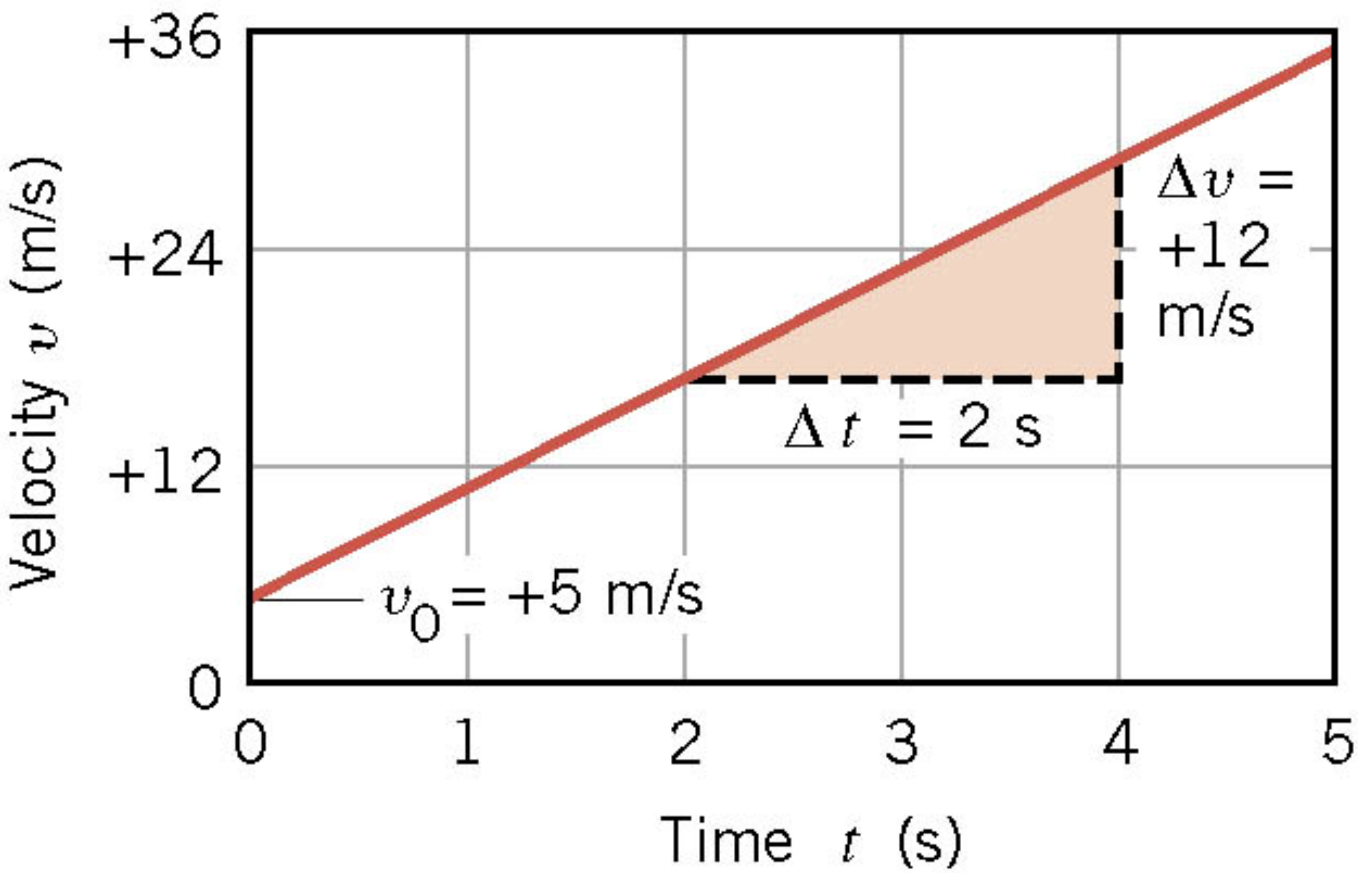
2.7 Graphical Analysis of Velocity and Acceleration



2.7 Graphical Analysis of Velocity and Acceleration



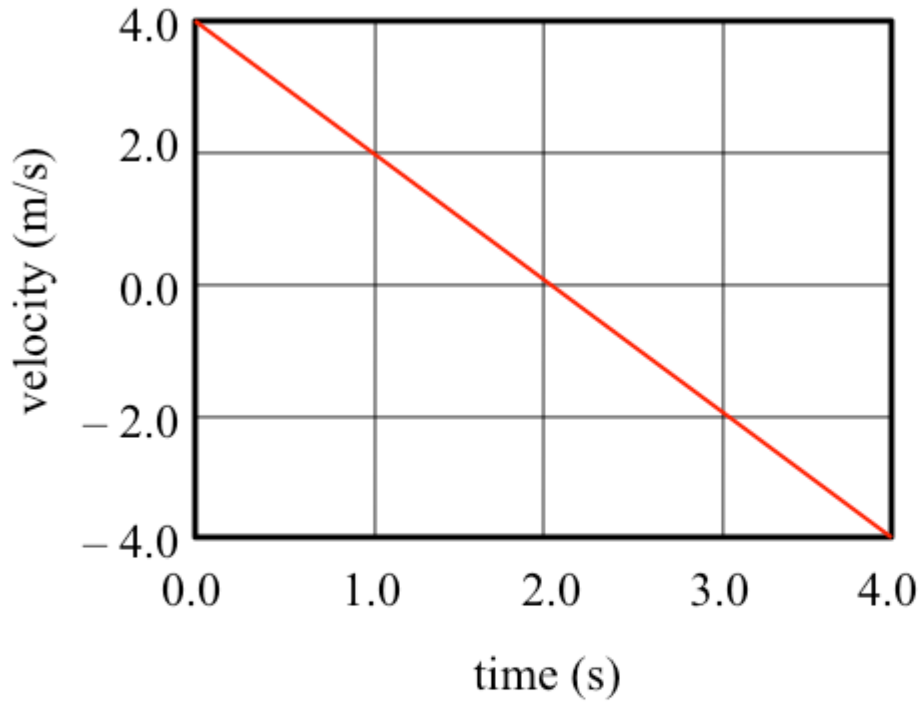
2.7 Graphical Analysis of Velocity and Acceleration



$$\text{Slope} = \frac{\Delta v}{\Delta t} = \frac{+12\ \text{m/s}}{2\ \text{s}} = +6\ \text{m/s}^2$$

Clicker Question 2.4

Consider the motion shown in the plot.



What is the acceleration at the time when $v = 0$ m/s ?

a) $a = 0$

b) $a = +2\ \text{m/s}^2$

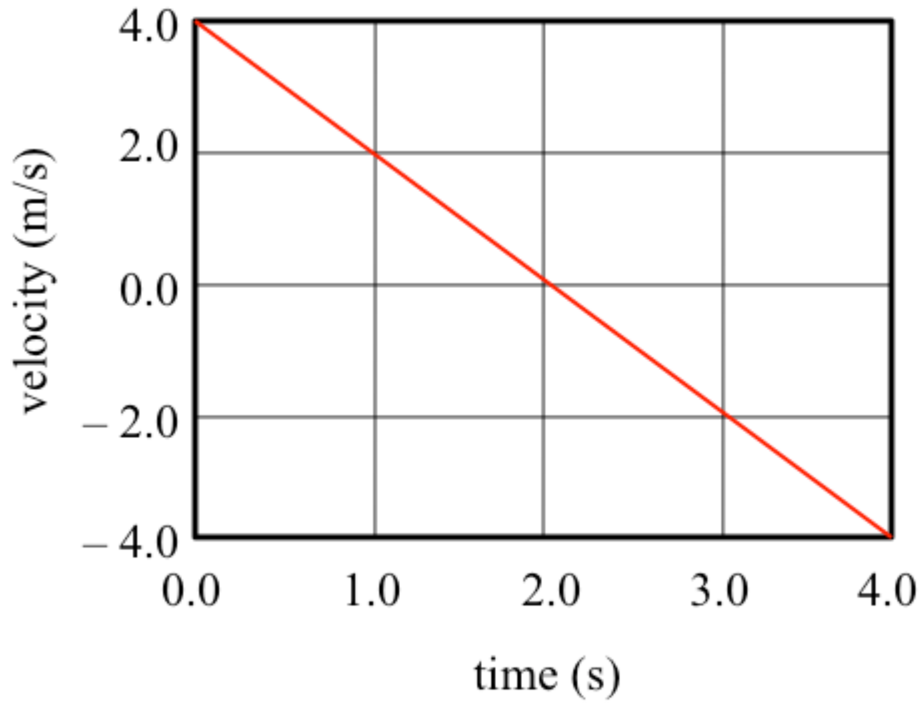
c) $a = +4\ \text{m/s}^2$

d) $a = -2\ \text{m/s}^2$

e) $a = -4\ \text{m/s}^2$

Clicker Question 2.4

Consider the motion shown in the plot.



What is the acceleration at the time when $v = 0$ m/s ?

a) $a = 0$

b) $a = +2 \text{ m/s}^2$

c) $a = +4 \text{ m/s}^2$

d) $\boxed{a = -2 \text{ m/s}^2}$

e) $a = -4 \text{ m/s}^2$

Chapter 3

Kinematics in Two Dimensions

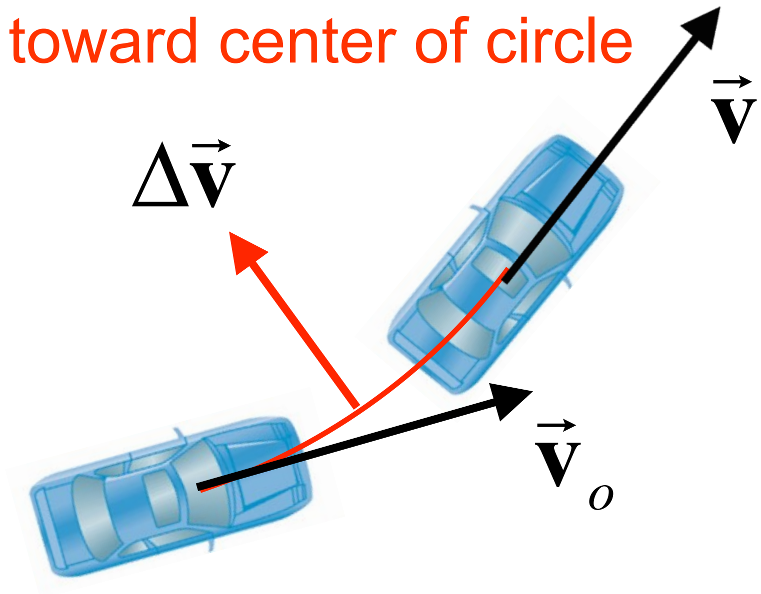
3.1 Displacement, Velocity, and Acceleration

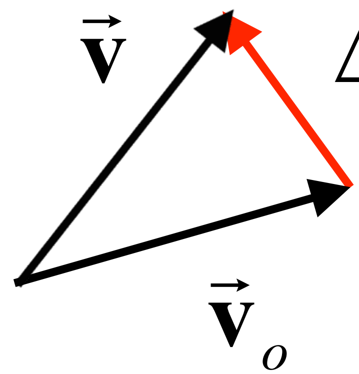
DEFINITION OF AVERAGE ACCELERATION

$$\overline{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_o}{t - t_o} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$

Consider a velocity that keeps the same magnitude (speed) but changes its direction, e.g., car going around a curve.

acceleration points toward center of circle

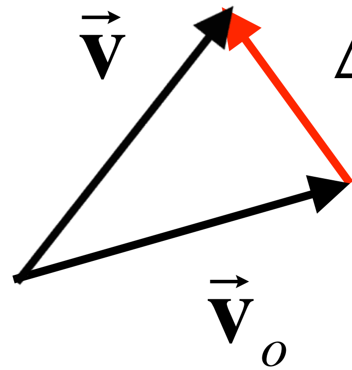




$$\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}} - \vec{\mathbf{v}}_0$$

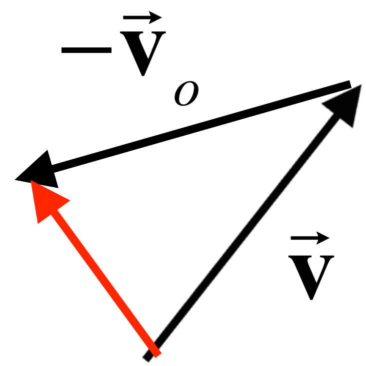
How to see this:

$$\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}} - \vec{\mathbf{v}}_0$$

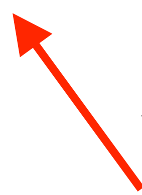


$$\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}} - \vec{\mathbf{v}}_0$$

$$= \vec{\mathbf{v}} + \left(-\vec{\mathbf{v}}_0\right)$$



acceleration points
toward center of circle



$$\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}} - \vec{\mathbf{v}}_0$$

Need in Chapter 5 when uniform circular motion is discusssed.

3.2 Equations of Kinematics in Two Dimensions

Equations of 1D Kinematics

$$v = v_0 + at$$

$$x = \tfrac{1}{2}\left(v_0 + v\right)t$$

$$v^2 = v_0^2 + 2ax$$

$$x = v_0 t + \tfrac{1}{2}at^2$$

Clicker Question 3.1

A car starting at rest maintains a constant acceleration, *a*. After a time, *t*, its displacement and velocity are, *x* and *v*. What are the displacement and velocity at time 2*t* ?

a) $2x$ and $2v$

b) $2x$ and $4v$

c) $4x$ and $2v$

d) $4x$ and $4v$

e) $8x$ and $4v$

Clicker Question 3.1

A car starting at rest maintains a constant acceleration, *a*. After a time, *t*, its displacement and velocity are, *x* and *v*. What are the displacement and velocity at time 2*t* ?

a) $2x$ and $2v$

b) $2x$ and $4v$

$\boxed{\text{c) } 4x \text{ and } 2v}$

d) $4x$ and $4v$

e) $8x$ and $4v$

$$v_0 = 0, \quad x, v \text{ at time } t, \quad x', v' \text{ at time } t' = 2t$$

acceleration, a, is the same at all time.

$$x = \tfrac{1}{2}at^2 \Rightarrow \quad a = \frac{2x}{t^2}$$

$$x' = \tfrac{1}{2}at'^2 \Rightarrow \quad a = \frac{2x'}{t'^2}$$

$$\frac{2x'}{(2t)^2} = \frac{2x'}{4t^2}$$

$$\boxed{\frac{x}{t^2} = \frac{x'}{4t^2} \quad \Rightarrow \quad \underline{x' = 4x}}$$

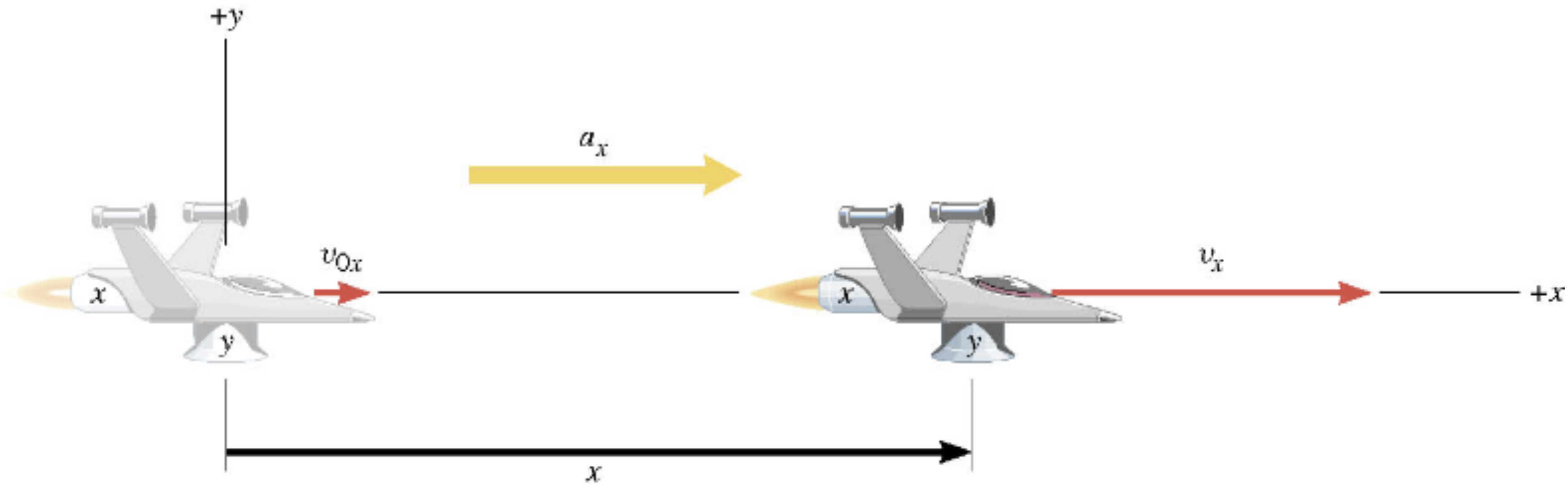
$$a = \frac{v}{t}$$

$$a = \frac{v'}{t'} = \frac{v'}{2t}$$

$$\boxed{\frac{v}{t} = \frac{v'}{2t} \Rightarrow \underline{v' = 2v}}$$

3.2 Equations of Kinematics in Two Dimensions

Except for time, motion in x and y directions are INDEPENDENT.



Motion in x direction.

$$v_x = v_{0x} + a_x t$$

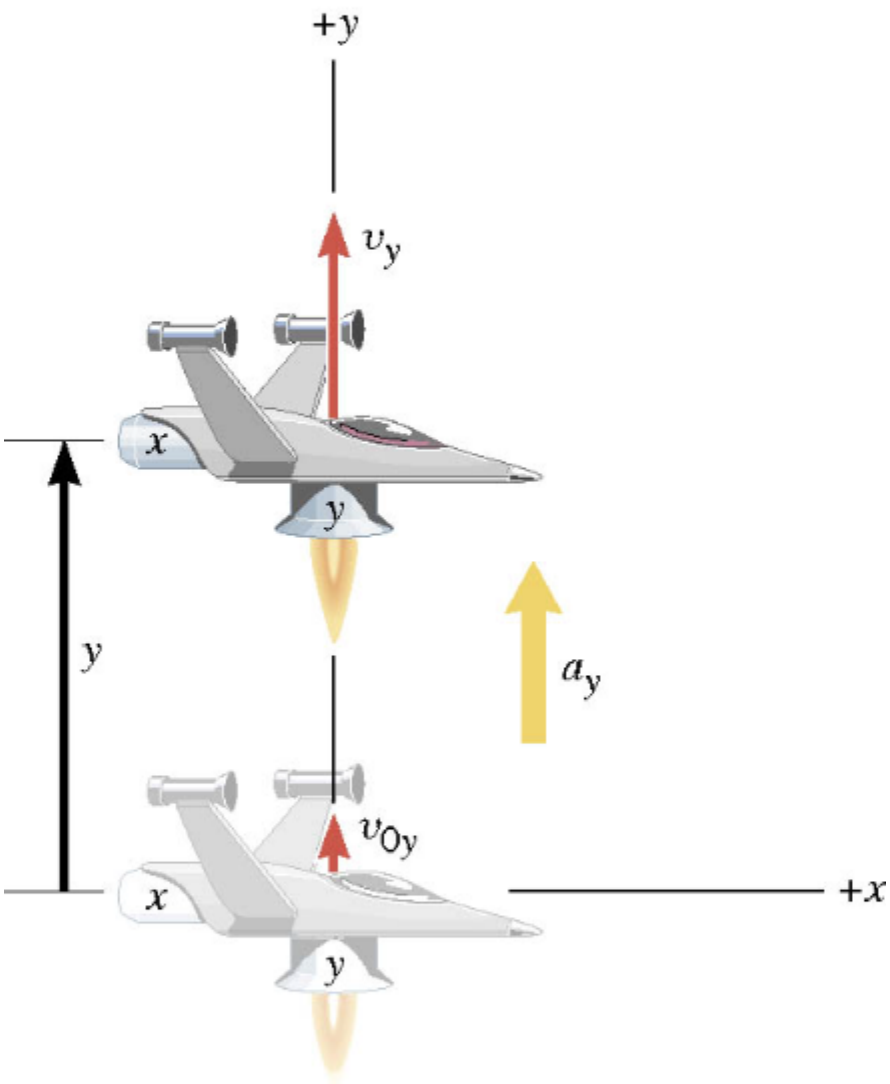
$$x = \tfrac{1}{2}\left(v_{0x} + v_x\right)t$$

$$x = v_{0x}t + \tfrac{1}{2}a_x t^2$$

$$v_x^2 = v_{0x}^2 + 2a_x x$$

3.2 Equations of Kinematics in Two Dimensions

Except for time, motion in x and y directions are INDEPENDENT.



Motion in y direction.

$$v_y = v_{0y} + a_y t$$

$$y = v_{0y} t + \tfrac{1}{2} a_y t^2$$

$$y = \tfrac{1}{2}\left(v_{0y} + v_y\right) t$$

$$v_y^2 = v_{0y}^2 + 2 a_y y$$

3.2 *Equations of Kinematics in Two Dimensions*

Reasoning Strategy

1. Make a drawing.

2. Decide which directions are to be called positive (+) and negative (-).

3. Write down the values that are given for any of the five kinematic variables associated with each direction.

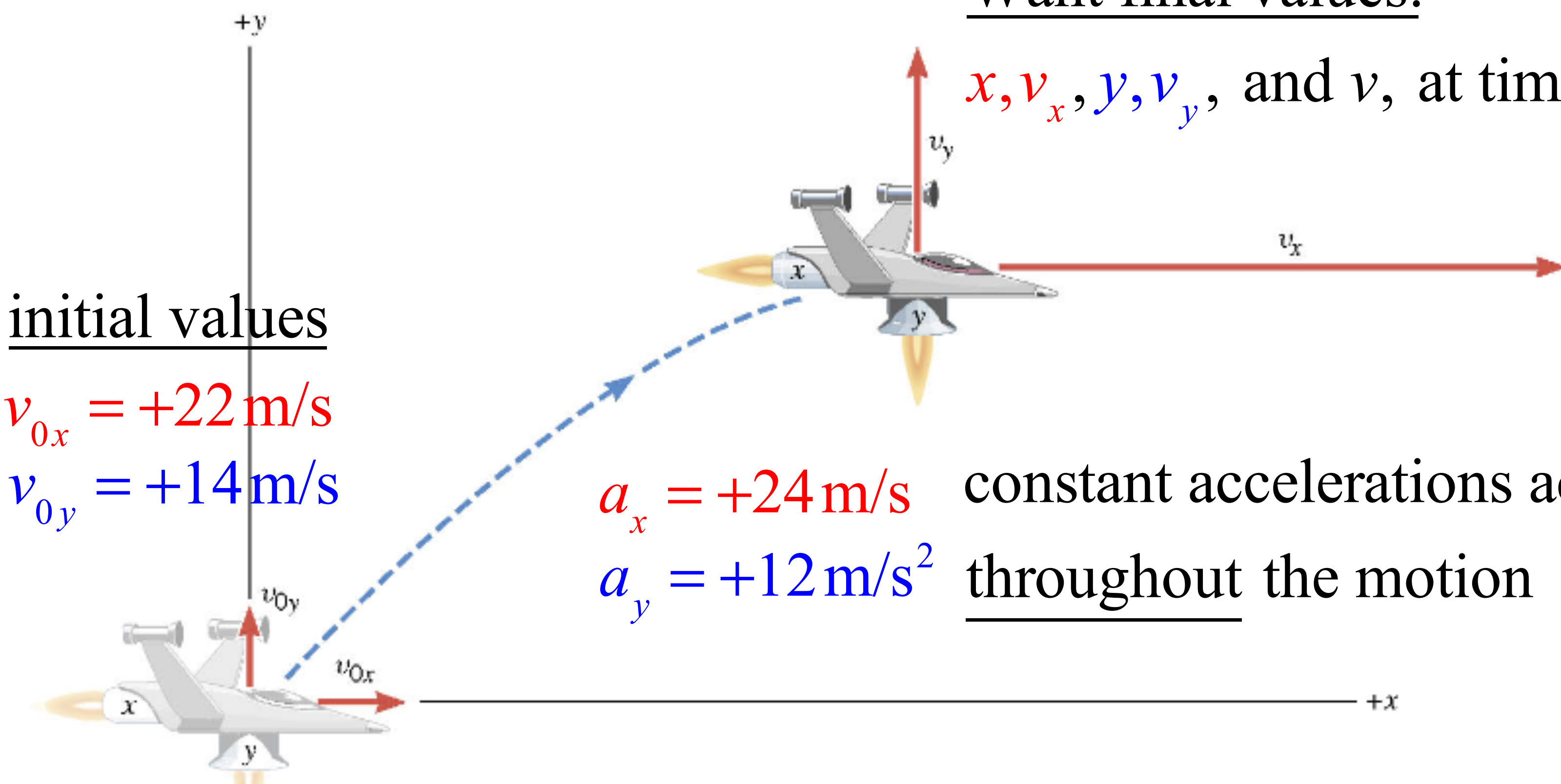
3.2 Equations of Kinematics in Two Dimensions

Example 1 A Moving Spacecraft

In the *x* direction, the spacecraft has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) *x* and v_x, (b) *y* and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.

Want final values:

x, v_x, y, v_y, and v, at time $t = 7$s.



initial values

$v_{0x} = +22\,\text{m/s}$

$v_{0y} = +14\,\text{m/s}$

$a_x = +24\,\text{m/s}$

$a_y = +12\,\text{m/s}^2$

constant accelerations act throughout the motion

3.2 *Equations of Kinematics in Two Dimensions*

Reasoning Strategy

1. Make a drawing.

2. Decide which directions are to be called positive (+) and negative (-).

3. Write down the values that are given for any of the five kinematic variables associated with each direction.

4. Verify that the information contains values for at least three of the kinematic variables. Do this for x and y. Select the appropriate equation.

5. When the motion is divided into segments, remember that the final velocity of one segment is the initial velocity for the next.

6. Keep in mind that there may be two possible answers to a kinematics problem.

3.2 *Equations of Kinematics in Two Dimensions*

Example 1 A Moving Spacecraft:

x direction motion

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22 m/s	7.0 s

$$x = v_{ox}t + \tfrac{1}{2}a_x t^2$$

$$= (22\,\text{m/s})(7.0\ \text{s}) + \tfrac{1}{2}(24\,\text{m/s}^2)(7.0\ \text{s})^2 = +740\ \text{m}$$

$$v_x = v_{ox} + a_x t$$

$$= (22\,\text{m/s}) + (24\,\text{m/s}^2)(7.0\ \text{s}) = +190\,\text{m/s}$$

3.2 *Equations of Kinematics in Two Dimensions*

Example 1 A Moving Spacecraft:

y direction motion

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14 m/s	7.0 s

$$y = v_{oy}t + \tfrac{1}{2}a_y t^2$$

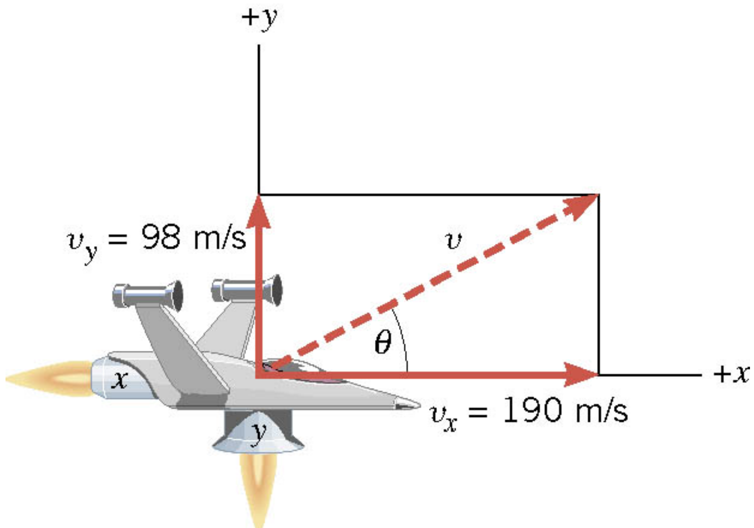
$$= (14\,\text{m/s})(7.0\text{ s}) + \tfrac{1}{2}(12\,\text{m/s}^2)(7.0\text{ s})^2 = +390\text{ m}$$

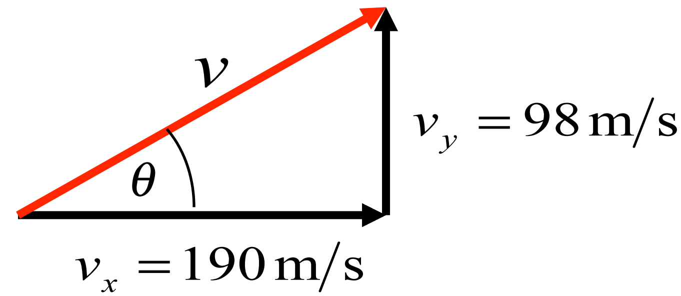
$$v_y = v_{oy} + a_y t$$

$$= (14\,\text{m/s}) + (12\,\text{m/s}^2)(7.0\text{ s}) = +98\,\text{m/s}$$

3.2 Equations of Kinematics in Two Dimensions

Can also find final speed and direction (angle) at $t = 7$s.





$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{(190\,\mathrm{m/s})^2 + (98\,\mathrm{m/s})^2} = 210\,\mathrm{m/s}$$

$$\theta = \tan^{-1}(98/190) = 27^\circ$$

3.3 Projectile Motion

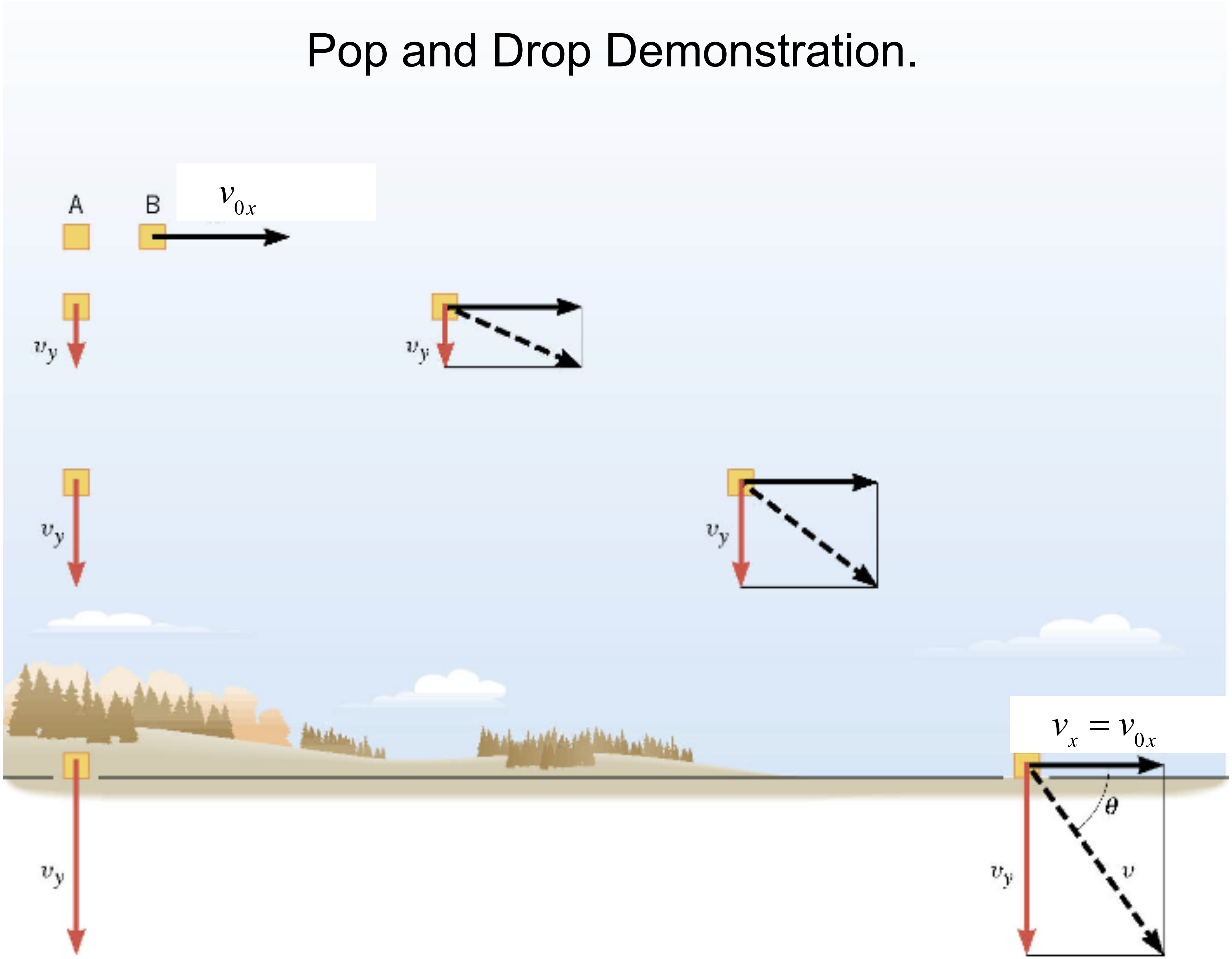
Under the influence of gravity alone, an object near the surface of the Earth will accelerate downwards at 9.80m/s^2.

$$a_y = -9.80\,\mathrm{m/s^2} \qquad a_x = 0$$

Great simplification for projectiles !

$$\Rightarrow \quad v_x = v_{ox} = \text{constant}$$

Pop and Drop Demonstration.

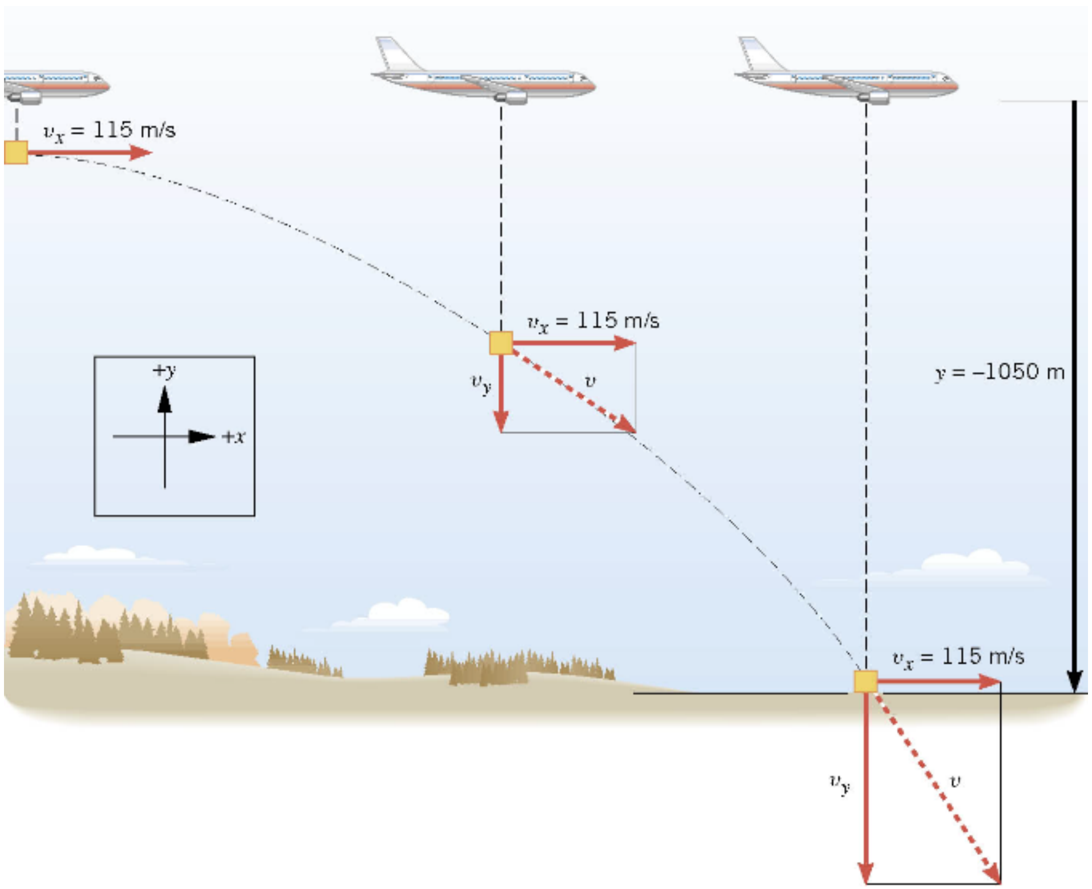


3.3 ***Projectile Motion***

Example 3 A Falling Care Package

The airplane is moving horizontally with a constant velocity of +115 m/s at an altitude of 1050m. Determine the time required for the care package to hit the ground.

Time to hit the ground depends ONLY on vertical motion



$$v_{oy} = 0$$

$$a_y = -9.8 \text{ m/s}^2$$

$$y = 1050 \text{ m}$$

3.3 Projectile Motion

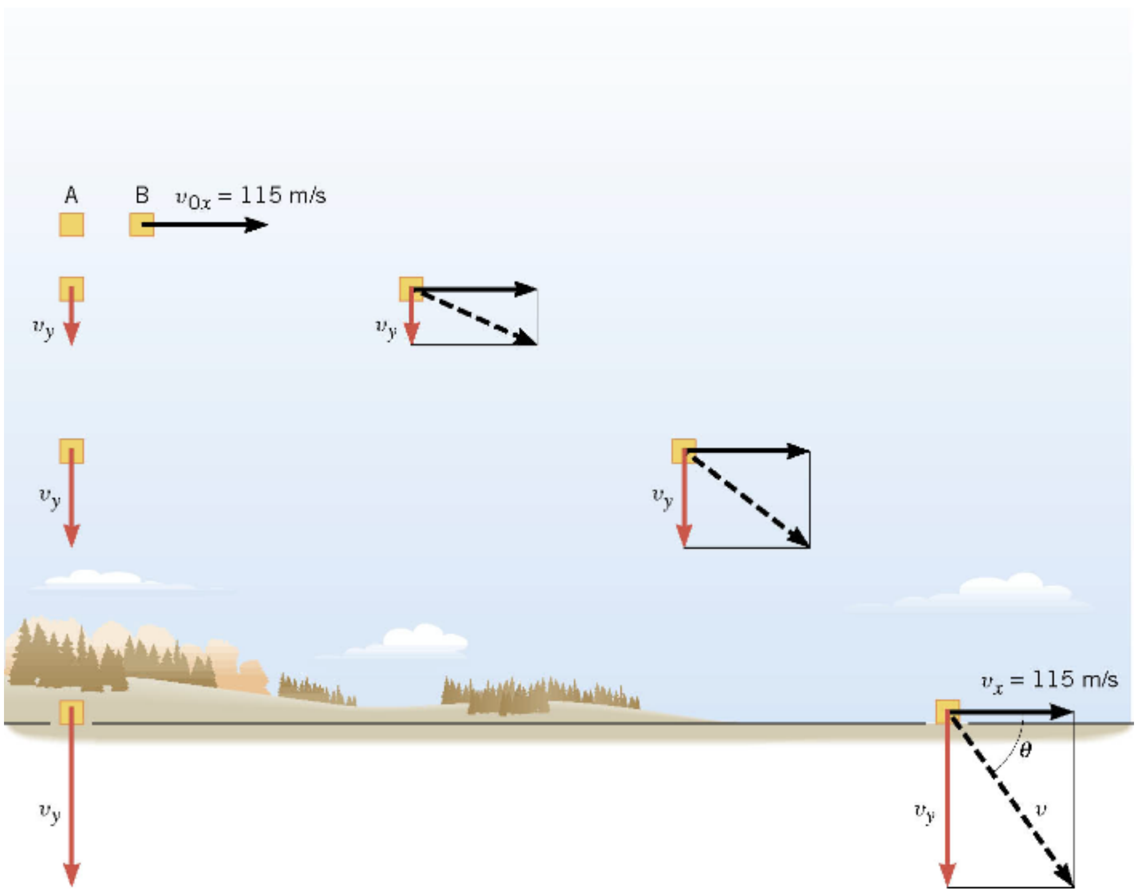
y	a_y	v_y	v_{oy}	t
-1050 m	-9.80 m/s^2		0 m/s	?

$$y = v_{oy}t + \tfrac{1}{2}a_y t^2 \quad \Longrightarrow \quad y = \tfrac{1}{2}a_y t^2$$

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-1050\,\mathrm{m})}{-9.80\,\mathrm{m/s^2}}} = 14.6\,\mathrm{s}$$

Example 4 The Velocity of the Care Package

What are the magnitude and direction of the final velocity of the care package?



$$v_{0y} = 0$$

$$a_y = -9.8 \text{ m/s}^2$$

$$y = 1050 \text{ m}$$

$$t = 14.6 \text{ s}$$

$$v_{0x} = +115 \text{ m/s}$$

$$a_x = 0$$

$$v_x = v_{0x} = +115 \text{ m/s}$$

x-component does not change

3.3 Projectile Motion

y	a_y	v_y	v_{oy}	t
-1050 m	-9.80 m/s^2	?	0 m/s	14.6 s

$$v_y = v_{oy} + a_y t = 0 + \left(-9.80\,\mathrm{m/s^2}\right)\left(14.6\ \mathrm{s}\right)$$

$$= -143\,\mathrm{m/s}$$ y-component of final velocity.

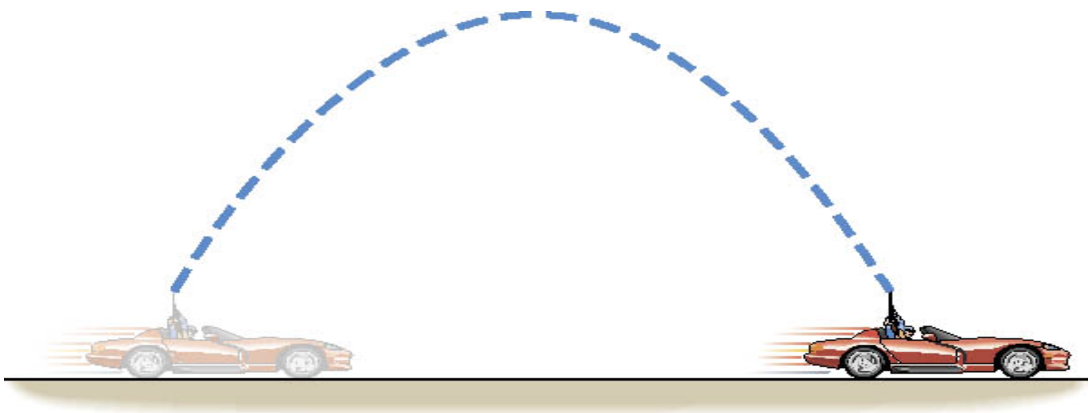
$$v_x = v_{ox} = +115\,\mathrm{m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = 184\ \mathrm{m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-143}{+115}\right) = -51^\circ$$

Conceptual Example 5 I Shot a Bullet into the Air...

Suppose you are driving a convertible with the top down. The car is moving to the right at constant velocity. You point a rifle straight up into the air and fire it. In the absence of air resistance, where would the bullet land – behind you, ahead of you, or in the barrel of the rifle?



Ballistic Cart Demonstration

3.3 Projectile Motion

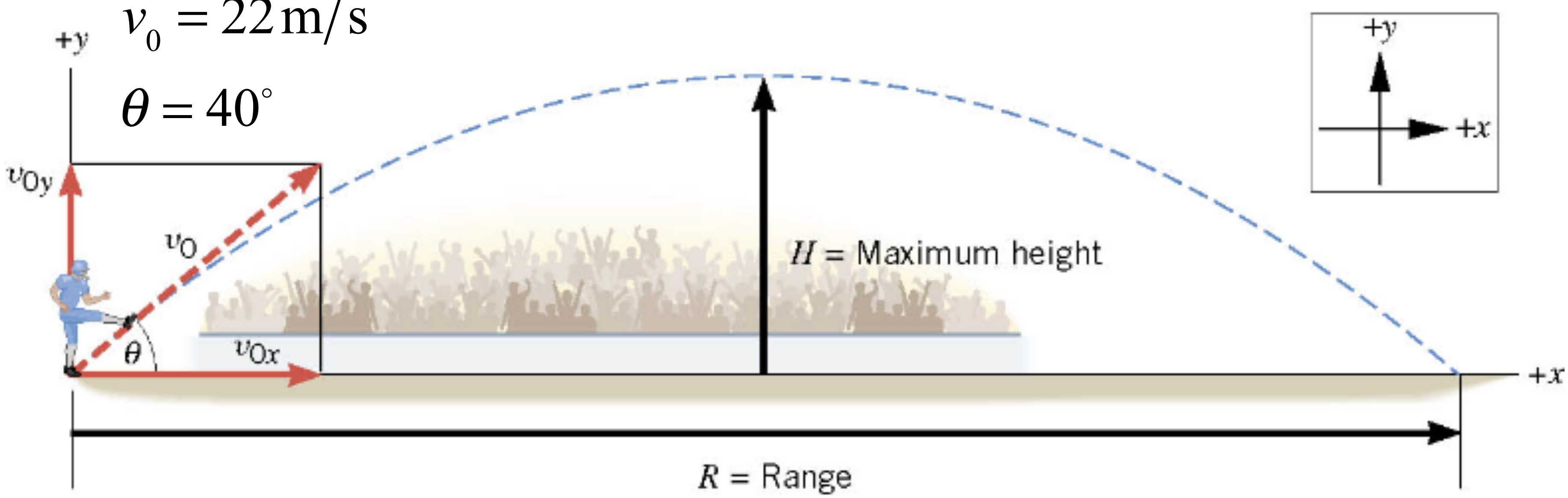
Example 6 The Height of a Kickoff

A placekicker kicks a football at and angle of 40.0 degrees and the initial speed of the ball is 22 m/s. Ignoring air resistance, determine the maximum height that the ball attains.

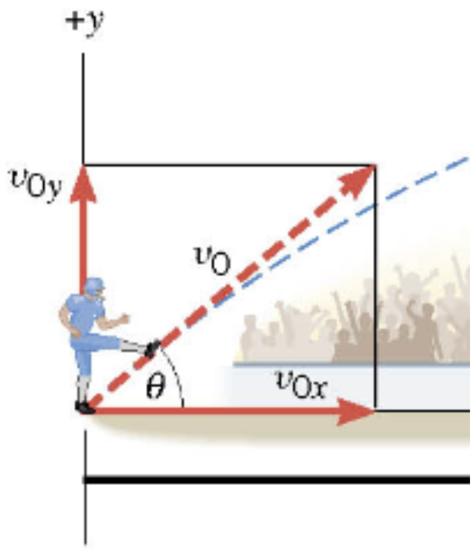
maximum height and "hang time" depend only on the y-component of initial velocity

$$v_0 = 22\,\mathrm{m/s}$$

$$\theta = 40^\circ$$

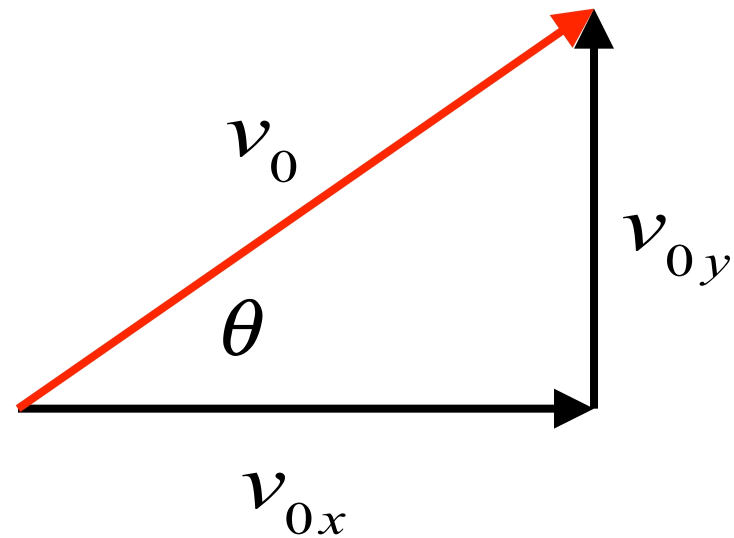


3.3 Projectile Motion



$$v_0 = 22\,\mathrm{m/s}$$

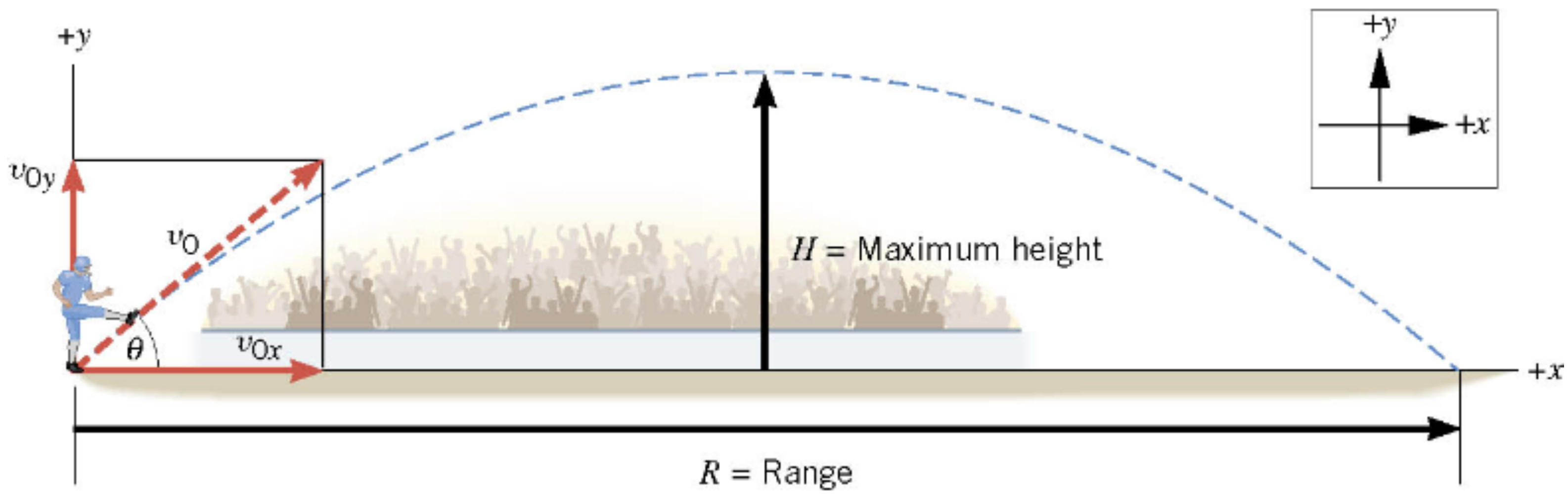
$$\theta = 40^\circ$$



$$v_{0y} = v_0 \sin\theta = (22\,\mathrm{m/s})\sin 40^\circ = 14\,\mathrm{m/s}$$

$$v_{0x} = v_0 \sin\theta = (22\,\mathrm{m/s})\cos 40^\circ = 17\,\mathrm{m/s}$$

3.3 Projectile Motion



y	a_y	v_y	v_{oy}	t
?	-9.80 m/s^2	0	14 m/s	

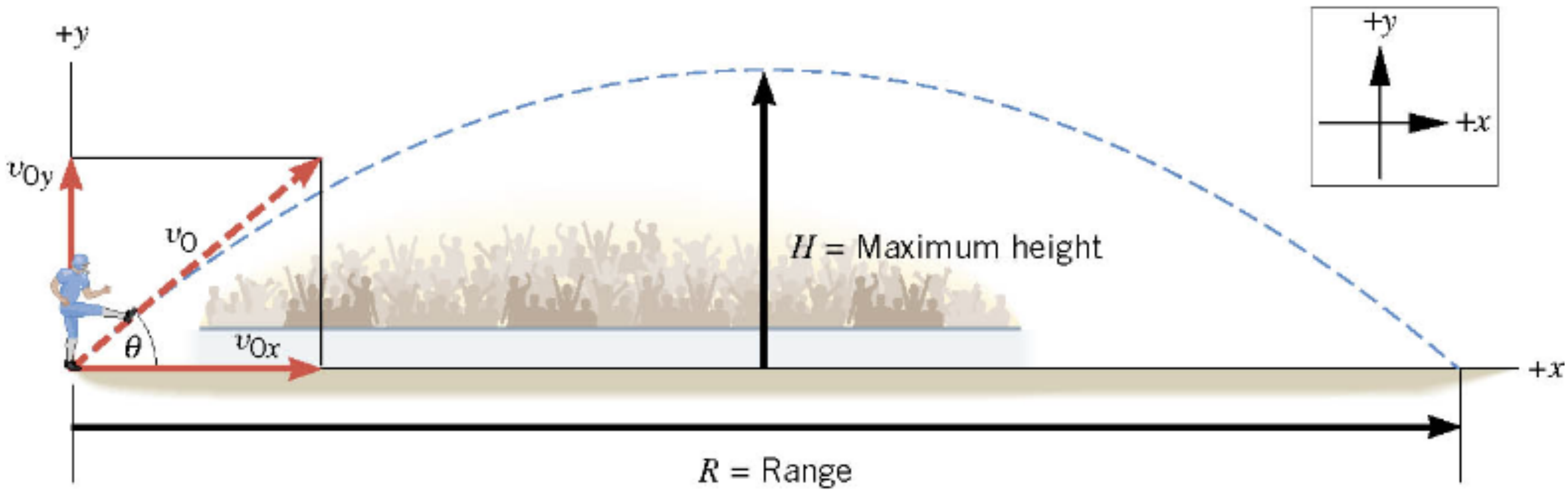
$$v_y^2 = v_{0y}^2 + 2a_y y \quad \Longrightarrow \quad y = \frac{v_y^2 - v_{0y}^2}{2a_y}$$

$$y = \frac{0 - (14\,\mathrm{m/s})^2}{2(-9.8\,\mathrm{m/s^2})} = +10\,\mathrm{m}$$

3.3 Projectile Motion

Example 7 The Time of Flight of a Kickoff

What is the time of flight between kickoff and landing?



y	a_y	v_y	v_{oy}	t
0	-9.80 m/s^2		14 m/s	?

3.3 Projectile Motion

y	a_y	v_y	v_{oy}	t
0	-9.80 m/s^2		14 m/s	?

$$y = v_{0y}t + \tfrac{1}{2}a_y t^2$$

$$0 = \left(14\,\mathrm{m/s}\right)t + \tfrac{1}{2}\left(-9.80\,\mathrm{m/s^2}\right)t^2$$

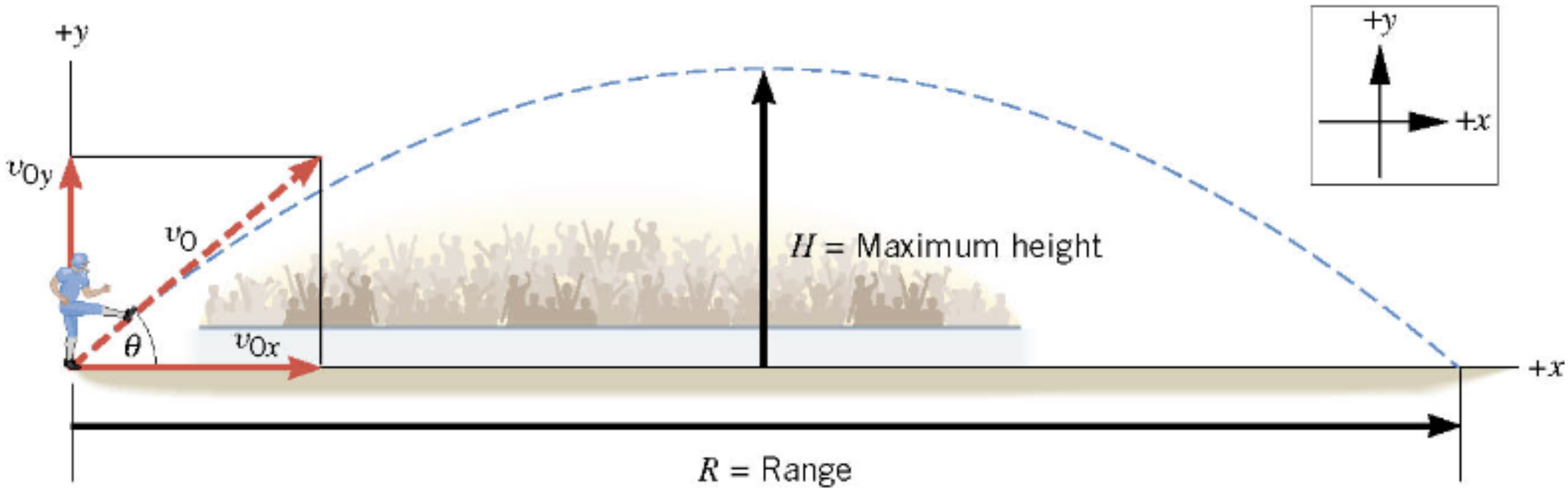
$$0 = 2\left(14\,\mathrm{m/s}\right) + \left(-9.80\,\mathrm{m/s^2}\right)t$$

$$t = 2.9\,\mathrm{s}$$

Example 8 The Range of a Kickoff

Calculate the range R of the projectile.

Range depends on the hang time and x-component of initial velocity

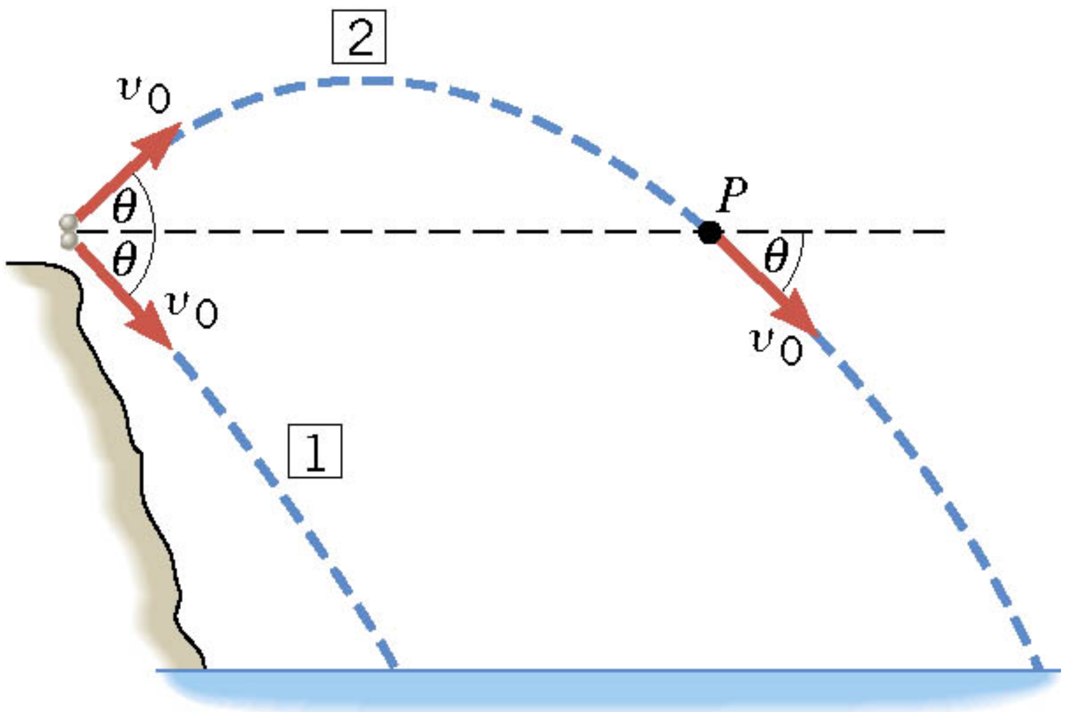


$$x = v_{ox}t + \tfrac{1}{2}a_x t^2 = v_{ox}t$$

$$= (17\ \mathrm{m/s})(2.9\ \mathrm{s}) = +49\ \mathrm{m}$$

Conceptual Example 10 Two Ways to Throw a Stone

From the top of a cliff, a person throws two stones. The stones have identical initial speeds, but stone 1 is thrown downward at some angle above the horizontal and stone 2 is thrown at the same angle below the horizontal. Neglecting air resistance, which stone, if either, strikes the water with greater velocity?

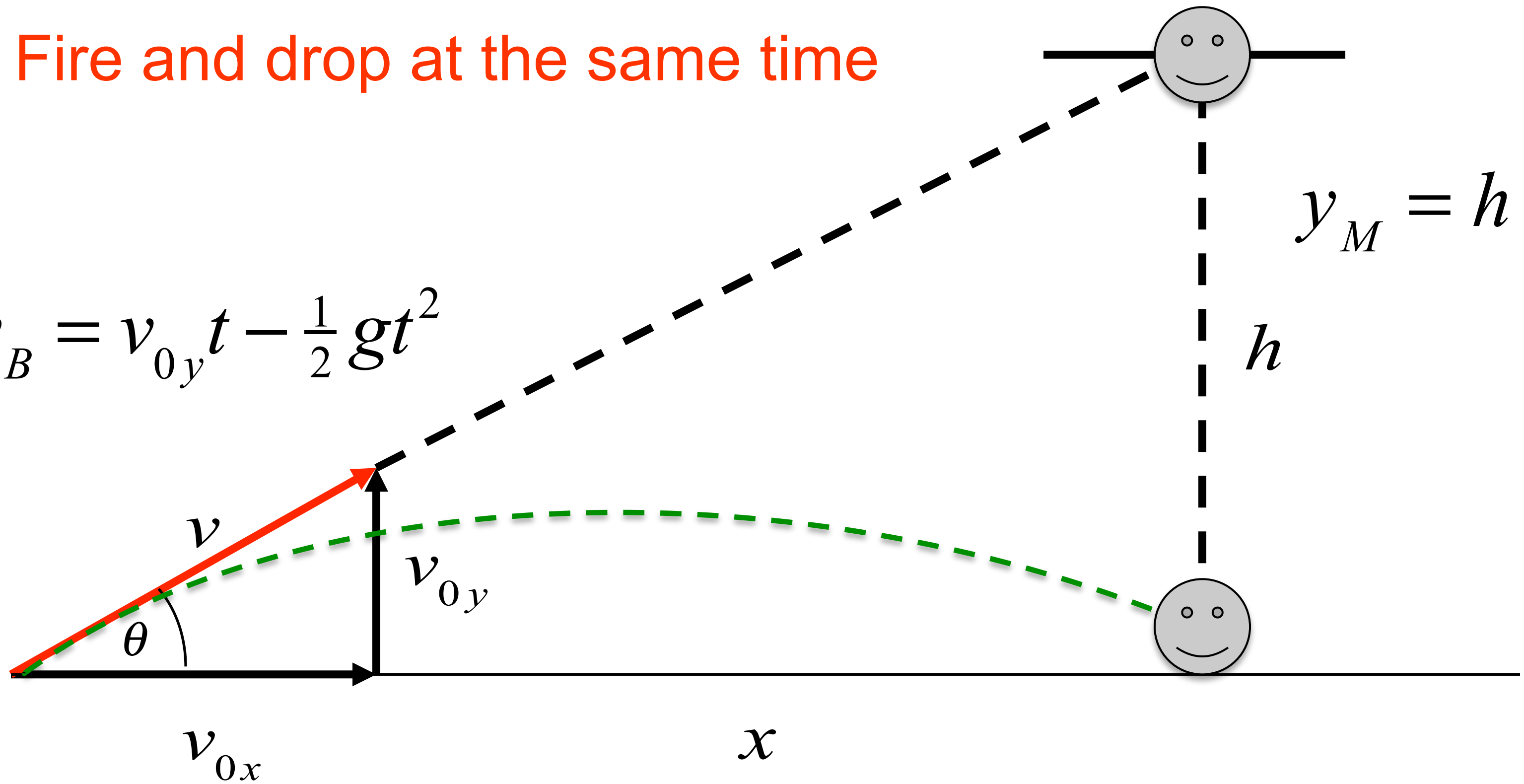


Shoot the Monkey Demonstration

Fire and drop at the same time

$$y_M = h - \tfrac{1}{2}gt^2$$

$$y_B = v_{0y}t - \tfrac{1}{2}gt^2$$



Shoot at the Monkey !

Hit height: $y_B = y_M \quad \Rightarrow \quad v_{0y}t = h$

Hit time: $t = \dfrac{x}{v_{0x}}$ $\qquad \dfrac{v_{0y}}{v_{0x}}x = h$

$$\boxed{\frac{v_{0y}}{v_{0x}} = \frac{h}{x} = \tan\theta}$$