Exam 1 Solutions

Kinematics and Newton's laws of motion

4. In a tug-of-war, each man on a 5-man team pulls with an average force of 500 N. What is the tension in the center of the rope?

A) zero newtons
B) 100 N
C) 500 N
D) 2500 N
E) 5000 N

Each team pulls with a force of 2500 N.

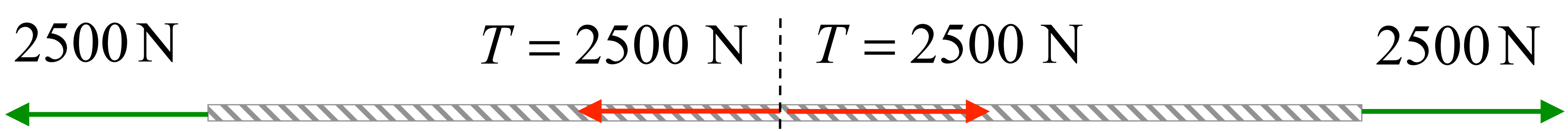
The rope pulls back with a Tension = 2500 N on each team.

Split the rope at the midpoint.

Attach the ends together with hooks.

How hard does each hook pull on the other?

That is the tension at the midpoint.



"Action – Reaction" pair of forces of Newton's 3rd law.

5. In which one of the following situations does the car have a westward acceleration?

A) The car travels westward at constant speed.

B) The car travels eastward and speeds up.

C) The car travels westward and slows down.

D) The car travels eastward and slows down.

E) The car starts from rest and moves toward the east.

east(+), initial velocity $= +v_0$, final velocity $= +v$

Slows down, $v_0 > v$

$$\text{Acceleration} = \frac{v - v_0}{t} \text{ is negative (west)}$$

What about C)?

west(+), initial velocity $= +v_0$, final velocity $= +v$

Slows down, $v_0 > v$

EASTWARD

$$\text{Acceleration} = \frac{v - v_0}{t} \text{ is negative (east)}$$

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C) The car travels westward and slows down.

D) The car travels eastward and slows down.

E) The car starts from rest and moves toward the east.

In C) What if you choose west as negative?

west(–), initial velocity $= -v_0$, final velocity $= -v$

Slows down, $v_0 > v$

$$\text{Acceleration} = \frac{(-v)-(-v_0)}{t} = \frac{v_0 - v}{t} \text{ is positive (east)}$$

STILL EASTWARD

If object travels in one direction and slows down, the acceleration is in the opposite direction!

13. What is the acceleration of the "two block" system?

A) 1 m/s^2

B) 2 m/s^2

C) 3 m/s^2

D) 6 m/s^2

E) 15 m/s^2

$$\boxed{a = \frac{F}{m} = \frac{30\text{ N}}{15\text{ kg}} = 2\text{ m/s}^2}$$

14. What is the force of static friction between the top and bottom blocks of the previous problem?

A) zero newtons

B) 10 N

C) 20 N

D) 25 N

E) 30 N

Acceleration of 5kg block must be the same as the combination.

$$\boxed{f_5 = ma = (5\text{ kg})\,(2\text{ m/s}^2) = 10\text{ N}}$$

14. What is the force of static friction between the top and bottom blocks of the previous problem?

A) zero newtons

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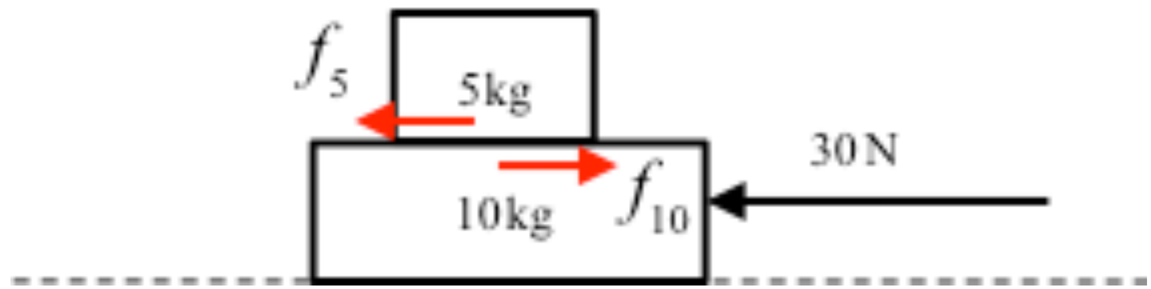
$$\boxed{f_5 = ma = (5\text{ kg})\,(2\text{ m/s}^2) = 10\text{ N}}$$



Also, the 10 kg block must have the same acceleration

f_5 and f_{10} are an "action-reaction" pair from Newton's 3rd law.

Equal magnitudes, opposite directions on two objects in contact.

left is +, $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} = (10\text{ kg})(+2\text{ m/s}^2) = +20\text{ N}$

$$= +\mathbf{f}_{10} + (30\text{N}) = +20\text{ N}$$

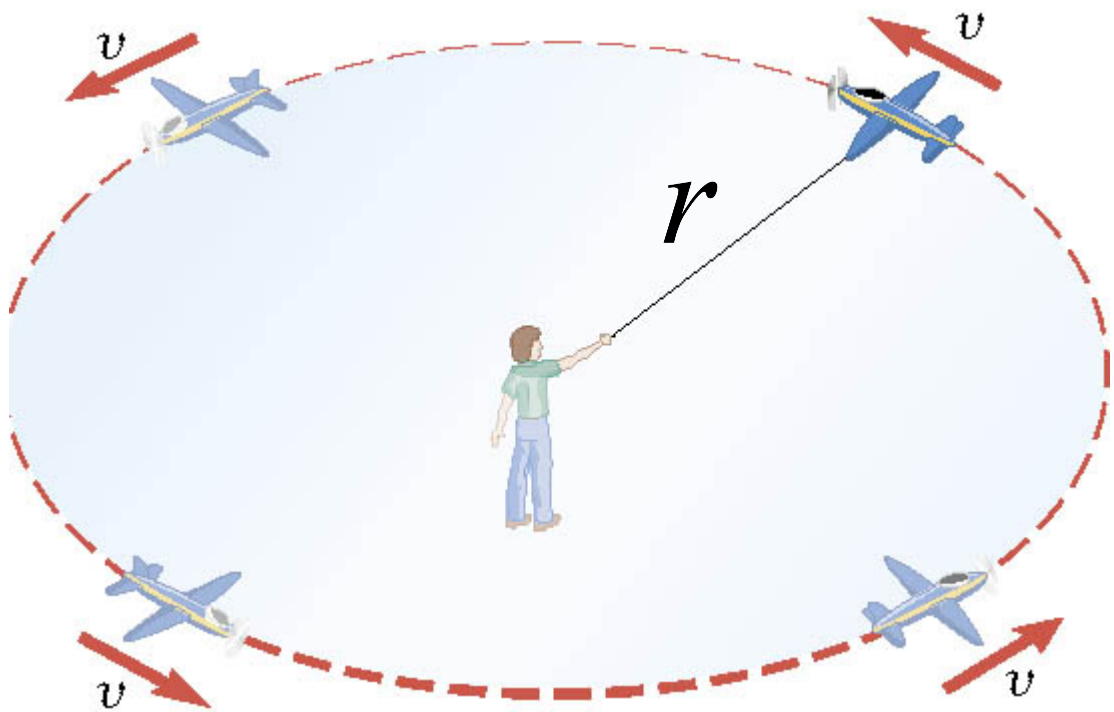
$$\mathbf{f}_{10} = -10\text{ N}$$

Chapter 5

Dynamics of Uniform Circular Motion

DEFINITION OF UNIFORM CIRCULAR MOTION

Uniform circular motion is the motion of an object traveling at a constant speed on a circular path.

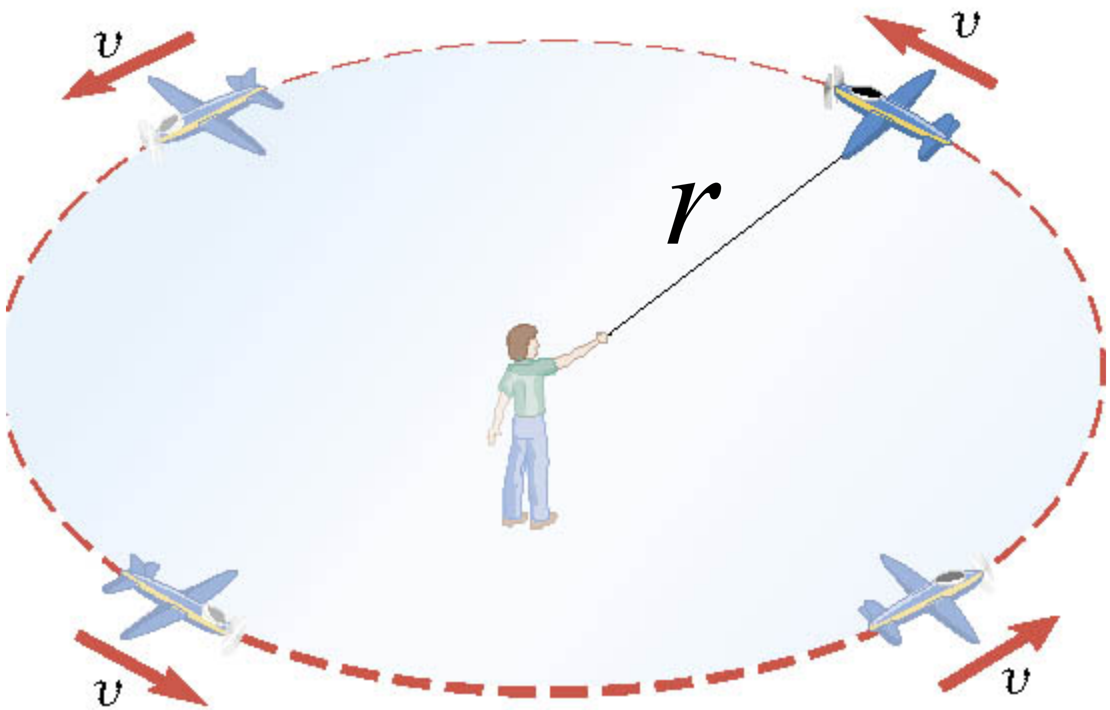


Circumference of the circle is $2\pi r$.

5.1 Uniform Circular Motion

The time it takes the object to travel once around the circle is T (a.k.a. the period)

$$\text{Speed around the circle is, } v = \frac{2\pi r}{T}.$$



Example 1: A Tire-Balancing Machine

The wheel of a car has a radius of 0.29m and it being rotated at 830 revolutions per minute on a tire-balancing machine. Determine the speed at which the outer edge of the wheel is moving.

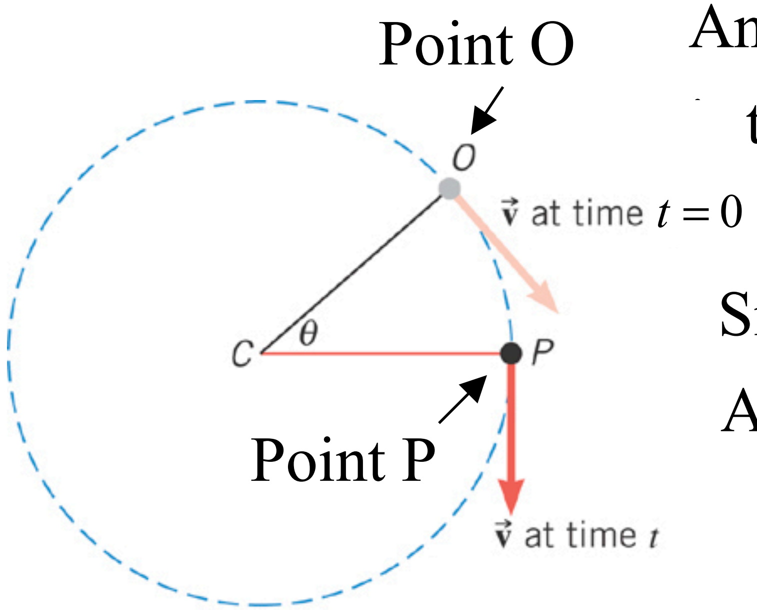
$$\frac{1}{830\,\text{revolutions/min}} = 1.2\times 10^{-3}\ \text{min/revolution}$$

$$T = 1.2\times 10^{-3}\ \text{min} = 0.072\,\text{s}$$

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.29\,\text{m})}{0.072\,\text{s}} = 25\,\text{m/s}$$

5.2 Centripetal Acceleration

In uniform circular motion, the speed is *constant*, but the direction of the velocity vector is *not constant.*



(a)

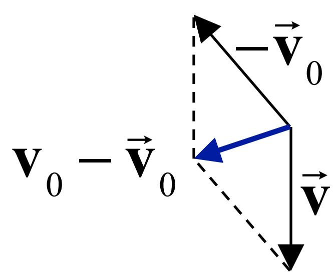
Angle between point O and point P the same as between $\vec{\mathbf{v}}_0$ and $\vec{\mathbf{v}}$.

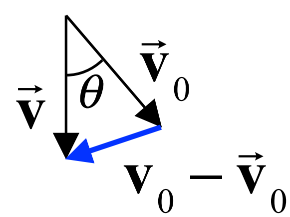
Since velocity vector changes direction Acceleration vector is NOT ZERO.

$$\mathbf{a} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_0}{t}$$

Need to understand: $\vec{\mathbf{v}} - \vec{\mathbf{v}}_0$

NOTE: $\vec{\mathbf{v}} - \vec{\mathbf{v}}_0$ and $\vec{\mathbf{a}}$ point in toward center of circle!





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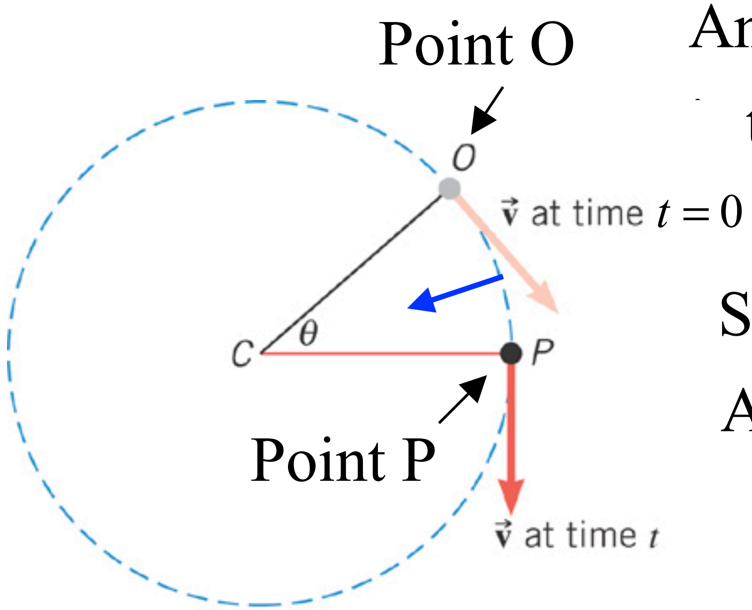

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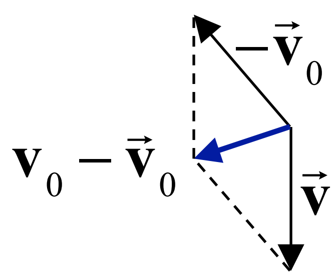
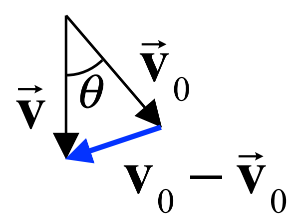
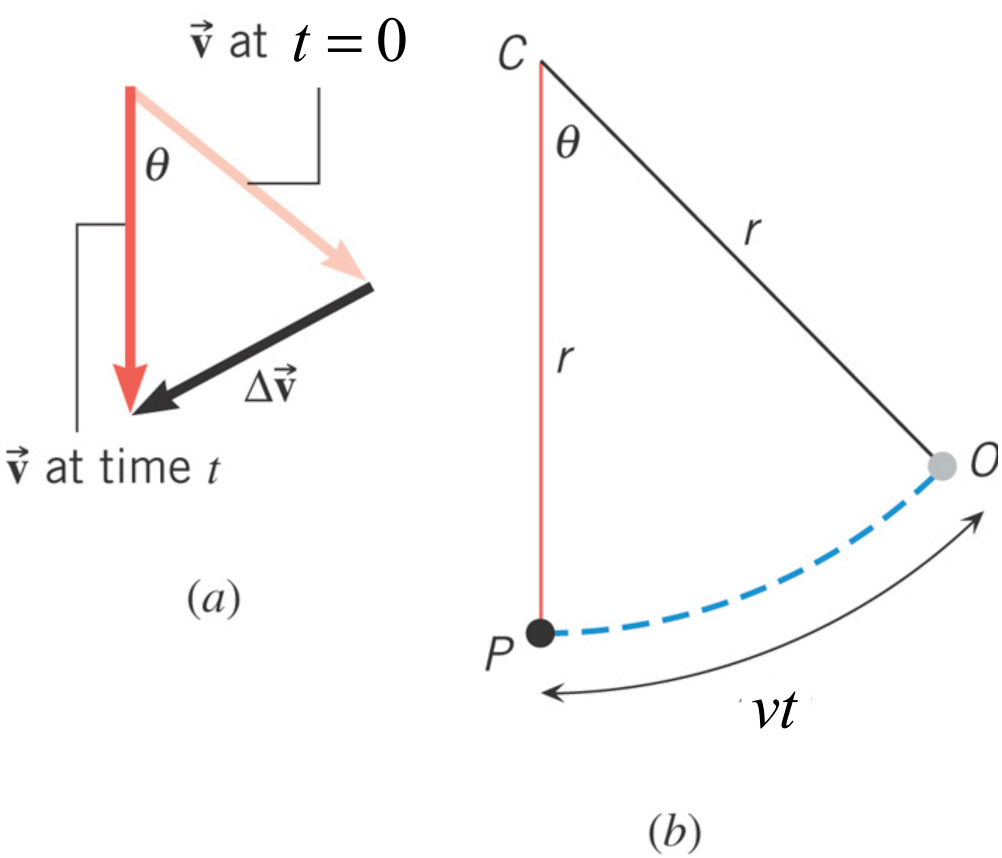




5.2 Centripetal Acceleration

Compare geometry of velocity vectors and the portion of the circle.

$$\theta = \frac{\Delta v}{v} \qquad \theta = \frac{vt}{r}$$



Magnitudes

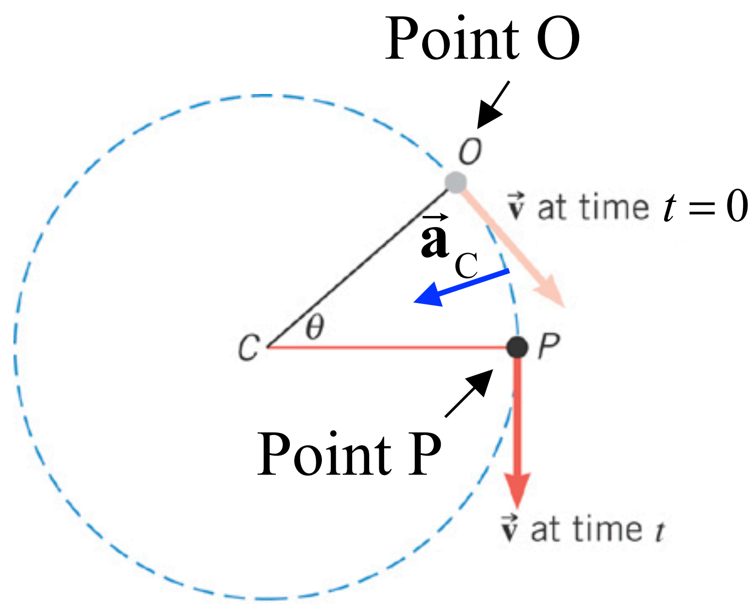
$$\frac{\Delta v}{v} = \frac{vt}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

$$\boxed{a_c = \frac{v^2}{r}}$$

5.2 Centripetal Acceleration

The direction of the centripetal acceleration is towards the center of the circle; in the same direction as the change in velocity.



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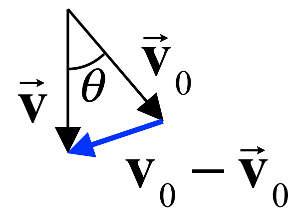


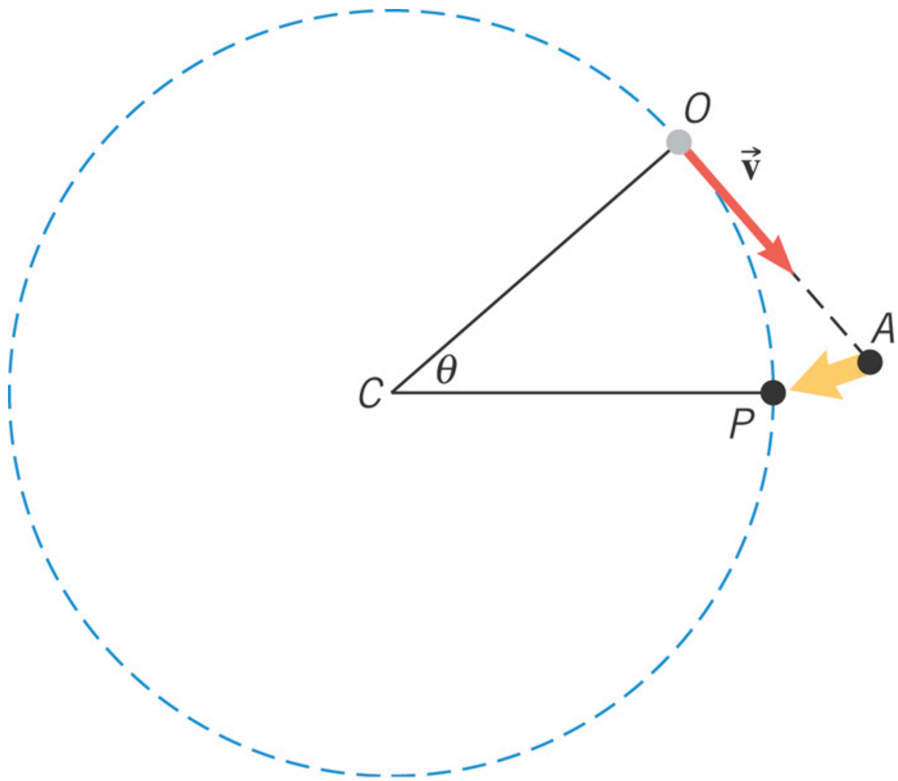
$$a_C = \frac{v^2}{r}$$

Centripetal acceleration vector points *inward* at ALL points on the circle

Conceptual Example 2: Which Way Will the Object Go?

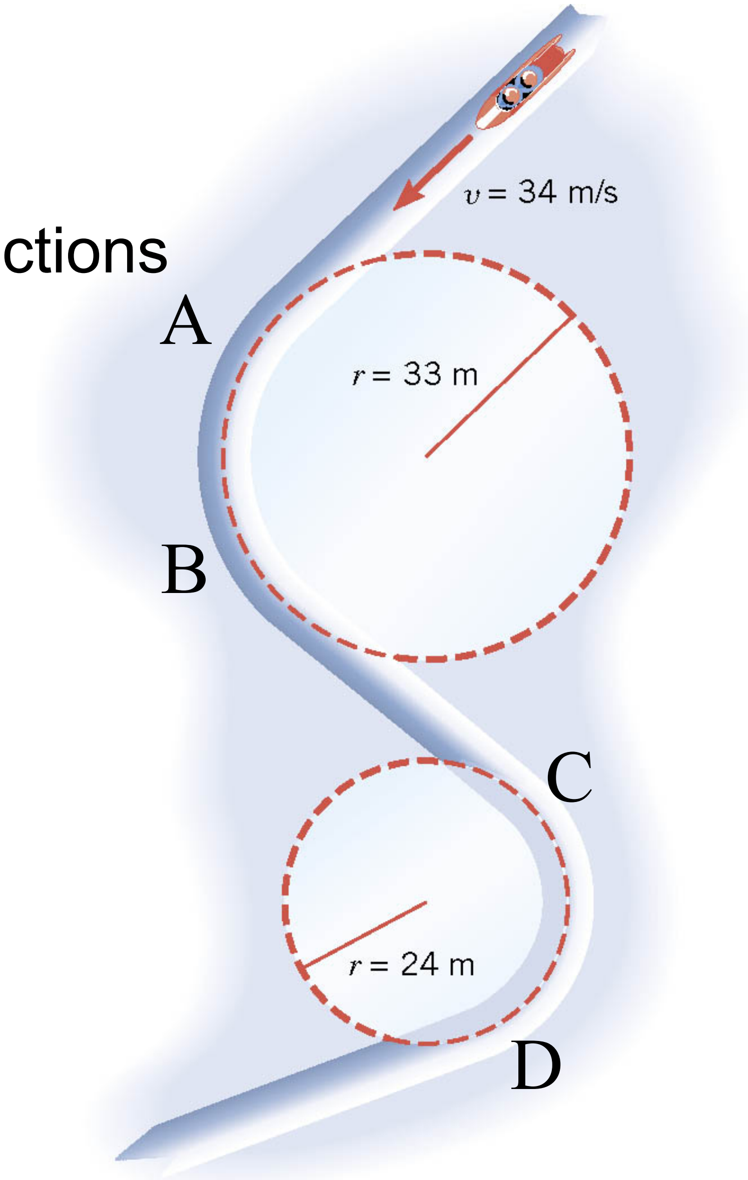
An object (•) is in uniform circular motion. At point *O* it is released from its circular path.

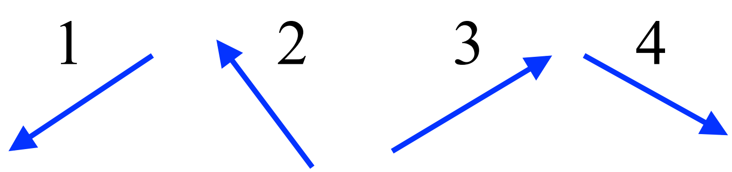
Does the object move along the
(A) Straight path between *O* and *A*
or
(B) Along the circular arc between points *O* and *P* ?



Example 3: The Effect of Radius on Centripetal Acceleration

The bobsled track contains turns with radii of 33 m and 24 m.
Match the acceleration vector directions below to the points A,B,C,D.





A –

B –

C –

D –

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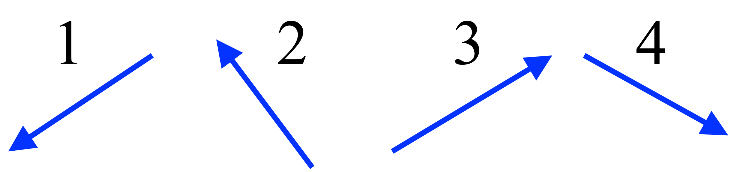
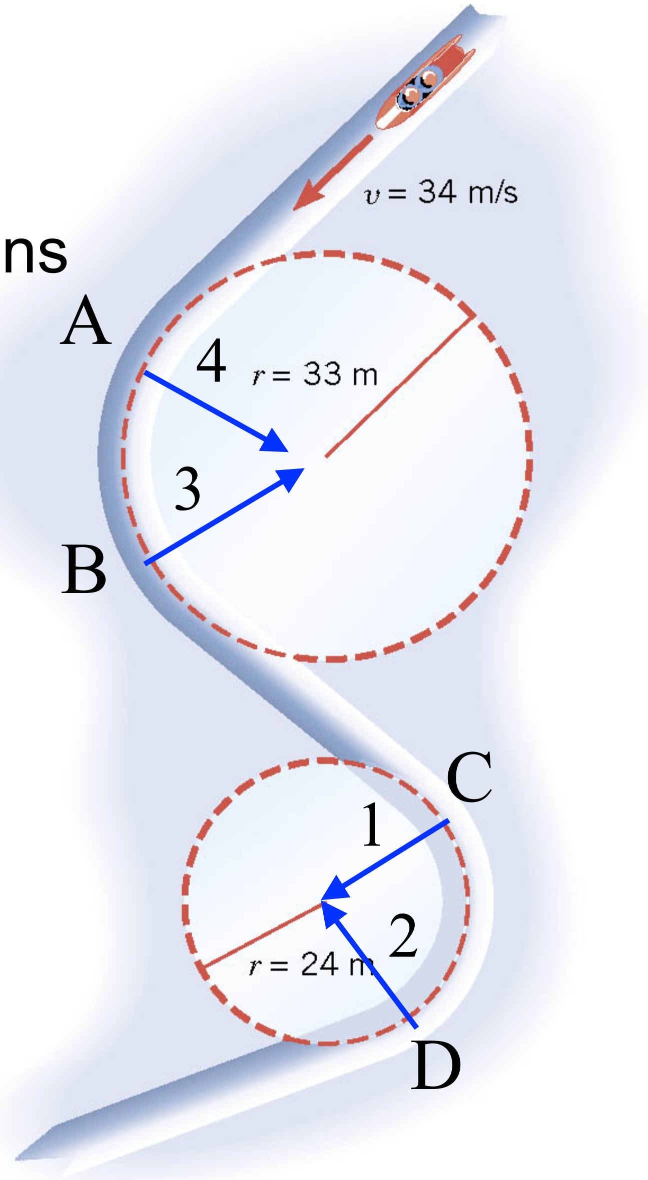


A – 4

B – 3

C – 1

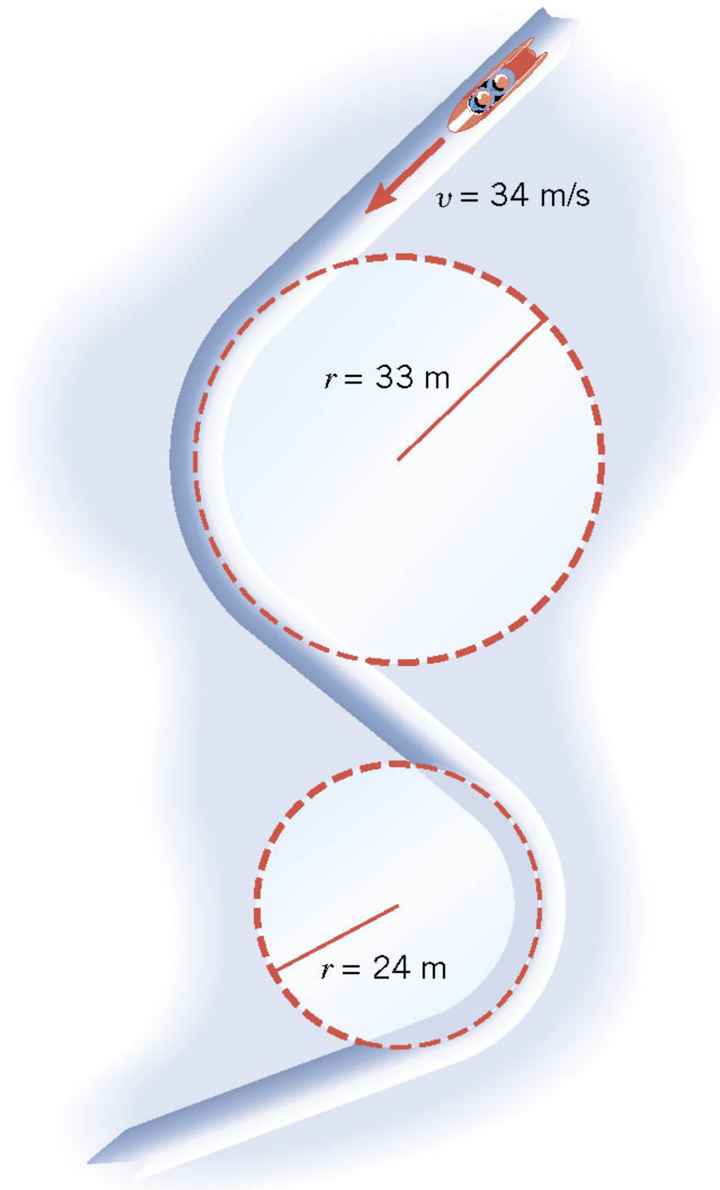
D – 2



5.2 Centripetal Acceleration

$$a_C = v^2/r$$

Find the centripetal acceleration at each turn for a speed of 34 m/s. Express answers as multiples of $g = 9.8\,\mathrm{m/s^2}$.



$$a_C = \frac{(34\,\mathrm{m/s})^2}{33\ \mathrm{m}} = 35\,\mathrm{m/s^2} = 3.6g$$

$$a_C = \frac{(34\,\mathrm{m/s})^2}{24\ \mathrm{m}} = 48\,\mathrm{m/s^2} = 4.9g$$

Clicker Question 5.1

Clicker Question 5.2

Newton's Second Law

When a net external force acts on an object of mass *m*, the acceleration that results is directly proportional to the net force and has a magnitude that is inversely proportional to the mass. The direction of the acceleration is the same as the direction of the net force.

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} \qquad \sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

Vector Equations

5.3 Centripetal Force

Thus, in uniform circular motion there must be a net force to produce the centripetal acceleration.

The centripetal force is the name given to the net force required to keep an object moving on a circular path.

The direction of the centripetal force always points toward the center of the circle and continually changes direction as the object moves.

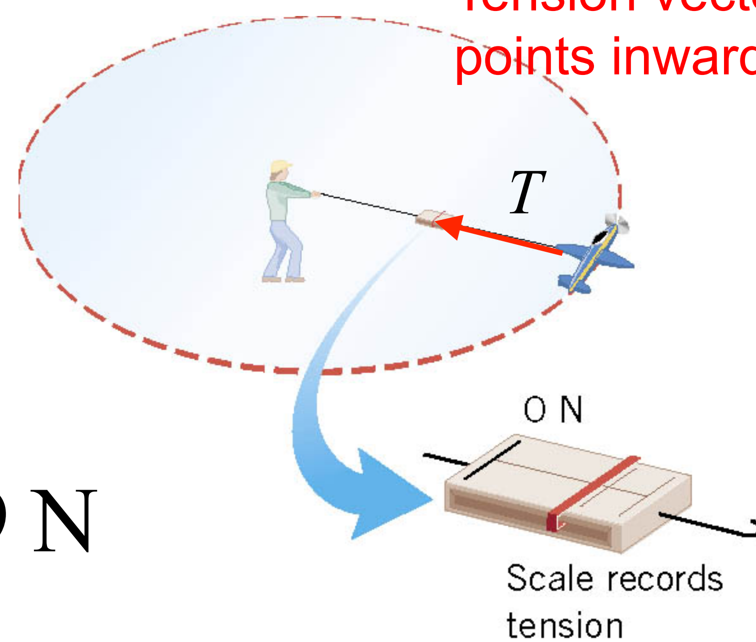
$$F_C = ma_C = m\frac{v^2}{r}$$

Magnitudes

Example 5: The Effect of Speed on Centripetal Force

The model airplane has a mass of 0.90 kg and moves at constant speed on a circle that is parallel to the ground. The path of the airplane and the guideline lie in the same horizontal plane because the weight of the plane is balanced by the lift generated by its wings. Find the tension in the 17 m guideline for a speed of 19 m/s.

Tension vector points inward!



$$T = F_C = m\frac{v^2}{r}$$

$$T = (0.90\text{ kg})\frac{(19\text{ m/s})^2}{17\text{ m}} = 19\text{ N}$$

Example 5: The Effect of Speed on Centripetal Force

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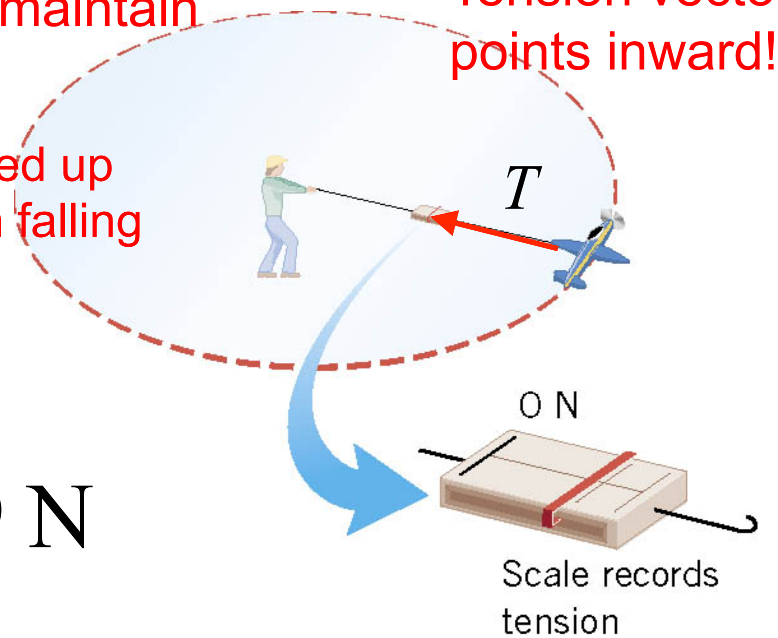
Tension is the centripetal force necessary to maintain airplane in the circle

Tension vector points inward!

$$T = F_C = m\frac{v^2}{r}$$

engine keeps speed up
wings keep it from falling



$$T = (0.90\text{ kg})\frac{(19\text{ m/s})^2}{17\text{ m}} = 19\text{ N}$$

Conceptual Example 6: A Trapeze Act

In a circus, a man hangs upside down from a trapeze, legs bent over and arms downward, holding his partner. Is it harder for the man to hold his partner when the partner hangs straight down and is stationary or when the partner is swinging through the straight-down position?

Tension in arms maintains circular motion but also must counter the gravitational force (weight)

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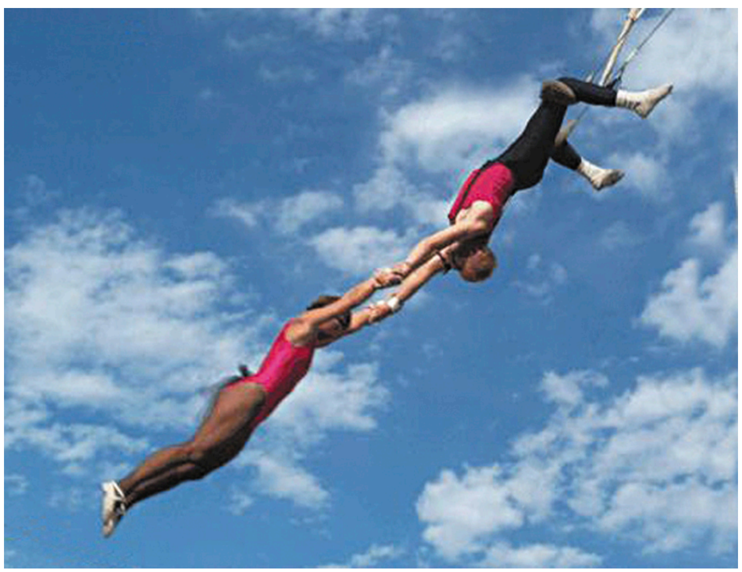
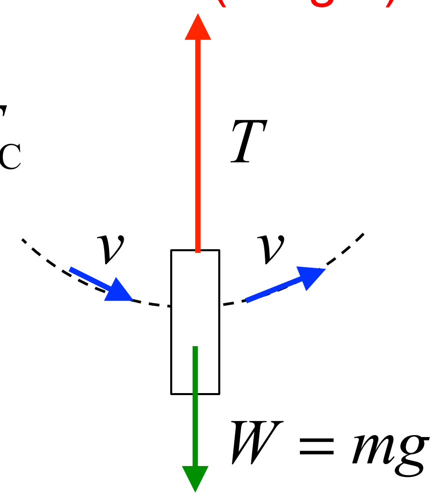
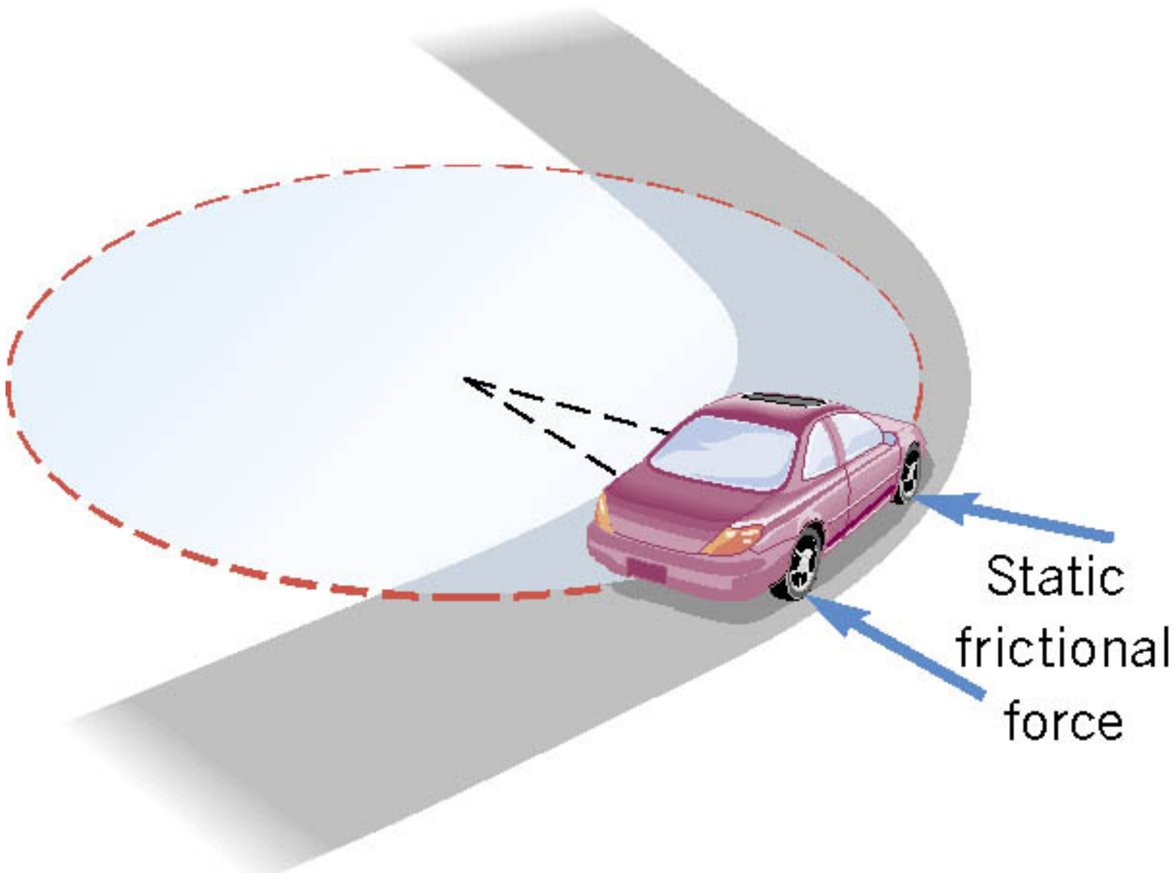


$$\sum \vec{\mathbf{F}} = +T - W = F_C$$

$$T = W + F_C$$

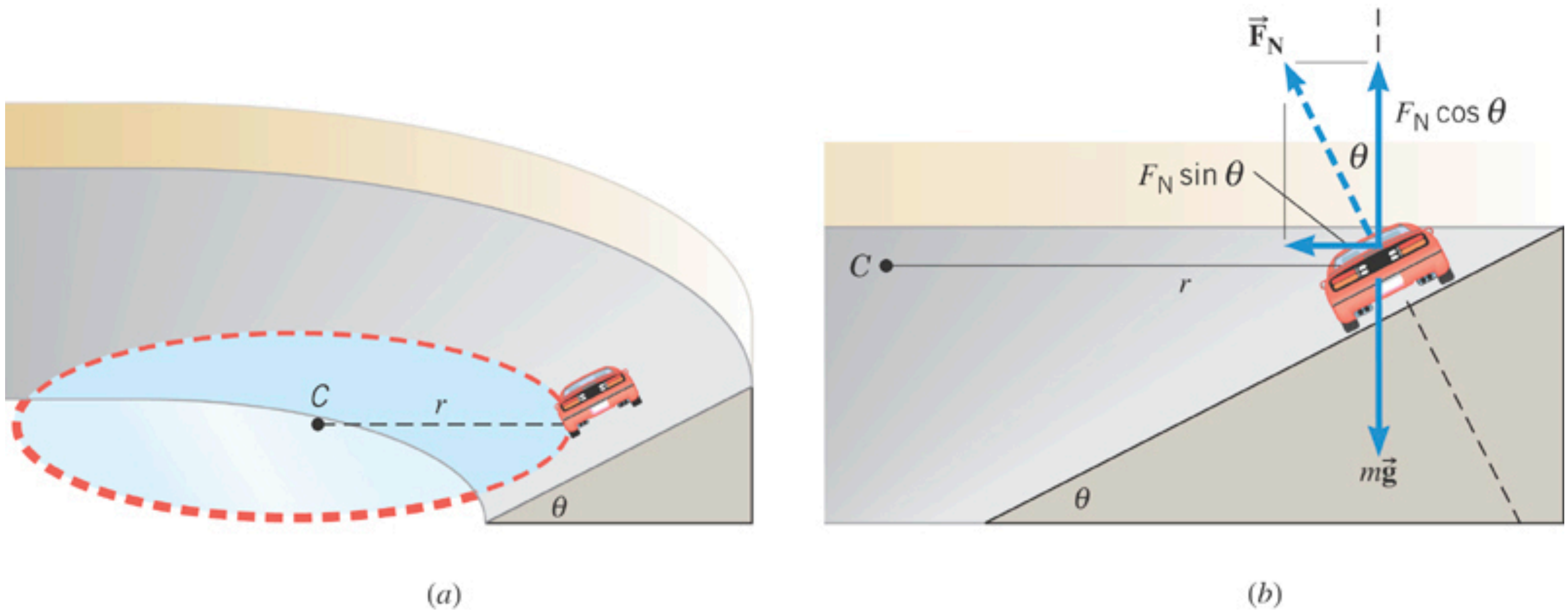


On an unbanked curve, the static frictional force provides the centripetal force.



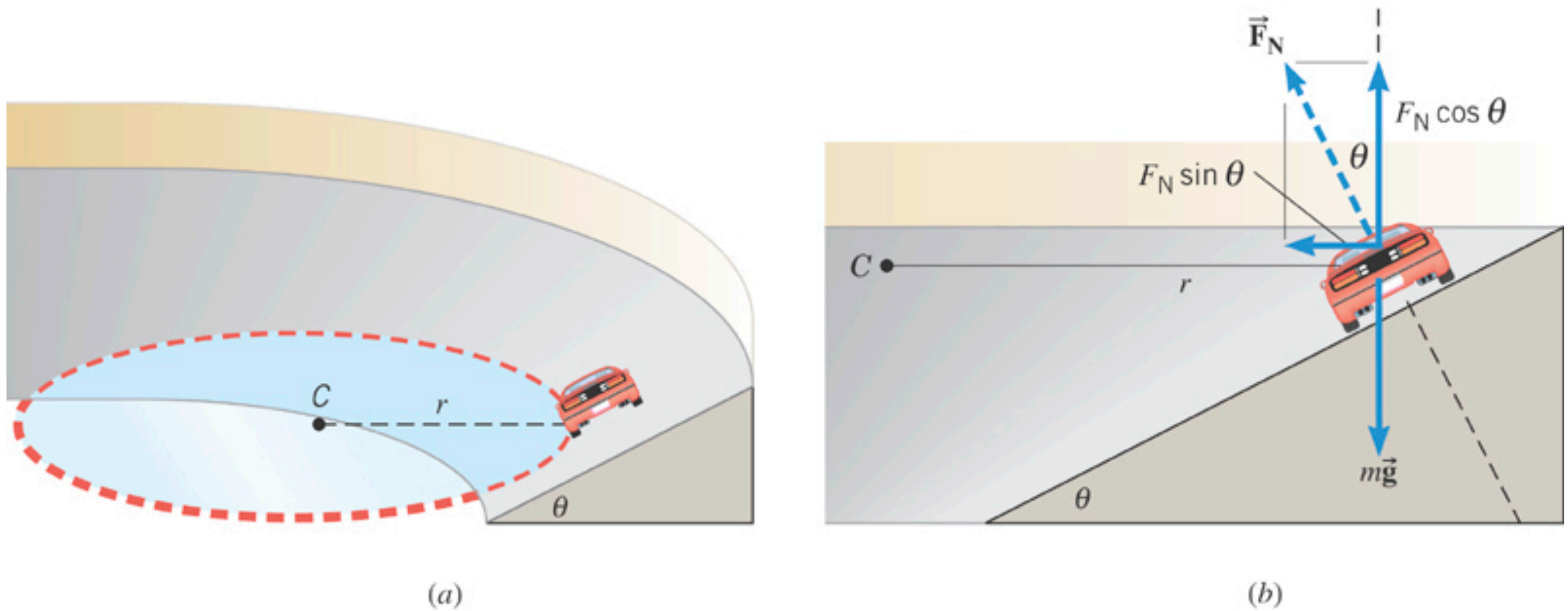
5.4 Banked Curves

On a frictionless banked curve, the centripetal force is the horizontal component of the normal force. The vertical component of the normal force balances the car's weight.



Compression of the banked road provides the normal force. The normal force pushes against the car to 1) support the weight and 2) provide the centripetal force to keep the car moving in a circle.

5.4 Banked Curves



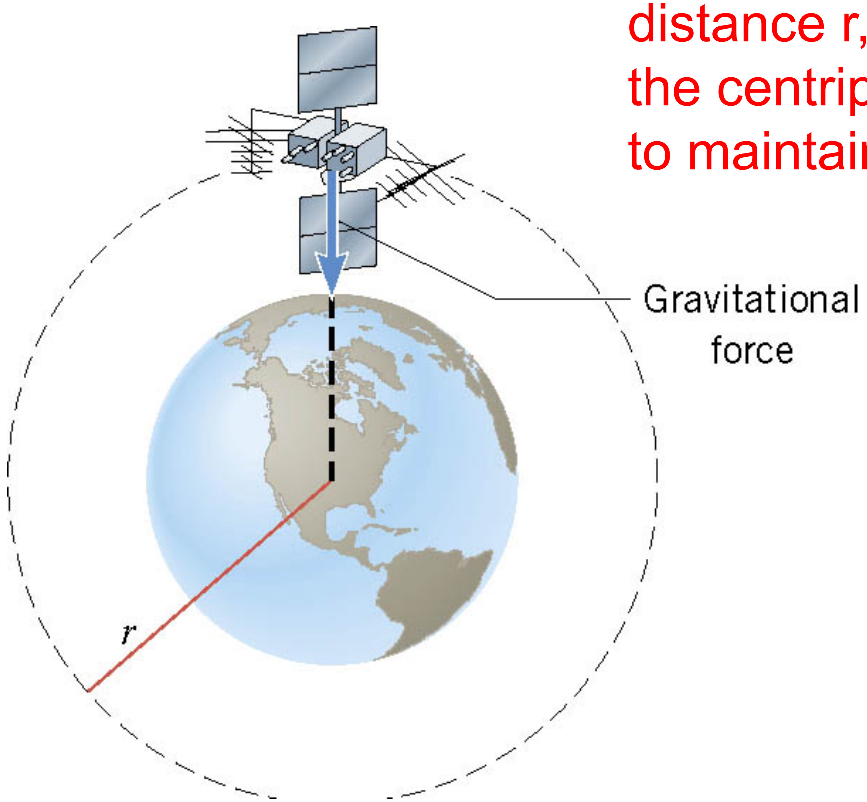
$$F_C = F_N \sin\theta = m\frac{v^2}{r}$$

$$F_N \cos\theta = mg$$

Combining the two relationships can determine the speed necessary to keep the car on the track with the given angle

5.5 Satellites in Circular Orbits

There is only one speed that a satellite can have if the satellite is to remain in an orbit with a fixed radius.



Gravitational force at the distance r, is the source of the centripetal force necessary to maintain the circular orbit.

5.5 Satellites in Circular Orbits

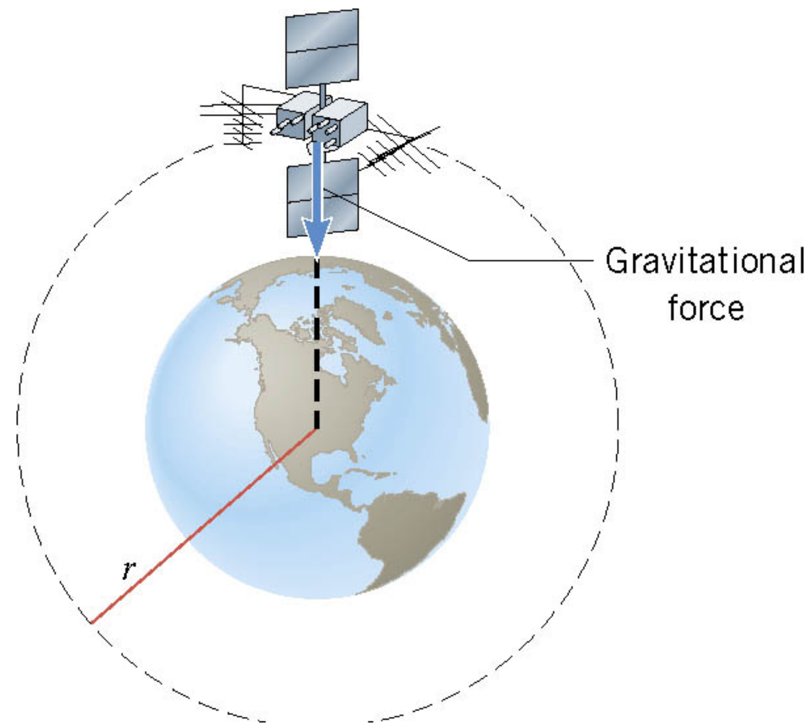
Gravitational force at the distance r

Centripetal force

$$F_C = G\frac{mM_E}{r^2} = m\frac{v^2}{r}$$

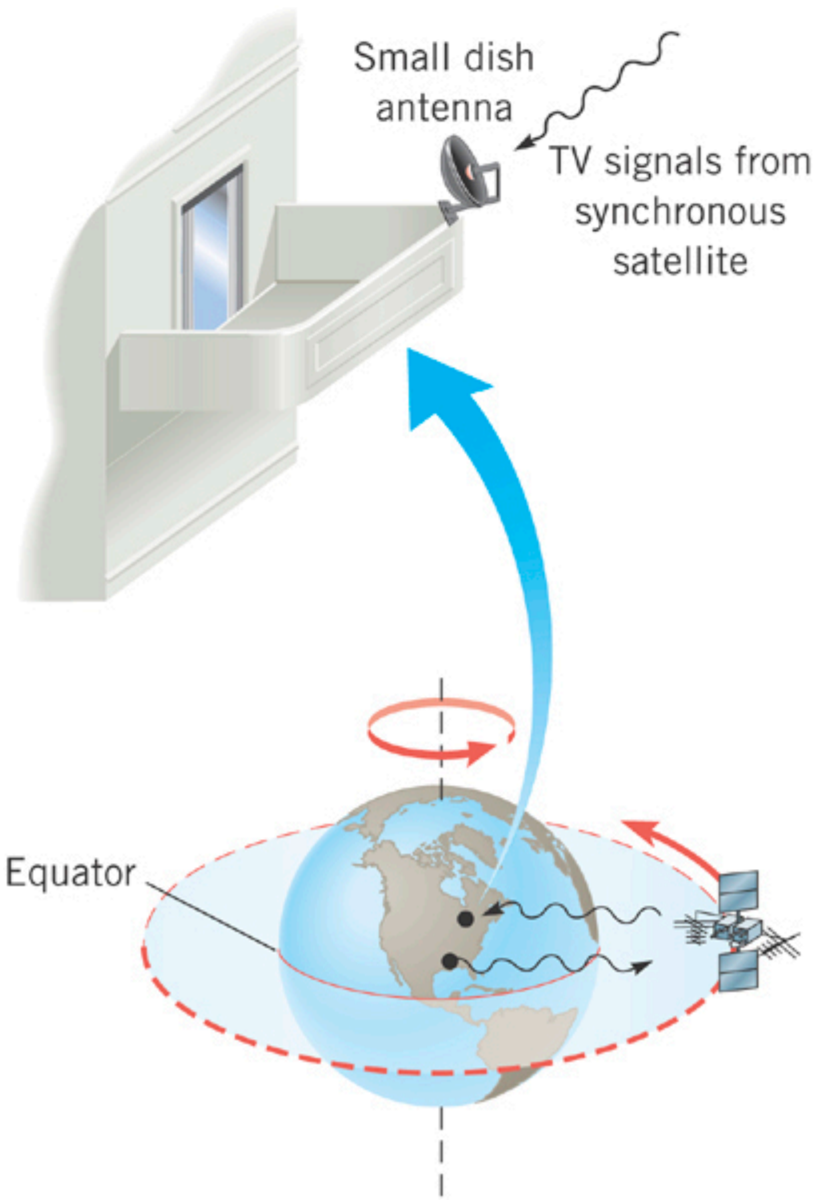
$$v = \sqrt{\frac{GM_E}{r}}$$

Speed to keep satellite in the orbit with radius r.



5.5 Satellites in Circular Orbits

There is a radius where the speed will make the satellite go around the earth in exactly 24 hours. This keeps the satellite at a fixed point in the sky.



5.6 *Apparent Weightlessness and Artificial Gravity*

Can you feel gravity? We previously determined that you can't.

1) Hanging from a 100 m high diving board
– your arms feel stretched by the bending of the board.

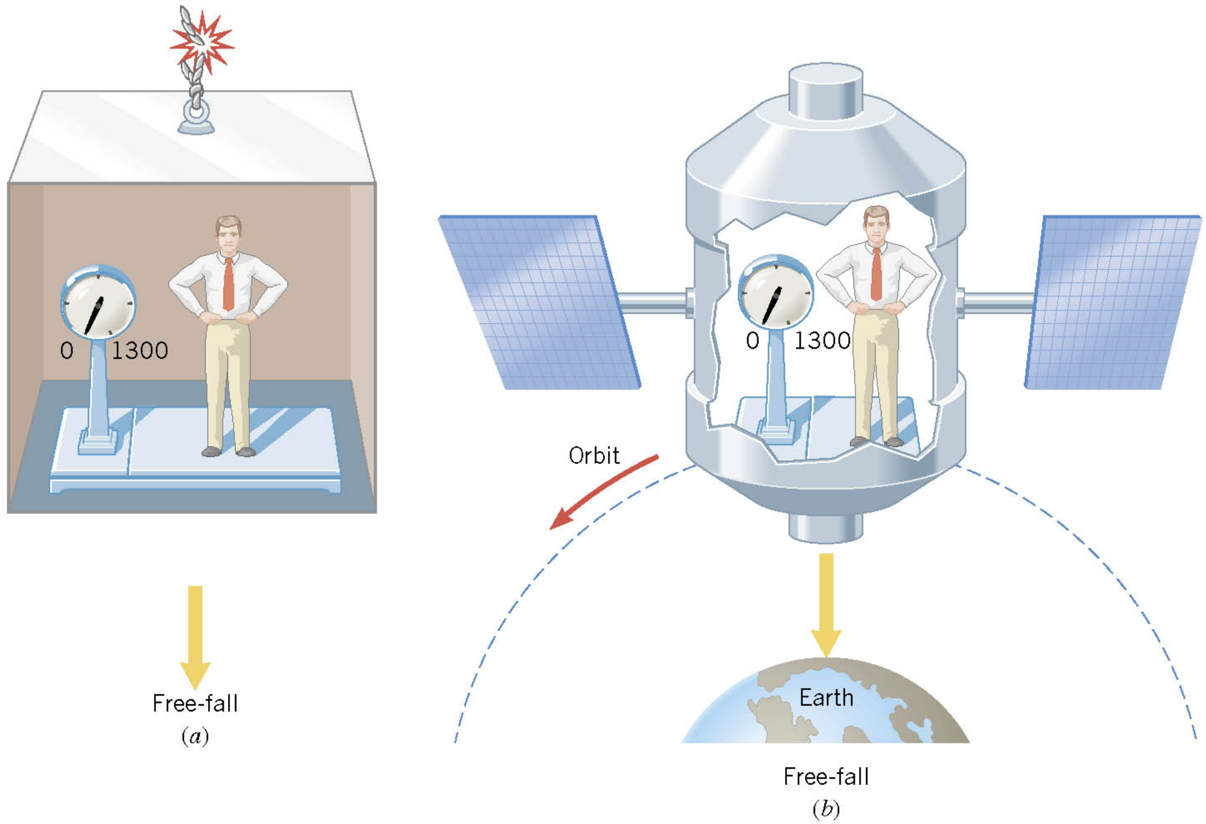
2) Standing on a bed – your legs feel compressed by the springs in the mattress.

The bent diving board or the compressed springs provide the force to balance the gravitational force on you.

When you let go of the diving board and before you hit the ground the ONLY force on you is gravity. It makes you accelerate downward, but it does not stretch or compress your body.

In free fall one cannot feel the force of gravity!

5.6 Apparent Weightlessness and Artificial Gravity



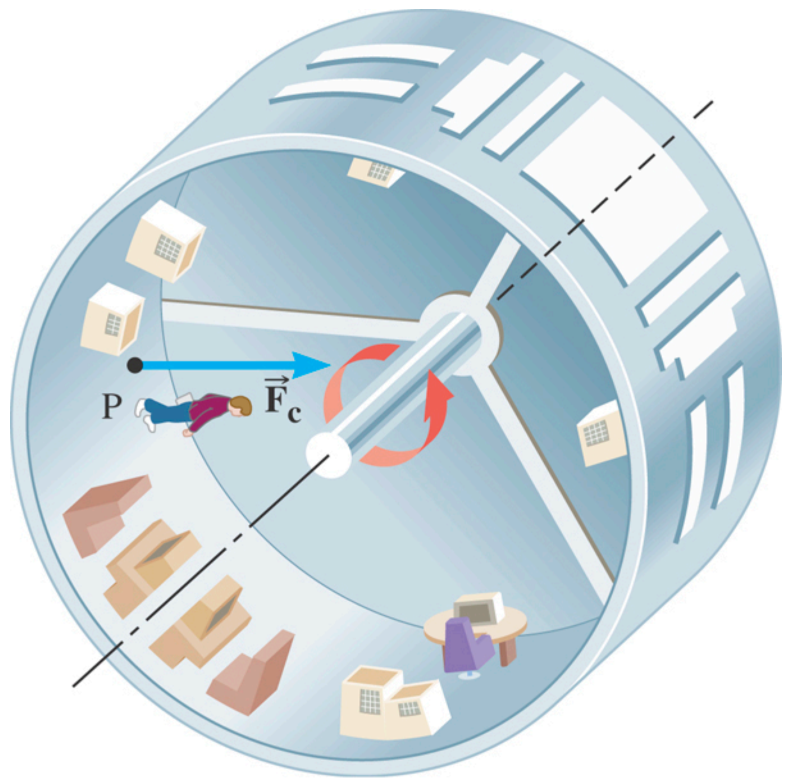
In each case, the weight recorded by the scale is ZERO.

Gravitational force acts on the body and on the satellite to provide the centripetal force necessary to keep both in orbit. Gravitational force makes both the elevator and the body fall with the same acceleration.

Example 13: Artificial Gravity

At what speed must the surface of a space station move so that an astronaut experiences a push on the feet equal to the weight on earth? The radius is 1700 m.

$$F_c = m\frac{v^2}{r} = mg$$



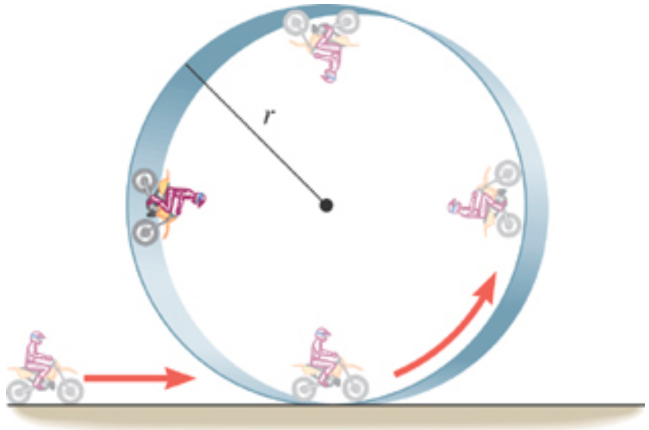
$$v = \sqrt{rg}$$

$$= \sqrt{(1700\text{ m})(9.80\,\text{m/s}^2)}$$

$$= 130\text{ m/s}$$

5.7 *Vertical Circular Motion*

Normal forces are created by stretching of the hoop.



$\frac{v_3^2}{r}$ must be $> g$

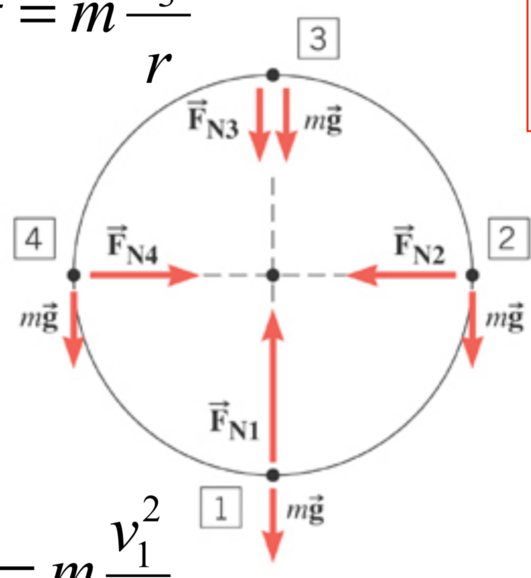
to stay on the track

$$F_{N3} + mg = m\frac{v_3^2}{r}$$

$$F_{N4} = m\frac{v_4^2}{r}$$

$$F_{N2} = m\frac{v_2^2}{r}$$

$$F_{N1} - mg = m\frac{v_1^2}{r}$$



(b)