Summary of Chapters 1-3

Equations of motion
for a
uniformly accelerating object

Quiz to follow

An unbalanced force acting on an object results in its acceleration

Accelerated motion in time, t, described with equations using

- vectors: location ($\vec{\mathbf{x}}$), displacement ($\Delta\vec{\mathbf{x}}$), velocity ($\vec{\mathbf{v}}$), acceleration ($\vec{\mathbf{a}}$)
- scalars: length (ℓ), magitude($\vec{\mathbf{v}}$) = speed v, magnitude($\vec{\mathbf{a}}$) = a, angle (θ)

Vectors have a magnitude (positive scalar) and a direction.

- motion along a straight line (1D): direction is a sign (+ or –)
- motion in 2 dimensions (2D): direction is an angle θ

Components of 2D vector $\vec{\mathbf{A}}$, with magnitude A, angle θ wrt x-axis,

$$A_x = A\cos\theta, \text{ and } A_y = A\sin\theta \quad \Leftrightarrow \quad A = \sqrt{A_x^2 + A_y^2}, \text{ and } \theta = \tan^{-1}\left(A_y / A_x\right)$$

2D motion is much EASIER using components in each 1D direction.

Equations of 1D Kinematics

$$v = v_0 + at$$

$$x = \tfrac{1}{2}\left(v_0 + v\right)t$$

$$v^2 = v_0^2 + 2ax$$

$$x = v_0 t + \tfrac{1}{2}at^2$$

Simplifications used

a) 1D vectors behave as scalars

direction is sign (+,–) of value

b) $x_0 = 0,\ \Delta x = x - x_0 \Rightarrow \Delta x = x$

displacement = location

c) $t_0 = 0,\ \Delta t = t - t_0 \Rightarrow \Delta t = t$

time interval = final time

Equations of 2D Kinematics

1D motion in x-direction

$$\left\{\begin{aligned} v_x &= v_{0x} + a_x t \\ x &= \tfrac{1}{2}\left(v_{0x} + v_x\right)t \\ v_x^2 &= v_{0x}^2 + 2a_x x \\ x &= v_{0x}t + \tfrac{1}{2}a_x t^2 \end{aligned}\right.$$

1D motion in y-direction

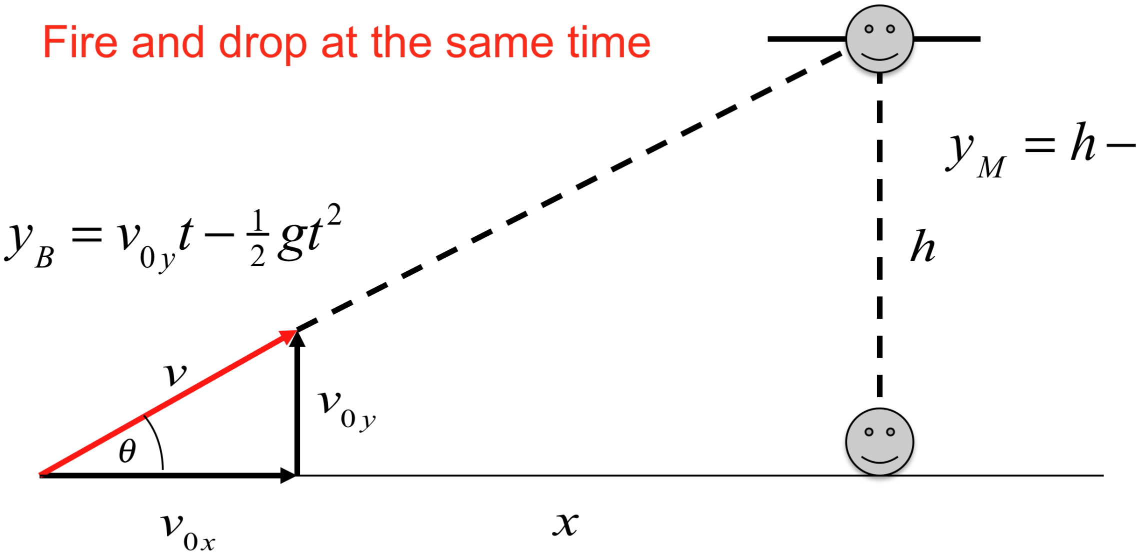
$$\left\{\begin{aligned} v_y &= v_{0y} + a_y t \\ y &= \tfrac{1}{2}\left(v_{0y} + v_y\right)t \\ v_y^2 &= v_{0y}^2 + 2a_y x \\ y &= v_{0y}t + \tfrac{1}{2}a_y t^2 \end{aligned}\right.$$

Objects in motion in the air, ignoring air friction effects, have a constant (–) acceleration in the vertical direction.

$$\boxed{\text{Defining up as positive, use: } a_y = -g = -9.80\ \text{m/s}^2, \quad a_x = 0}$$

Shoot the Monkey Demonstration

Fire and drop at the same time



$y_M = h - \frac{1}{2}gt^2$ (need $y_0 = h$, here)

$y_B = v_{0y}t - \frac{1}{2}gt^2$

Hit height: $y_B = y_M \quad \Rightarrow \quad v_{0y}t = h$

Hit time: $t = \dfrac{x}{v_{0x}}$ $\qquad \dfrac{v_{0y}}{v_{0x}}x = h$

Shoot at the Monkey !

$$\boxed{\frac{v_{0y}}{v_{0x}} = \frac{h}{x} = \tan\theta}$$

Quiz 2

1. **C&J page 89 (bottom), Check Your Understanding #2**

2. **Object I has an initial velocity of +12 m/s and acceleration of –3.0 m/s^2. Object II has an initial velocity of +12 m/s and acceleration of +3.0 m/s^2. What are the final *speeds* of the two objects 5 seconds later?**

	object 1	object 2
a)	3.0 m / s	15 m / s
b)	27 m / s	27 m / s
c)	3.0 m / s	27 m / s
d)	27 m / s	15 m / s
e)	3.0 m / s	3.0 m / s

3. **A pistol accelerates water from rest to a velocity of 26 cm/s while moving only +3.0 cm. What is water's acceleration during this time?**

a) 13 cm / s^2

b) 54 cm / s^2

c) 110 cm / s^2

d) 170 cm / s^2

e) 260 cm / s^2

4. In this velocity vs. time graph, at what time is the *magnitude* of the acceleration the greatest?

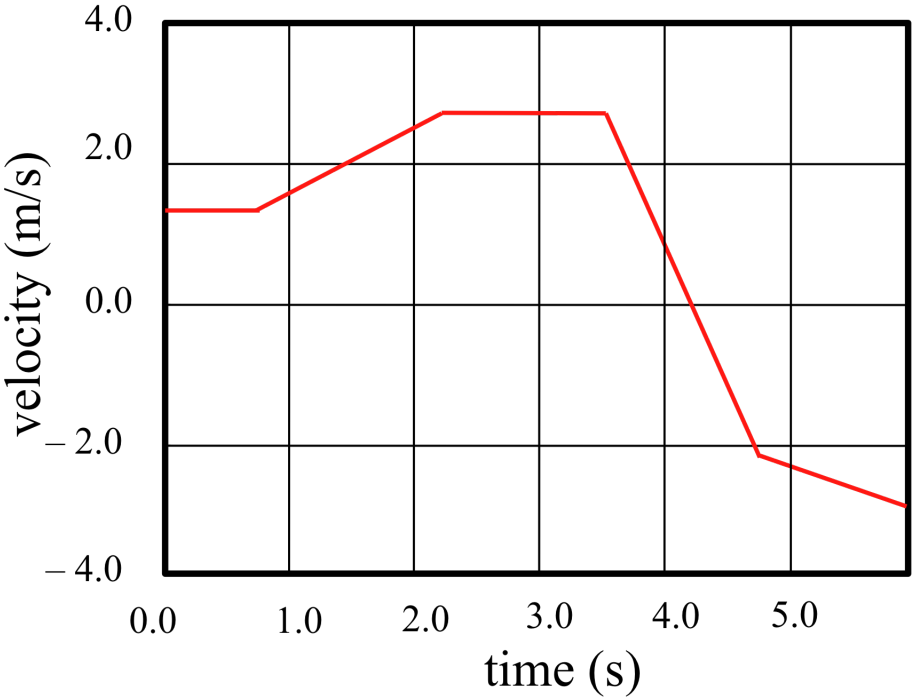
a) $t = 1.0\,\mathrm{s}$

b) $t = 2.0\,\mathrm{s}$

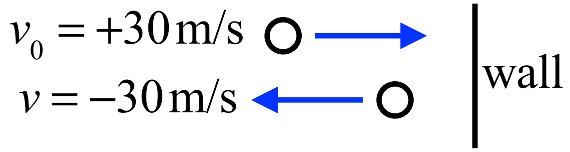
c) $t = 3.0\,\mathrm{s}$

d) $t = 4.0\,\mathrm{s}$

e) $t = 5.0\,\mathrm{s}$



5. A paddle ball travelling horizontally bounces off a wall. The *speed* of the ball was 30 m/s before and after hitting the wall. If the ball was in contact on the wall for 0.04 s, what was its average acceleration during this time?



a) $+1500\ \mathrm{m/s^2}$

b) $-1500\ \mathrm{m/s^2}$

c) $-150\ \mathrm{m/s^2}$

d) $0\ \mathrm{m/s^2}$

e) $+150\ \mathrm{m/s^2}$

Quiz 2

1. **C&J page 89 (bottom), Check Your Understanding #2**

c) At rest *or* at constant velocity

2. **Object I has an initial velocity of +12 m/s and acceleration of –3.0 m/s^2. Object II has an initial velocity of +12 m/s and acceleration of +3.0 m/s^2. What are the final *speeds* of the two objects 5 seconds later?**

	object 1	object 2
a)	3.0 m / s	15 m / s
b)	27 m / s	27 m / s
c)	3.0 m / s	27 m / s
d)	27 m / s	15 m / s
e)	3.0 m / s	3.0 m / s

$$v = v_0 + at$$

$$\text{I:}\ \ v = 12\ \text{m/s} + (-3.0\ \text{m/s}^2)(5\ \text{s})\ = -3\ \text{m/s}$$

$$\text{II:}\ v = 12\ \text{m/s} + (3.0\ \text{m/s}^2)(5\ \text{s})\ = +27\ \text{m/s}$$

$$\underline{\text{speeds}}\ \text{for I: 3 m/s;}\quad \text{for II: 27 m/s}$$

3. **A pistol accelerates water from rest to a velocity of 26 cm/s while moving only +2.0 cm. What is water's acceleration during this time?**

a) 13 cm / s^2

b) 54 cm / s^2

c) 104 cm / s^2

d) 170 cm / s^2

e) 260 cm / s^2

$$v^2 = v_0^2 + 2ax \Rightarrow a = v^2 / 2x$$

$$a = (26\,\text{cm/s})^2 \Big/ \left[2\left(2.0\,\text{cm}\right)\right] = 170\ \text{cm/s}^2$$

4. In this velocity vs. time graph, at what time is the *magnitude* of the acceleration the greatest?

a) $t = 1.0\,\mathrm{s}$

b) $t = 2.0\,\mathrm{s}$

c) $t = 3.0\,\mathrm{s}$

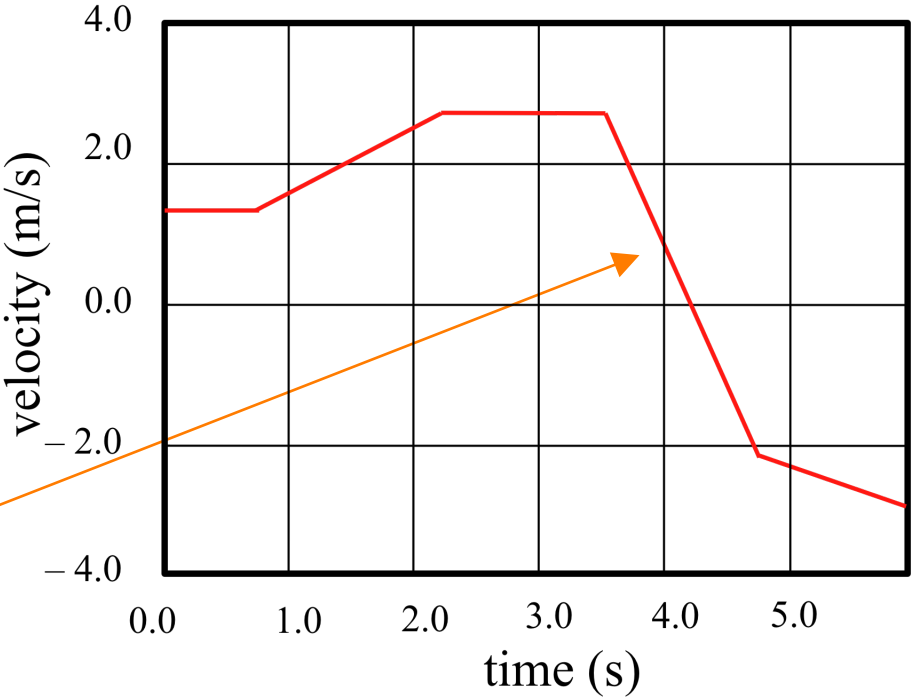
d) $t = 4.0\,\mathrm{s}$

e) $t = 5.0\,\mathrm{s}$

$a = \Delta v / \Delta t = \text{slope}$

steepest slope

is near $t = 4\,\mathrm{s}$.



5. A paddle ball travelling horizontally bounces off a wall. The *speed* of the ball was 30 m/s before and after hitting the wall. If the ball was in contact on the wall for 0.04 s, what was its average acceleration during this time?

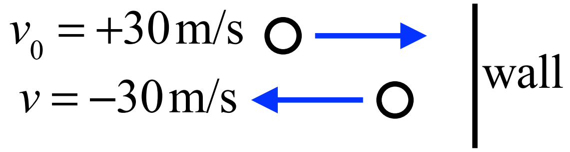
a) $+1500\ \mathrm{m/s^2}$

b) $-1500\ \mathrm{m/s^2}$

c) $-150\ \mathrm{m/s^2}$

d) $0\ \mathrm{m/s^2}$

e) $+150\ \mathrm{m/s^2}$



$v_0 = +30\,\mathrm{m/s}; \quad v = -30\,\mathrm{m/s}$

$$a = \frac{\Delta v}{\Delta t} = \frac{\left[-30\,\mathrm{m/s} - \left(+30\,\mathrm{m/s}\right)\right]}{0.04\,\mathrm{s}} = -1500\,\mathrm{m/s^2}$$

Chapter 4

Forces and Newton’s Laws of Motion

4.1 The Concepts of Force and Mass

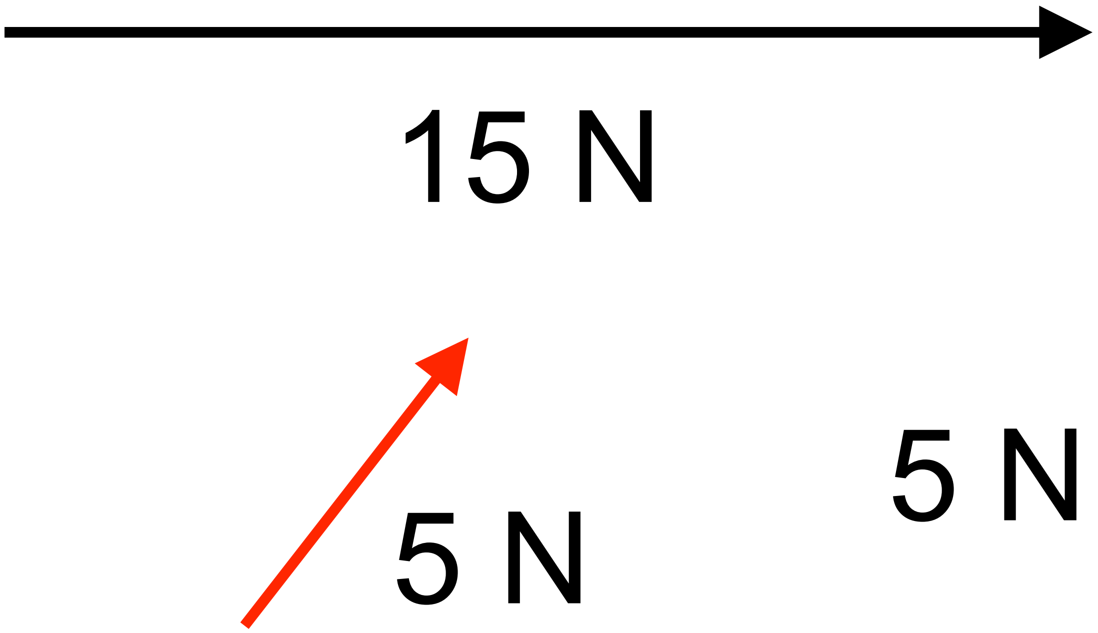
A ***force*** is a push or a pull acting on an object. A force is a vector!

Contact forces arise from physical contact, and are due to stretching or compressing at the point of contact.

Action-at-a-distance forces do not require contact and include gravity and electrical forces.

4.1 The Concepts of Force and Mass

Arrows are used to represent forces.
The length of the arrow
is proportional to the magnitude of the force.


15 N

5 N

5 N ~ 1 lb

4.1 The Concepts of Force and Mass

Mass of an object is a measure of the number and type of atoms within the object.

Mass can be measured without resorting to gravity/weight.

A spring will oscillate a mass with an oscillation period,

$T \propto \sqrt{m}$.   ($\propto$ means proportional to)

If the period is twice as long, the mass is 4 times bigger.

Device to measure a mass anywhere in the universe

a planet or moon
or a big spaceship (air-track unnecessary)

These springs can be taken anywhere in the universe and used to measure the mass of any cart. Also, the stretching of these springs can be used to define the unit of force.

SI Unit of Mass: kilogram (kg)

Newton’s First Law

An object continues in a state of rest or in a state of motion at a constant speed *along a straight line*, unless compelled to change that state by a net force.

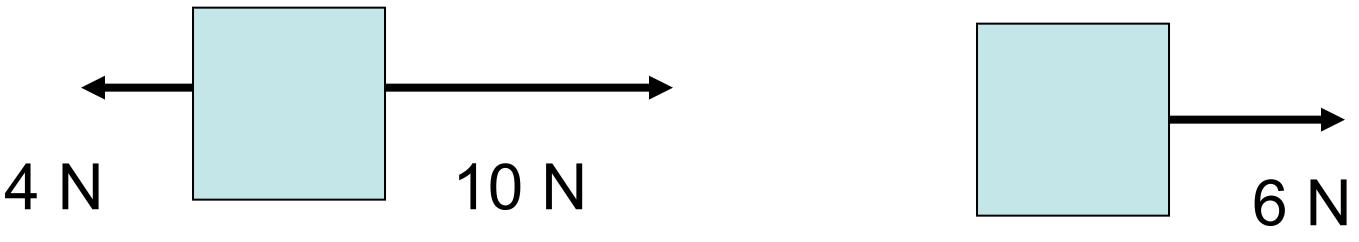
The *net force* is the vector sum of all of the forces acting on an object.

4.2 Newton’s First Law of Motion

The net force on an object is the vector sum of all forces acting on that object.

The SI unit of force is the Newton (N).

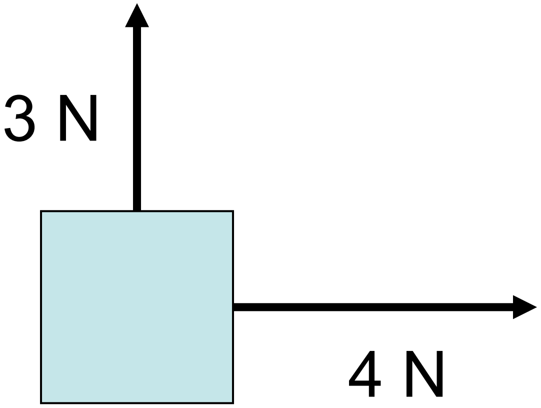
Individual Forces   Top views   Net Force



4.2 Newton’s First Law of Motion

Individual Forces

Top view

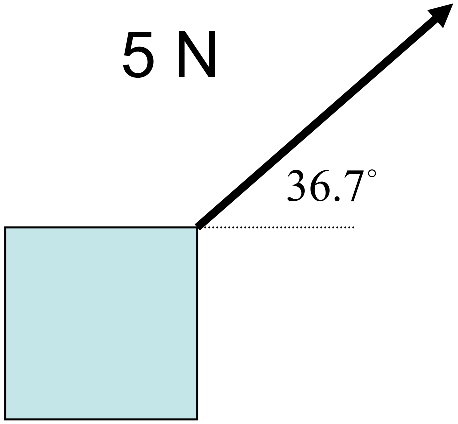


No friction

No Gravity

Net Force

Top view



θ is an angle with respect to x-axis

$$\tan\theta = \frac{F_y}{F_x} \quad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.7°$$

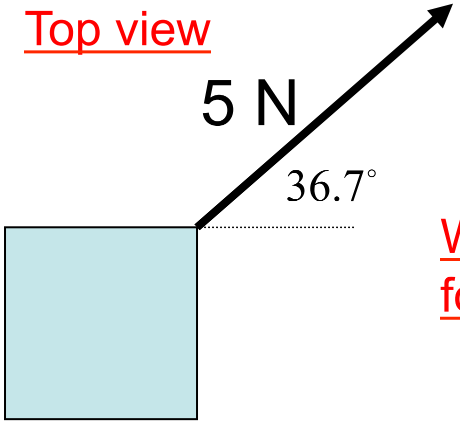
4.2 Newton's First Law of Motion

Net Force

Top view

No friction

No Gravity

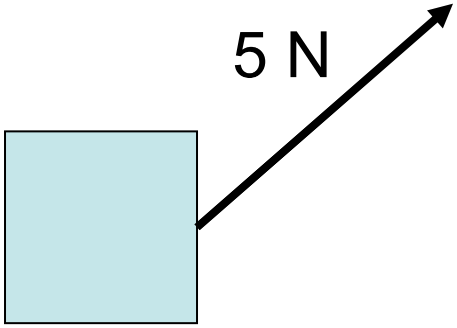


Why does the picture show the force vector attached to a corner ?

You will see this in most textbooks.

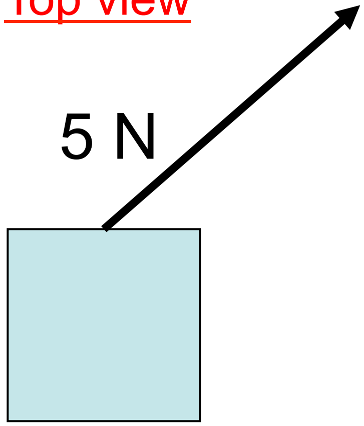
Why not

Top view



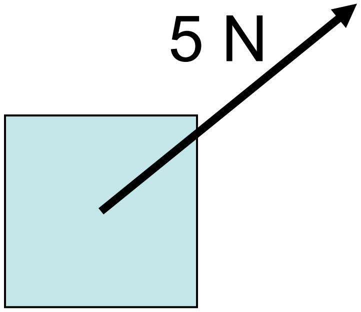
or this

Top view



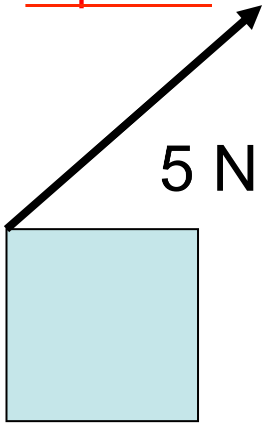
or this

Top view



or this

Top view



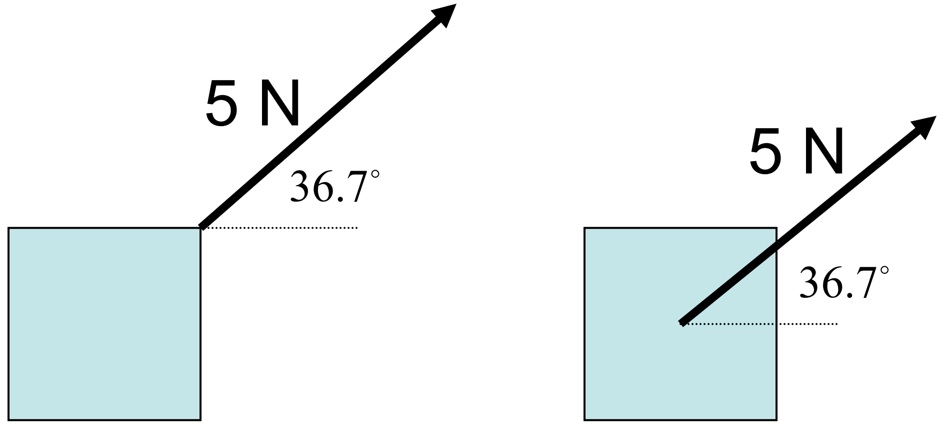
Best would be this,
attached to the center of object.

4.2 Newton's First Law of Motion

No friction

No Gravity

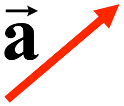
Top view



Both drawings lead to the same linear motion of the object

The object will not maintain a constant $\underbrace{\text{speed \& direction}}_{\text{velocity}}$.

The object will accelerate in this direction: $\vec{\mathbf{a}}$



4.2 Newton's First Law of Motion

Newton's 1st law: for an object to remain at rest, or move with constant speed & direction, the Net Force acting on it <u>must be</u> ZERO.

So

Newton's 1st law: if the Net Force acting on a object is NOT ZERO, the velocity (magnitude, or direction, or both) <u>must change</u>.

4.2 Newton's First Law of Motion

Newton's 1st law is often called
the law of inertia.

Inertia is the natural tendency of an object to remain at rest or in motion at a constant speed along a straight line.

The *mass* of an object is a quantitative measure of inertia.

4.2 Newton’s First Law of Motion

An *inertial reference frame* is one in which Newton’s law of inertia is valid.

All accelerating reference frames are non-inertial.

4.2 ***Newton’s First Law of Motion***

Warning:

Newton’s 1st law can appear to be violated if you don’t recognize the existence of contact forces.

Newton’s 1st law: for an object to *remain at rest*, or *move with constant speed & direction*, the Net Force acting on it must be ZERO.

4.2 Newton's First Law of Motion

Examples (4 clicker questions):

A mass hanging from a string.
A mass at rest on a table.
A mass at rest on a ramp.
A mass sliding on a table.

Clicker Question 4.1

A mass hanging from a string.

Gravity applies a 100 N gravitational force to the object. What force does the string apply to the object?

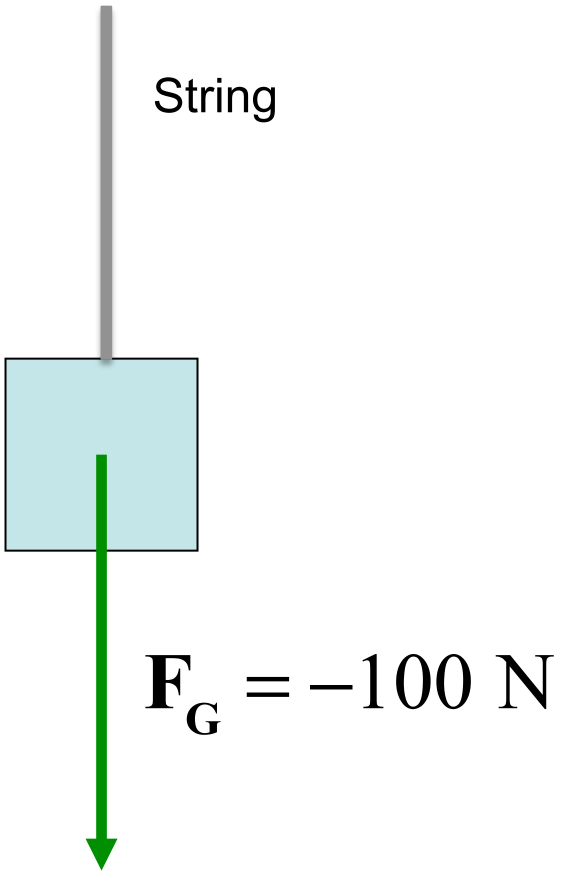
a) string $F_y = -100\text{ N}$

b) string $F_y = \quad 0\text{ N}$

c) string $F_y = +100\text{ N}$

d) string $F_y = \quad 1\text{ N}$

e) A string can't make a force



Clicker Question 4.1

A mass hanging from a string.

Gravity applies a 100 N gravitational force to the object. What force does the string apply to the object?

a) string $F_y = -100$ N

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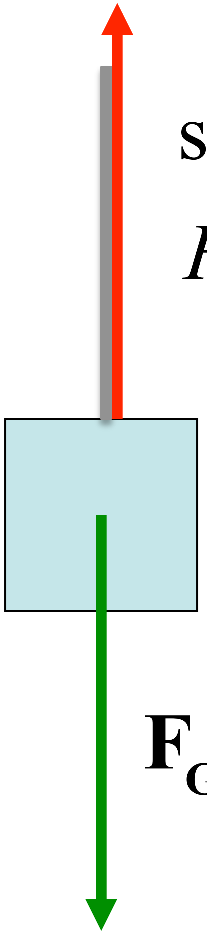
[✓ c) string $F_y = +100$ N]

d) string $F_y = 1$ N

e) A string can't make a force



string stretches

$F_y = +100$ N

$\mathbf{F_G} = -100$ N

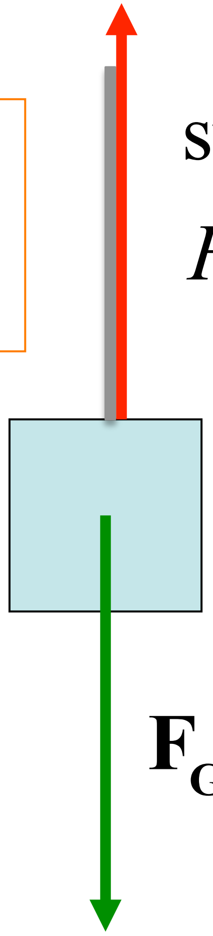
At rest: Net force must be zero

Clicker Question 4.1

A mass hanging from a string.

How does the string “know” how much force to apply to EXACTLY balance the gravity force?

As you slowly let the mass go, the string starts to stretch. The more it streches the harder it pulls up. When you let go, it has stretched just enough to pull back with EXACTLY the right amount of force.



string stretches

$F_y = +100\ \text{N}$

$\mathbf{F}_\mathbf{G} = -100\ \text{N}$

At rest: Net force must be zero

Clicker Question 4.2

A mass resting on a table.

At rest: Net force must be zero

Gravity applies a 100 N gravitational force to the object. What force does the table apply to the object?

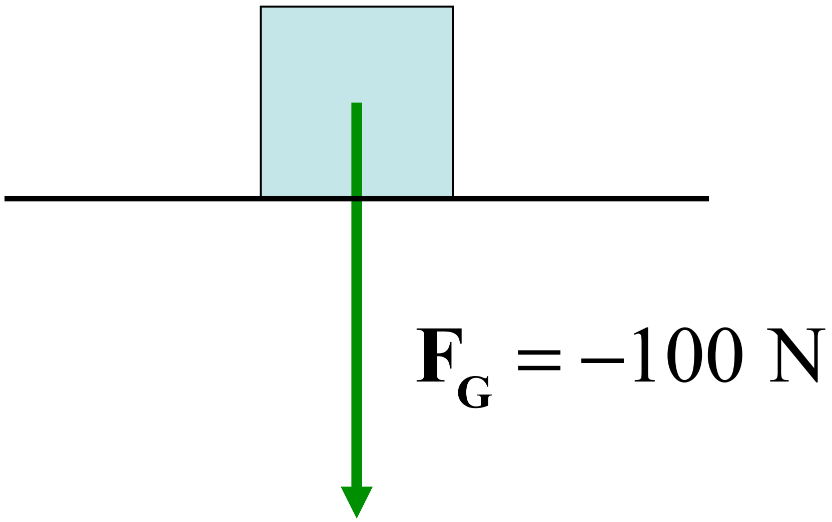
a) table $F_y = +100\ \mathrm{N}$

b) table $F_y = -100\ \mathrm{N}$

c) table $F_y = \quad 0\ \mathrm{N}$

d) table $F_y = \quad 1\ \mathrm{N}$

e) A table can't make a force



$\mathbf{F}_\mathbf{G} = -100\ \mathrm{N}$

Clicker Question 4.2

A mass resting on a table.

At rest: Net force must be zero

Gravity applies a 100 N gravitational force to the object. What force does the table apply to the object?

$\boxed{\text{a) table } F_y = +100 \text{ N}}$

b) table $F_y = -100$ N

c) table $F_y = \quad 0$ N

d) table $F_y = \quad 1$ N

e) A table can't make a force

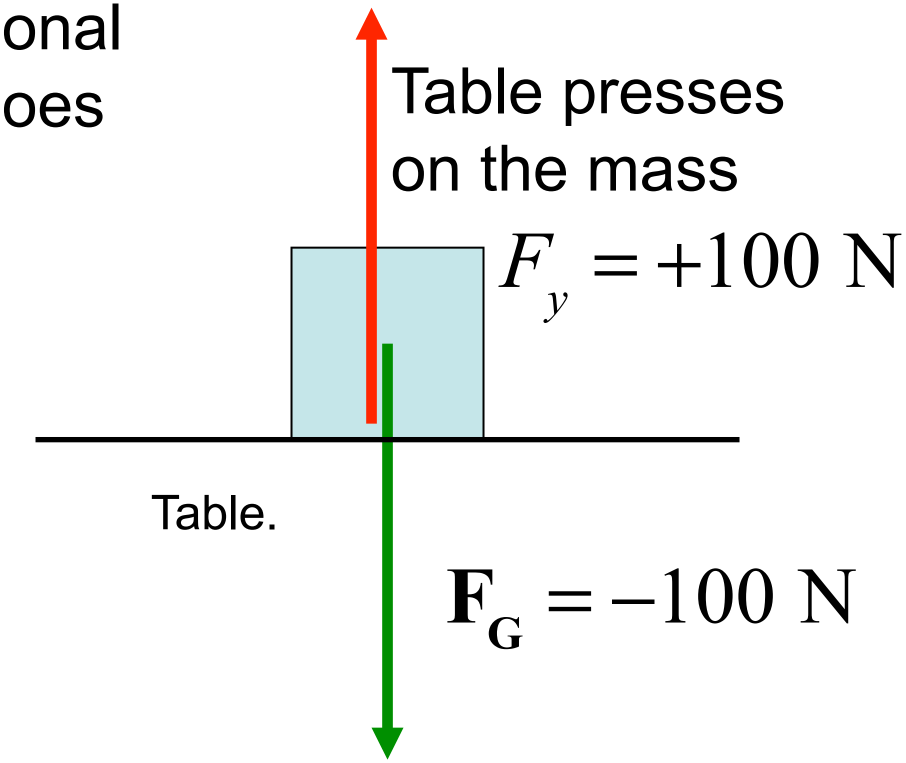


Table presses on the mass

$F_y = +100$ N

Table.

$\mathbf{F_G} = -100$ N

Clicker Question 4.2

A mass resting on a table.

At rest: Net force must be zero

How does the table “know” how much force to apply to EXACTLY balance the gravity force?

As you slowly put the mass on the table, it starts compress. The more it compresses the harder it pushes up. When you let go, it has compressed just enough to push back with EXACTLY the right amount of force.

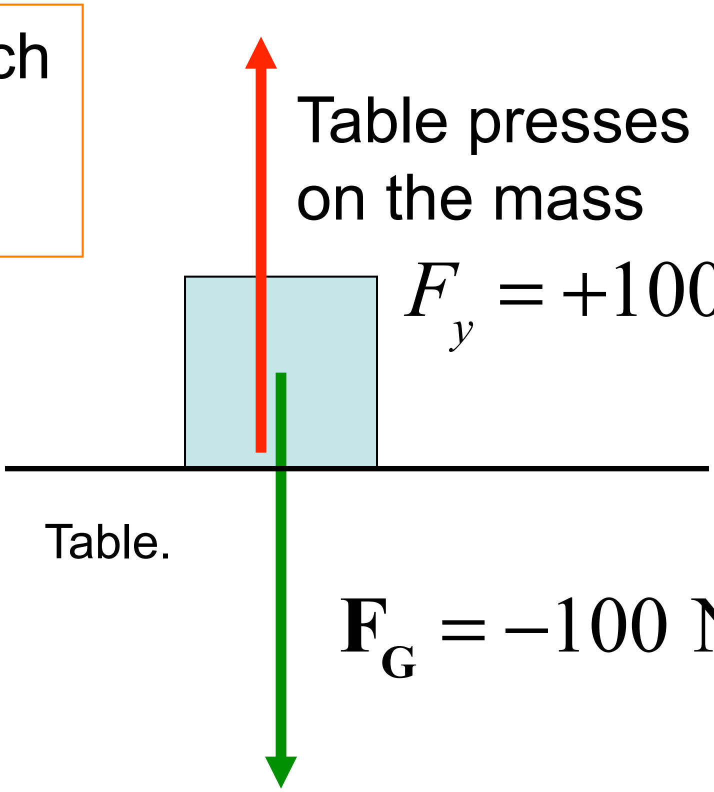


Table presses on the mass

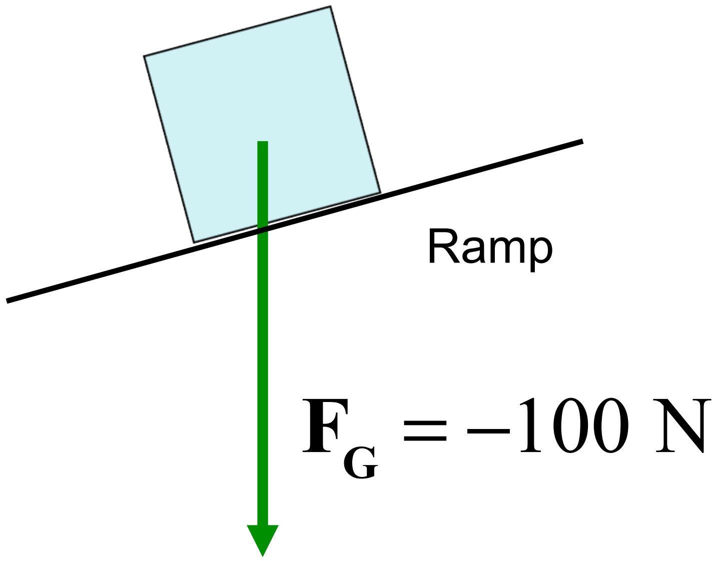
$F_y = +100\ \text{N}$

Table.

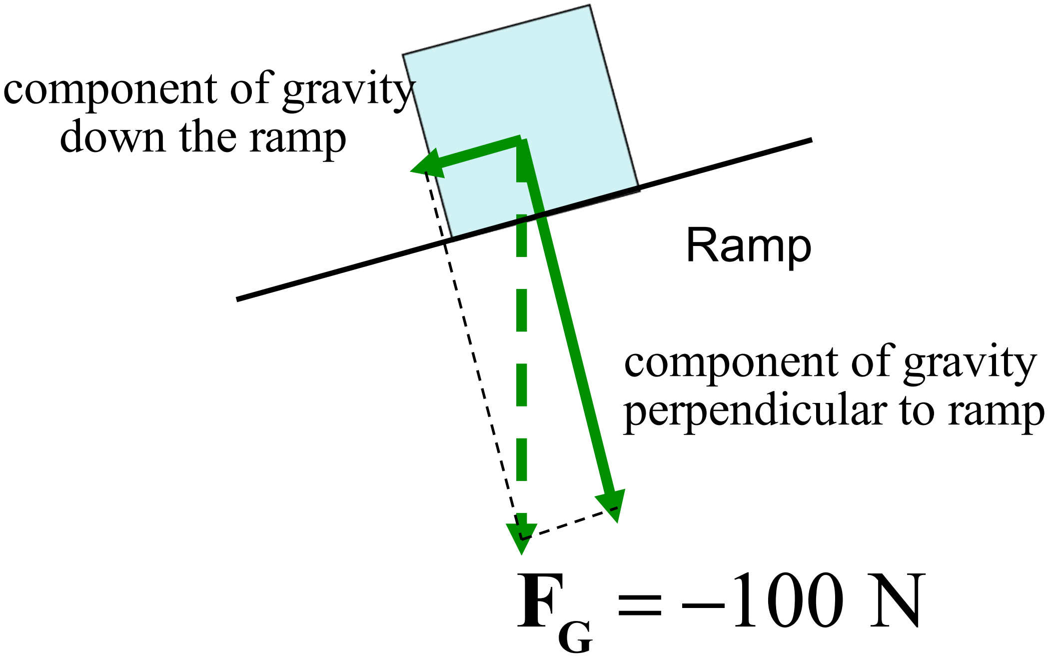
$\mathbf{F_G} = -100\ \text{N}$

A mass at rest on a ramp.

Gravity applies a 100 N gravitational force to an object at rest on a 15° ramp.



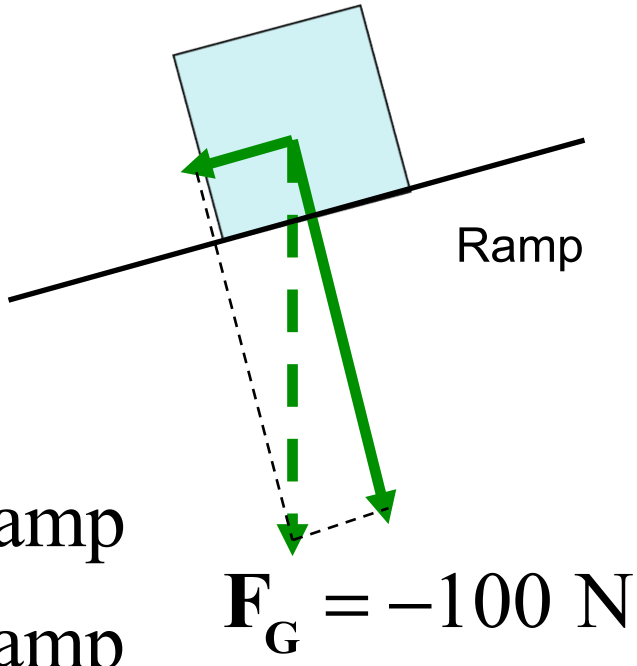
Component of gravity pulls the mass down the the ramp



Clicker Question 4.3

A mass at rest on a ramp.

Gravity applies a 100 N gravitational force to an object at rest on a 15° ramp. The friction between ramp and object applies a force on the mass in what direction?



$\mathbf{F}_G = -100\ \text{N}$

a) Frictional force > 100 N, up the ramp

b) Frictional force = 100 N, up the ramp

c) Frictional force < 100 N, up the ramp

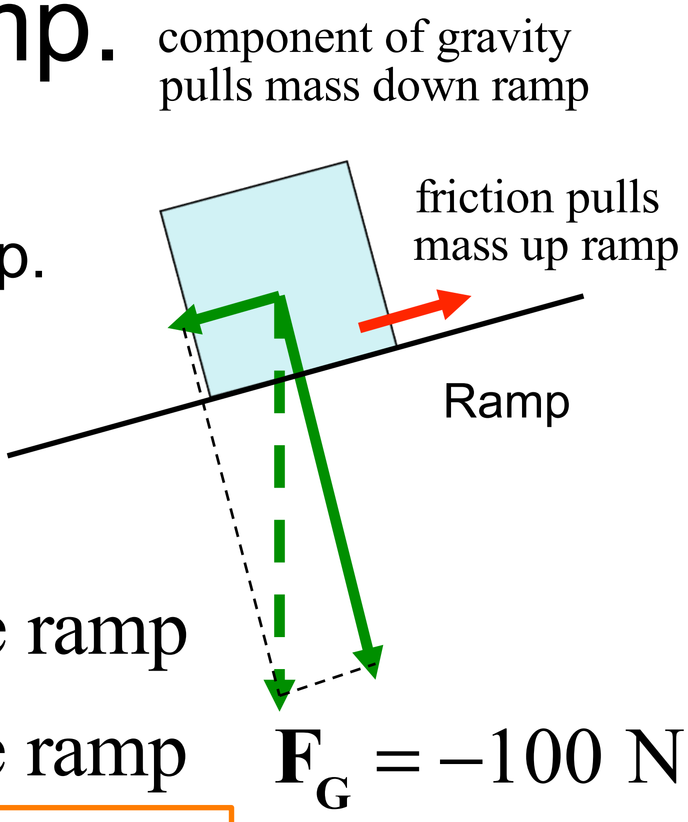
d) Frictional force = 100 N, down the ramp

e) A ramp can't make a force

At rest: Net force must be zero

Clicker Question 4.3

A mass at rest on a ramp.



Gravity applies a 100 N gravitational force to an object at rest on a 15° ramp. The friction between ramp and object applies a force on the mass of what magnitude and direction?

a) Frictional force > 100 N, up the ramp

b) Frictional force = 100 N, up the ramp

[✓ c) Frictional force < 100 N, up the ramp]

d) Frictional force = 100 N, down the ramp

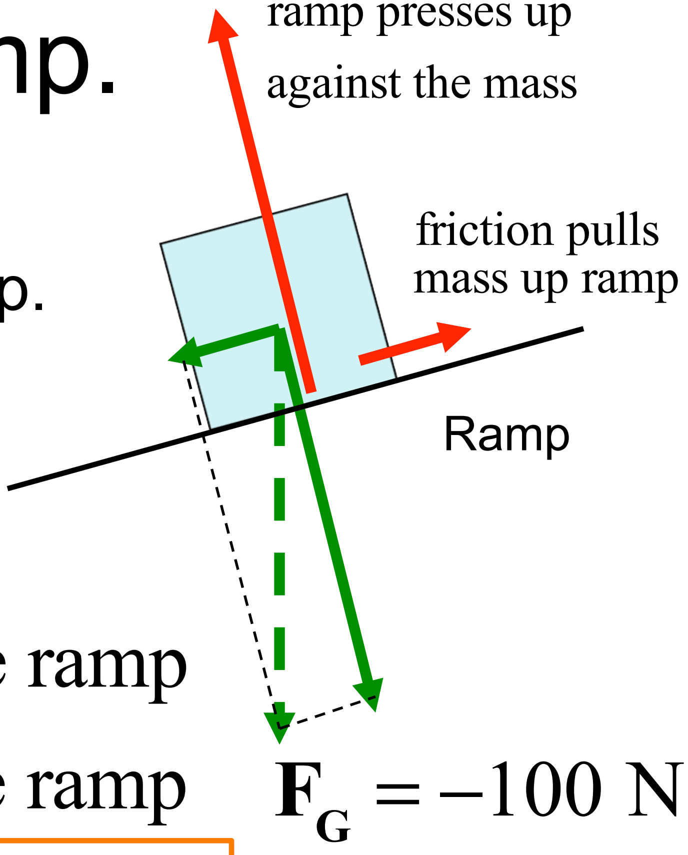
e) A ramp can't make a force

At rest: Net force must be zero

Clicker Question 4.3

A mass at rest on a ramp.

Gravity applies a 100 N gravitational force to an object at rest on a 15° ramp. The friction between ramp and object applies a force on the mass of what magnitude and direction?



a) Frictional force > 100 N, up the ramp

b) Frictional force = 100 N, up the ramp

c) Frictional force < 100 N, up the ramp

d) Frictional force = 100 N, down the ramp

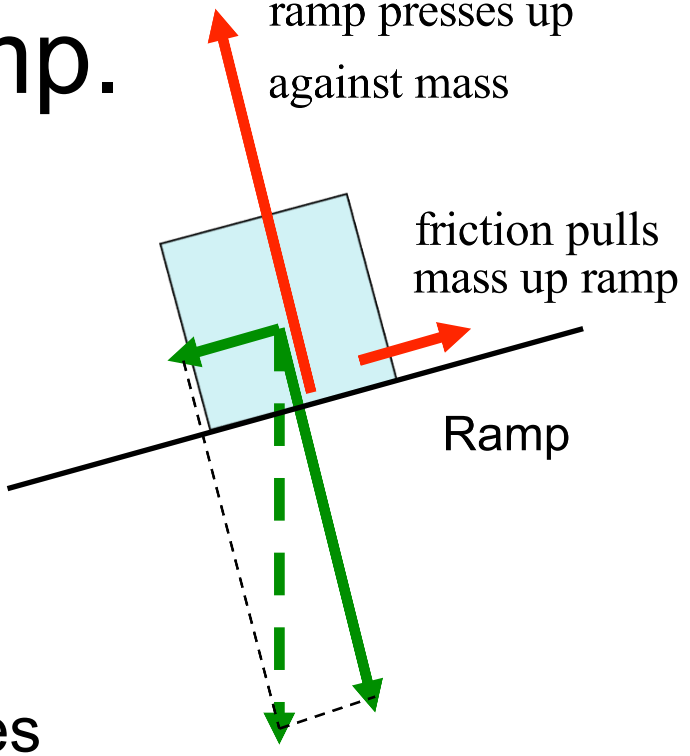
e) A ramp can't make a force

At rest: Net force must be zero

Clicker Question 4.3

A mass at rest on a ramp.

How does the friction between the mass and the table “know” how much force will EXACTLY balance the gravity force pulling the mass down the ramp?



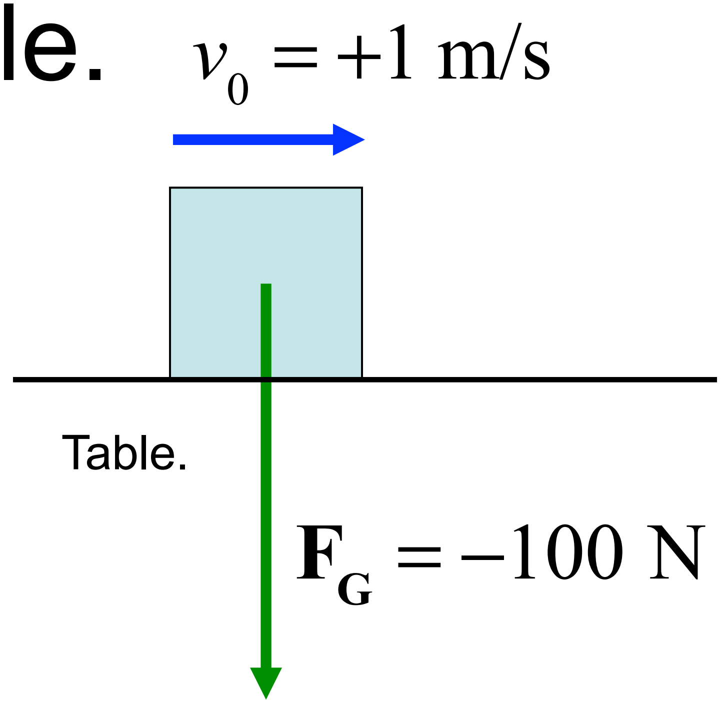
$$\mathbf{F}_{\mathbf{G}} = -100\ \mathrm{N}$$

As you slowly put the mass on the ramp, the ramp compresses & stretches along the ramp as gravity *tries* to slide the mass down the ramp. When you let go, the ramp has stretched enough to push on the mass with EXACTLY the right amount of force up the ramp.

At rest: Net force must be zero

Clicker Question 4.4

A mass sliding on a table.



Gravity applies a force to a mass.
It is sliding on a table with an initial velocity of +1 m/s. It slows while sliding to +0.5 m/s. To slow it the friction between them applies a force to the mass in what direction?

a) Upward

b) Downward

c) To the right (+)

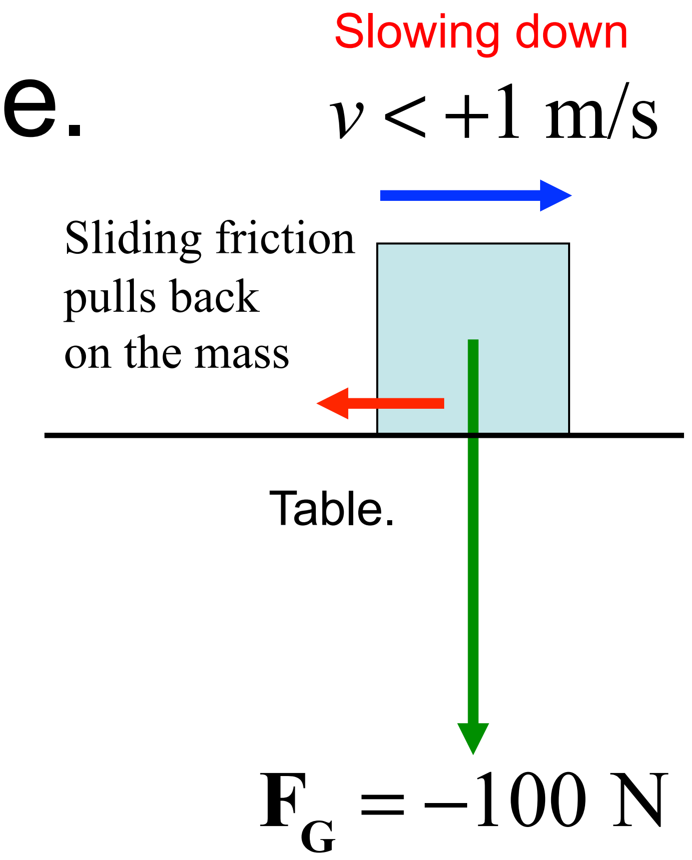
d) To the left (–)

e) A table can't make a force.

Clicker Question 4.4

A mass sliding on a table.

Gravity applies a force to a mass.
It is sliding on a table with an initial velocity of +1 m/s. It slows while sliding to +0.5 m/s. The friction between them applies a force in what direction?



Slowing down

$v < +1\ \mathrm{m/s}$

Sliding friction
pulls back
on the mass

Table.

$\mathbf{F}_\mathbf{G} = -100\ \mathrm{N}$

a) Upward

b) Downward

c) To the right (+)

d) To the left (–)

e) A table can't make a force.

Changing velocity means
Net Force is NOT ZERO

Net Force acting on ONE object

Mathematically, the net force is written as

$$\sum \vec{\mathbf{F}}$$

where the Greek letter sigma denotes the vector sum of all forces acting on an object.

ONE object!

Newton's Second Law

When a net external force acts on an object of mass *m*, the acceleration that results is directly proportional to the net force and has a magnitude that is inversely proportional to the mass. The direction of the acceleration is the same as the direction of the net force.

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} \qquad \sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

Sum of forces
acting on 1 object

SI Unit for Force

$$(\mathrm{kg})\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right)=\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2}$$

Note: it has the same units as ma.

This combination of units is called a *newton* (N).

$$1\,\mathrm{kg}\cdot\mathrm{m/s}^2 = 1\,\mathrm{N}$$

4.3 Newton's Second Law of Motion

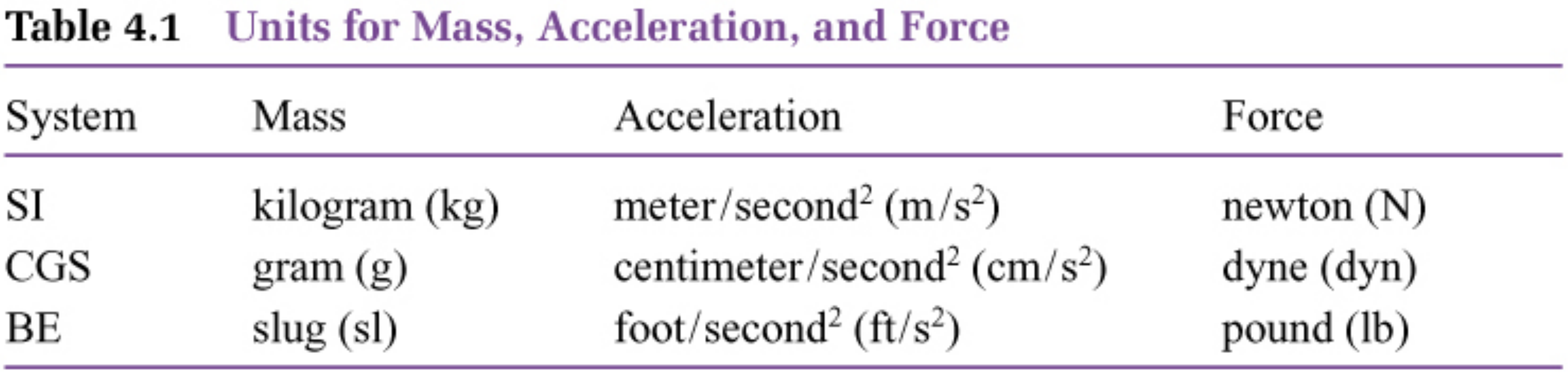


Table 4.1 Units for Mass, Acceleration, and Force

System	Mass	Acceleration	Force
SI	kilogram (kg)	meter/second^2 (m/s^2)	newton (N)
CGS	gram (g)	centimeter/second^2 (cm/s^2)	dyne (dyn)
BE	slug (sl)	foot/second^2 (ft/s^2)	pound (lb)

$\sim 5\,\mathrm{N} = 1\,\mathrm{lb}$