Chapter 6

Work and Energy

6.1 *Work Done by a Constant Force*

The concept of forces acting on a mass (one object) is intimately related to the concept of ENERGY production or storage.

- A mass accelerated to a non-zero speed carries energy (mechanical)
- A mass raised up carries energy (gravitational)
- The mass of an atom in a molecule carries energy (chemical)
- The mass of a molecule in a hot gas carries energy (thermal)
- The mass of the nucleus of an atom carries energy (nuclear)

  (The energy carried by radiation will be discussed in PHY232)

The road to energy is paved with forces acting on moving masses.

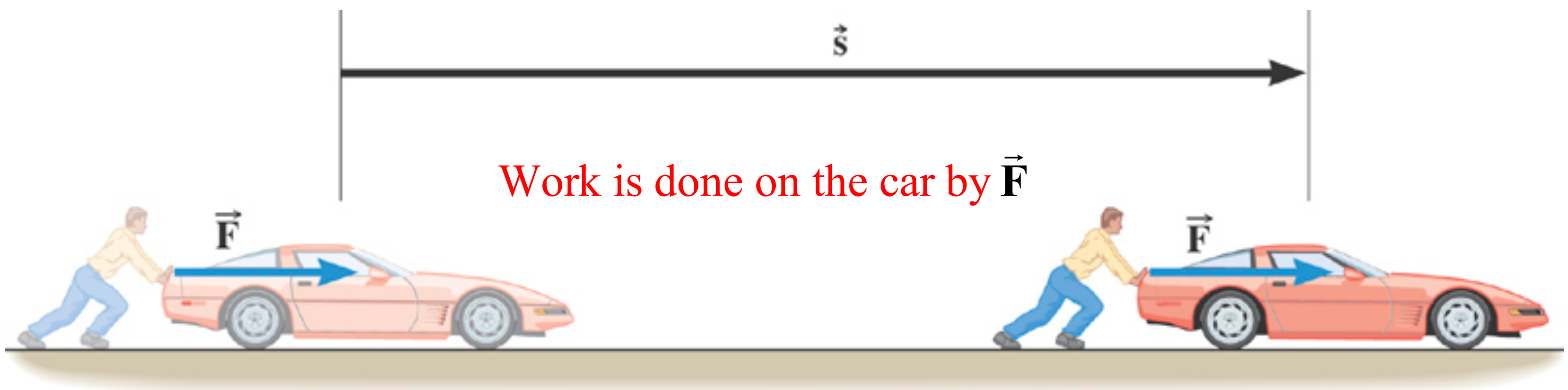
WORK

Sorry, but there is no other way to understand the concept of energy.

6.1 Work Done by a Constant Force

Work is *done on* a moving object (a mass) *by* the force component acting on the object, that is parallel to the displacement of the object.

Only acceptable definition.



The case shown is the simplest: the directions of $\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$ are the same.

F and s are the magnitudes of these vectors.

6.1 Work Done by a Constant Force

Only acceptable definition.

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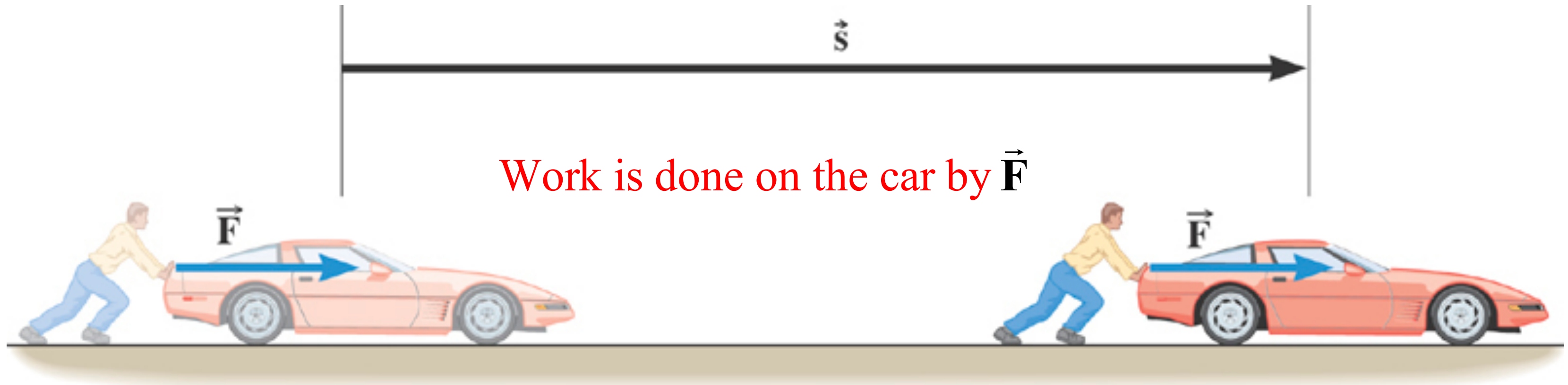


Sorry about using the symbol W again. Hard to avoid it.

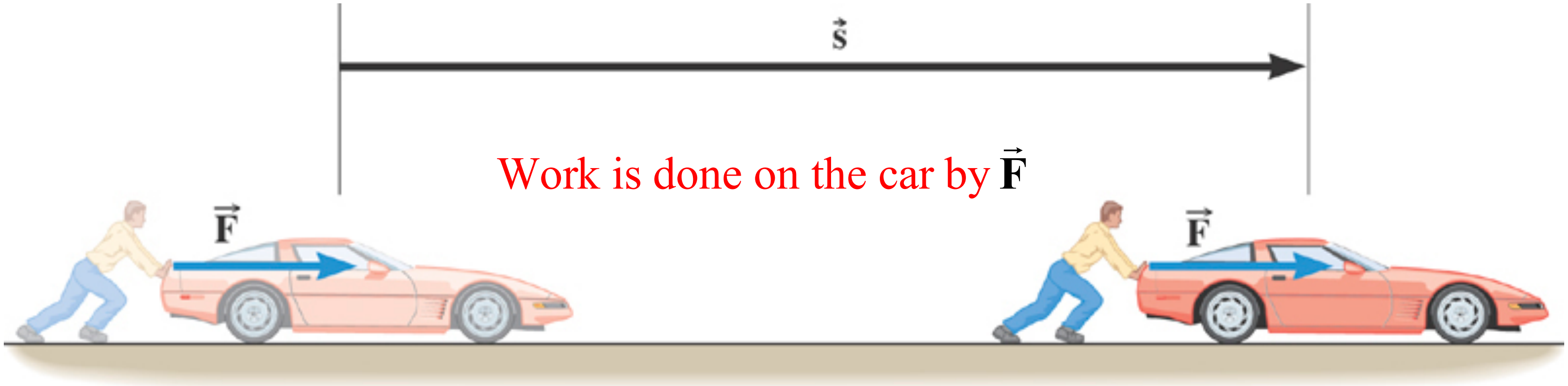
$$\boxed{W = Fs}$$

$$\boxed{1\ \mathrm{N}\cdot\mathrm{m} = 1\ \text{joule}\ (\mathrm{J})}$$

Work is a scalar
(Positive or Negative)

The nature (or source) of the force is a DIFFERENT issue, covered later.
Other forces may be doing work on the object at the same time.
The net amount of work done on the object is the result of the net force on it.

6.1 Work Done by a Constant Force



With only one force acting on the car (m_{Car}), the car must accelerate, and over the displacement s, the speed of the car will increase.

Newton's 2nd law: acceleration of the car, $a = F/m_{Car}$

Starting with velocity v_0, find the final speed.

$$v^2 = v_0^2 + 2as$$

$$\underline{v = \sqrt{v_0^2 + 2as}}$$

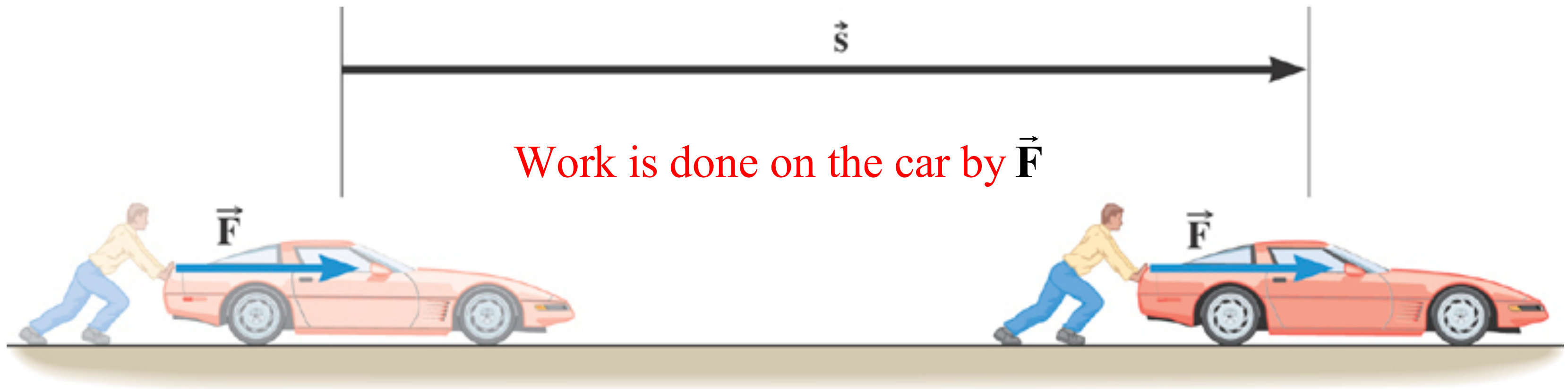
The work done on the car by the force:

$$\boxed{W = Fs} \qquad \boxed{1\ \mathrm{N \cdot m} = 1\ \mathrm{joule\ (J)}}$$

has increased the speed of the car.

6.1 Work Done by a Constant Force

Other forces may be doing work on the object at the same time.



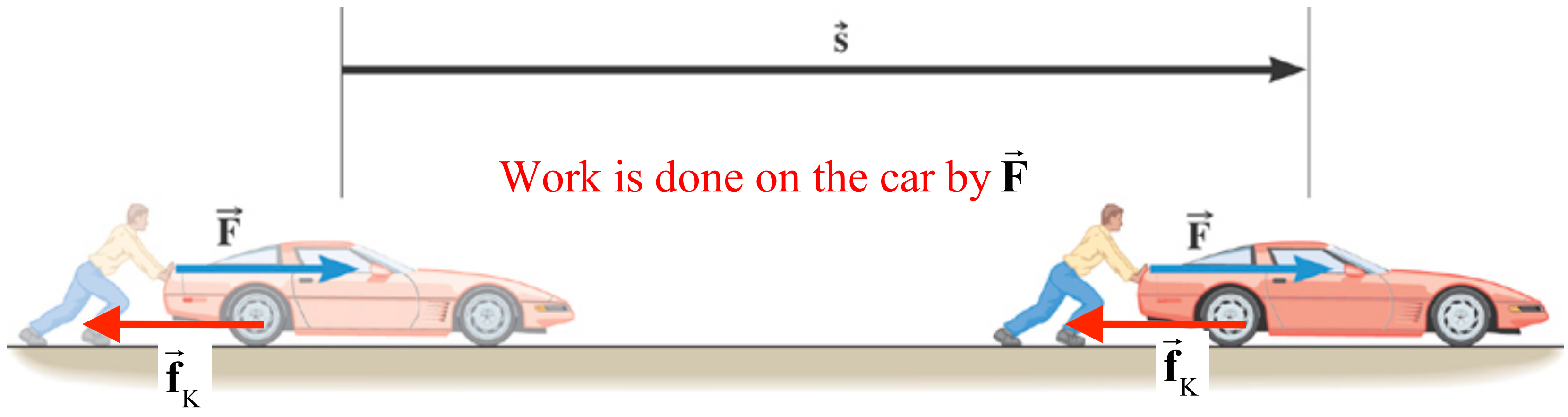
Example:

This time the car is not accelerating, but maintaining a constant speed, v_0.

Constant speed and direction: net force $\sum \mathbf{F} = 0$.

There must be at least one other force acting on the car !

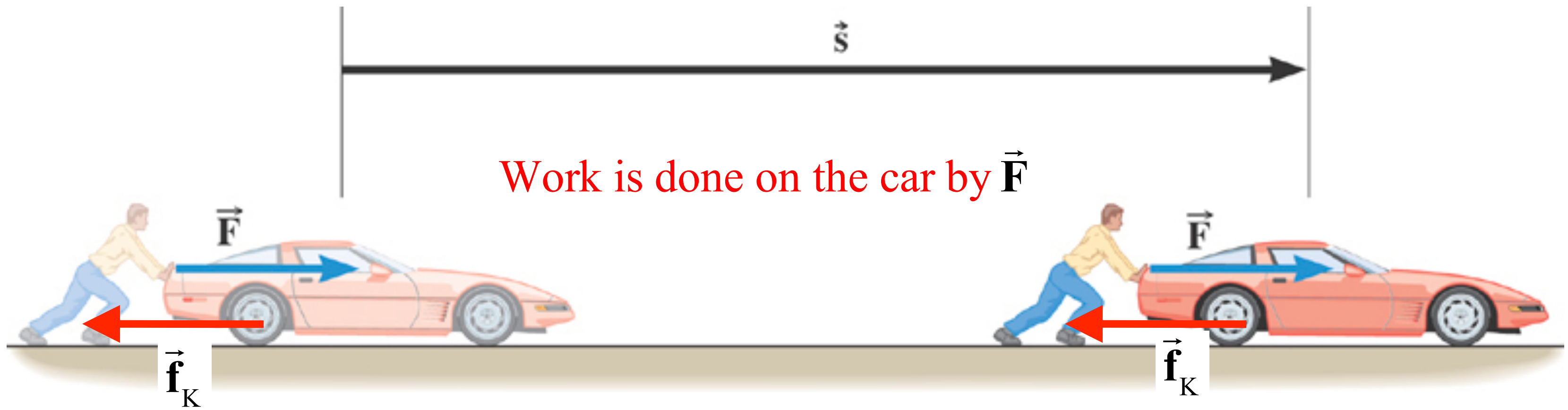
6.1 *Work Done by a Constant Force*



Also acting on the car is a kinetic friction force, $\vec{\mathbf{f}}_K = -\vec{\mathbf{F}}$.

Net force on car must be ZERO, because the car does not accelerate !

6.1 Work Done by a Constant Force



Also acting on the car is a kinetic friction force, $\vec{\mathbf{f}}_K = -\vec{\mathbf{F}}$.

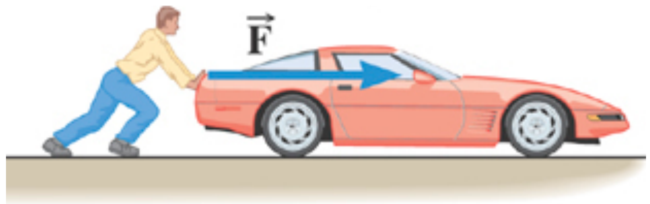
Net force on car must be ZERO, because the car does not accelerate !

$$W = Fs$$

$$W_f = -f_K s = -Fs$$

The work done on the car by $\vec{\mathbf{F}}$ was countered by the work done by the kinetic friction force, $\vec{\mathbf{f}}_K$.

6.1 Work Done by a Constant Force



Car's emergency brake was not released. What happens?

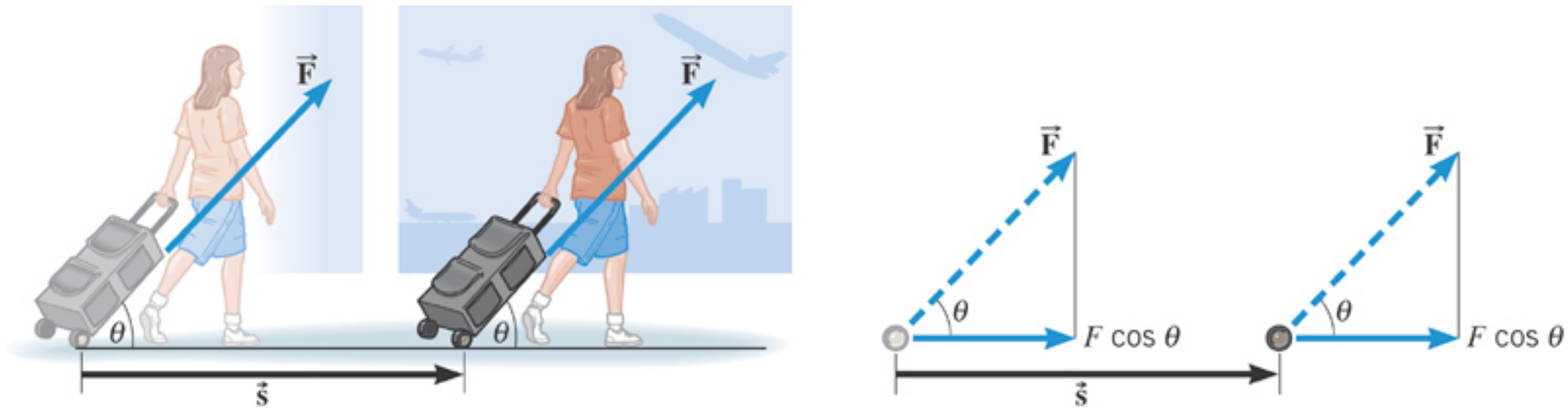
The car does not move. No work done on the car.

Work by force $\vec{\mathbf{F}}$ is zero. What about the poor person?

The person's muscles are pumping away but the attempt to do work on the car, has failed. What happens to the person we will discuss later.

What must concern us here is: if the car does not move the work done on the car by the force $\vec{\mathbf{F}}$ is ZERO.

6.1 Work Done by a Constant Force



(a) (b)

If the force and the displacement are not in the same direction, work is done by only the component of the force in the direction of the displacement.

$$\boxed{W = \left(F\cos\theta\right)s}$$

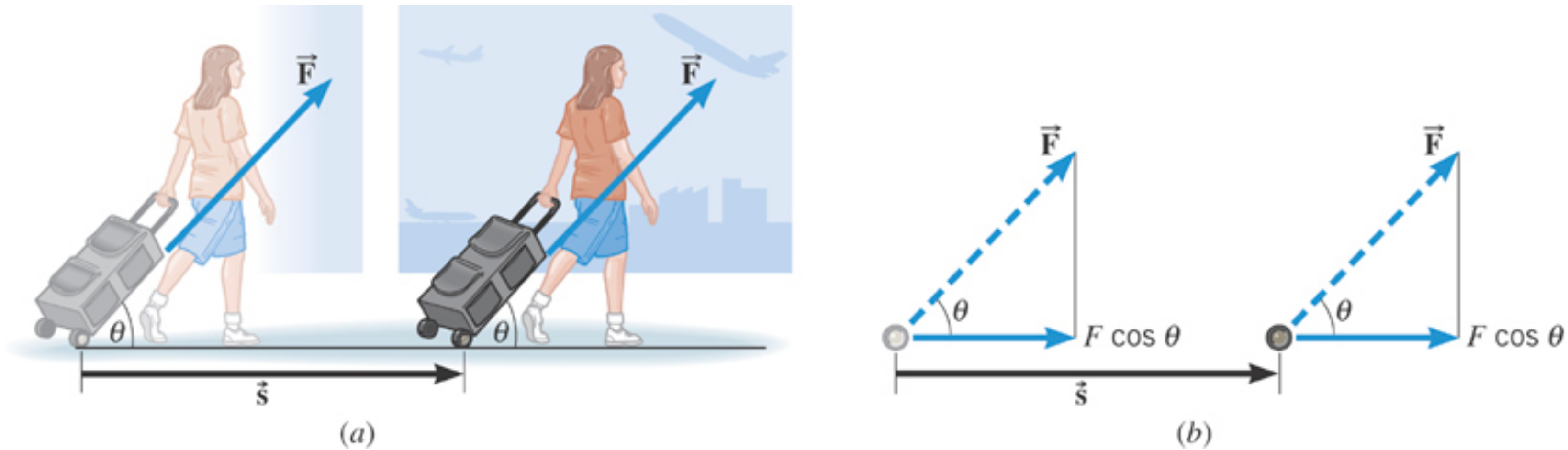
Works for all cases.

$\cos 0^\circ = 1$ $\quad$ $\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$ in the same direction. $\quad$ $W = Fs$

$\cos 90^\circ = 0$ $\quad$ $\vec{\mathbf{F}}$ perpendicular to $\vec{\mathbf{s}}$. $\quad$ $W = 0$

$\cos 180^\circ = -1$ $\quad$ $\vec{\mathbf{F}}$ in the opposite direction to $\vec{\mathbf{s}}$. $\quad$ $W = -Fs$

6.1 Work Done by a Constant Force



Example 1 Pulling a Suitcase-on-Wheels

Find the work done if the force is 45.0-N, the angle is 50.0 degrees, and the displacement is 75.0 m.

$$W = \left(F\cos\theta\right)s = \left[\left(45.0\ \mathrm{N}\right)\cos 50.0^{\circ}\right]\left(75.0\ \mathrm{m}\right)$$

$$= 2170\ \mathrm{J}$$

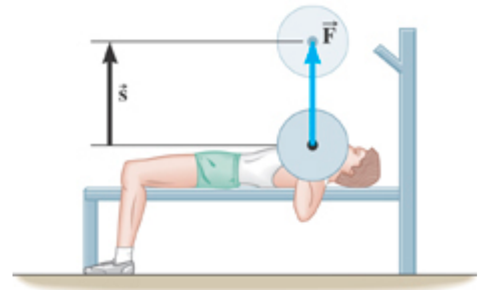
6.1 Work Done by a Constant Force

The bar bell (mass m) is moved slowly at a constant speed $\Rightarrow\; F = mg$.
The work done by the gravitational force will be discussed later.

(a)

Raising the bar bell, the displacement is up, and the force is up.

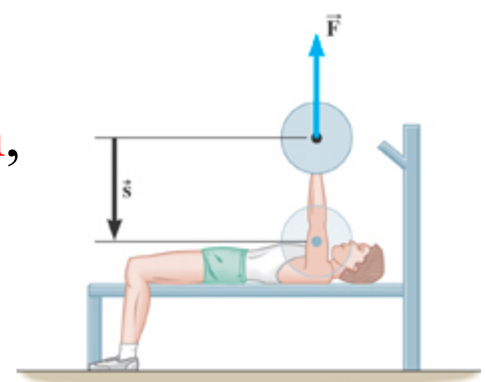
$$W = \left(F\cos 0\right)s = Fs$$



(b)

Lowering the bar bell, the displacement is down, and the force is (STILL) up.

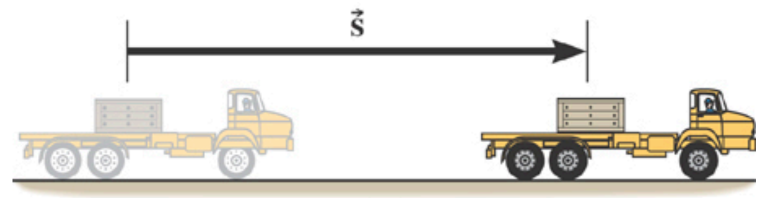
$$W = \left(F\cos 180\right)s = -Fs$$



(c)

6.1 Work Done by a Constant Force

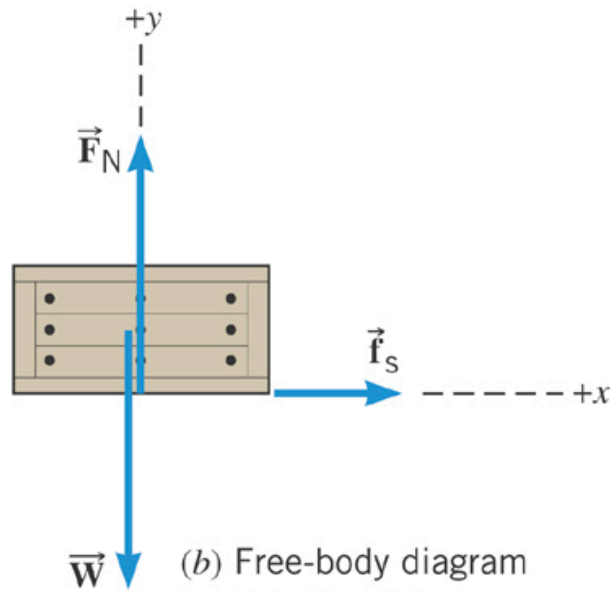
Example 3 Accelerating a Crate



(*a*)

The truck is accelerating at a rate of +1.50 m/s². The mass of the crate is 120-kg and it does not slip. The magnitude of the displacement is 65 m.

What is the total work done on the crate by all of the forces acting on it?



(*b*) Free-body diagram for the crate

$$\text{(normal force)}\; W = \left(F_N \cos 90^\circ\right)s = 0$$

$$\text{(gravity force)}\; W = \left(F_G \cos 90^\circ\right)s = 0$$

$$\text{(friction force)}\; W = \left(f_S \cos 0^\circ\right)s = f_S s$$

$$= (180\text{ N})(65\text{ m}) = 12\text{ kJ}$$

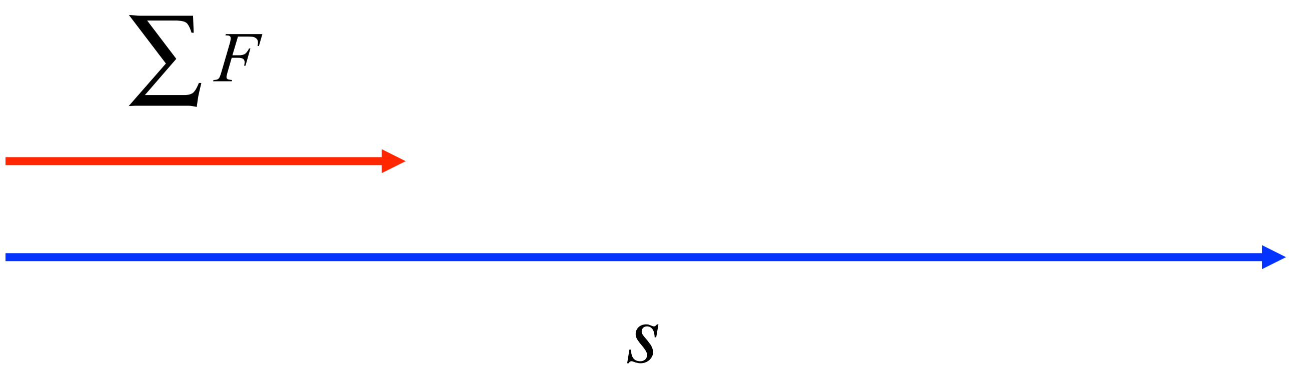
$$\boxed{\begin{aligned} f_S &= ma = (120\text{ kg})(1.50\text{ m/s}^2) \\ &= 180\,\text{N} \end{aligned}}$$

$$\boxed{1\text{ N}\cdot\text{m} \;=\; 1\text{ joule }(\text{J})}$$

6.2 The Work-Energy Theorem and Kinetic Energy

Consider a constant net external force acting on an object.

The object is displaced a distance *s*, in the same direction as the net force.



The work is simply $W = \left(\sum F\right)s = \left(ma\right)s$

6.2 The Work-Energy Theorem and Kinetic Energy

We have often used this 1D motion equation

but now using v_f for final velocity:

$$v_f^2 = v_o^2 + 2ax$$

Let $x = s$, and multiply equation by $\frac{1}{2}m$ (why?)

$$\tfrac{1}{2}mv_f^2 = \tfrac{1}{2}mv_o^2 + mas \qquad \text{but } F = ma$$

$$\tfrac{1}{2}mv_f^2 = \tfrac{1}{2}mv_o^2 + Fs \qquad \text{but work, } W = Fs$$

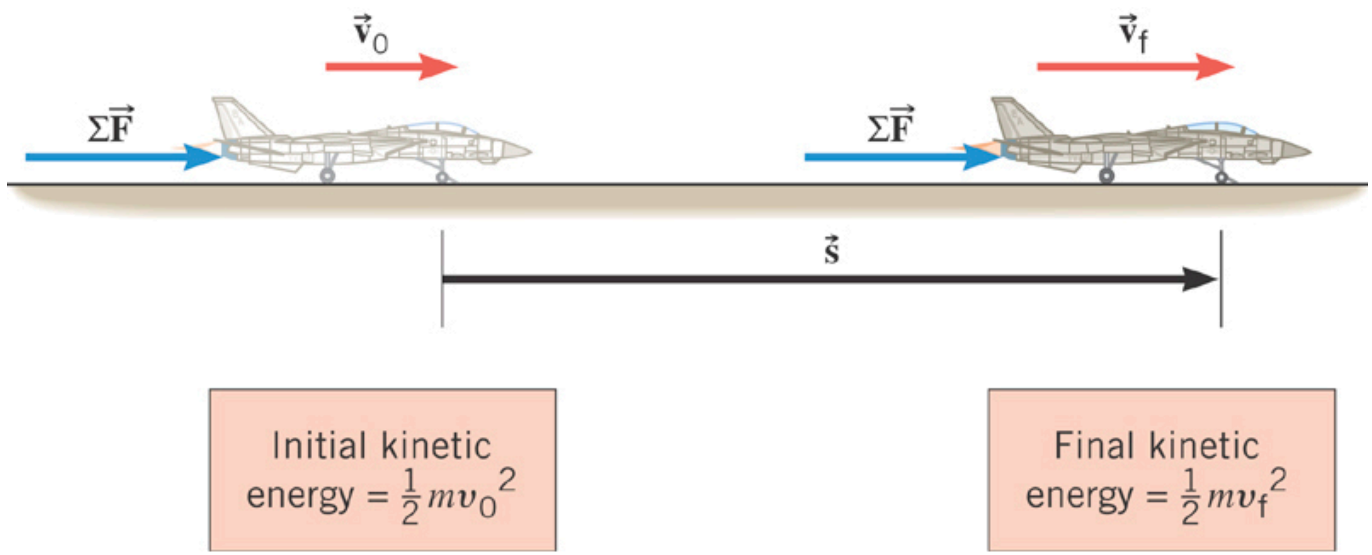
DEFINE KINETIC ENERGY of an object with mass m speed v: $\boxed{\mathrm{KE} = \tfrac{1}{2}mv^2}$

Now it says, Kinetic Energy of a mass changes due to Work:

$$\boxed{KE_f = KE_o + W}$$

or $\boxed{KE_f - KE_o = W}$ Work–Energy Theorem

6.2 The Work-Energy Theorem and Kinetic Energy



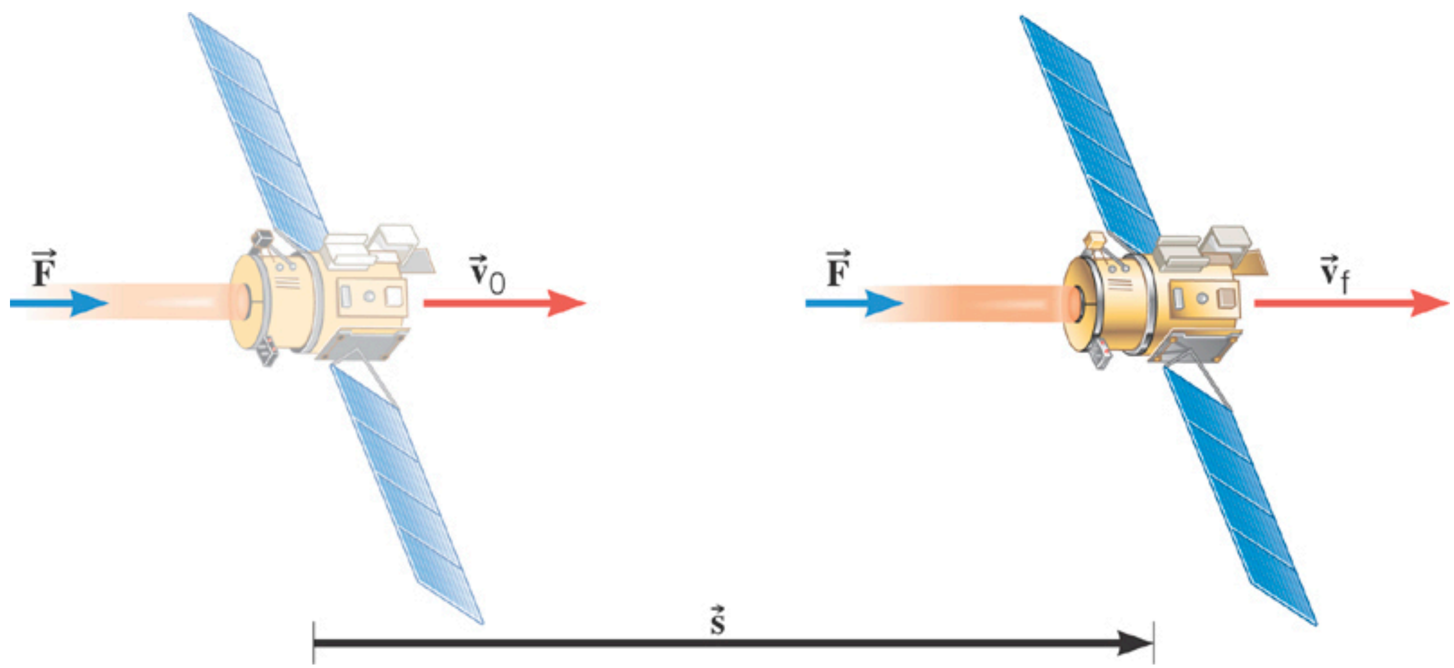
THE WORK-ENERGY THEOREM

When a net external force does work on and object, the kinetic energy of the object changes according to

$$W = \mathrm{KE_f} - \mathrm{KE_o} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2$$

Example 4 Deep Space 1

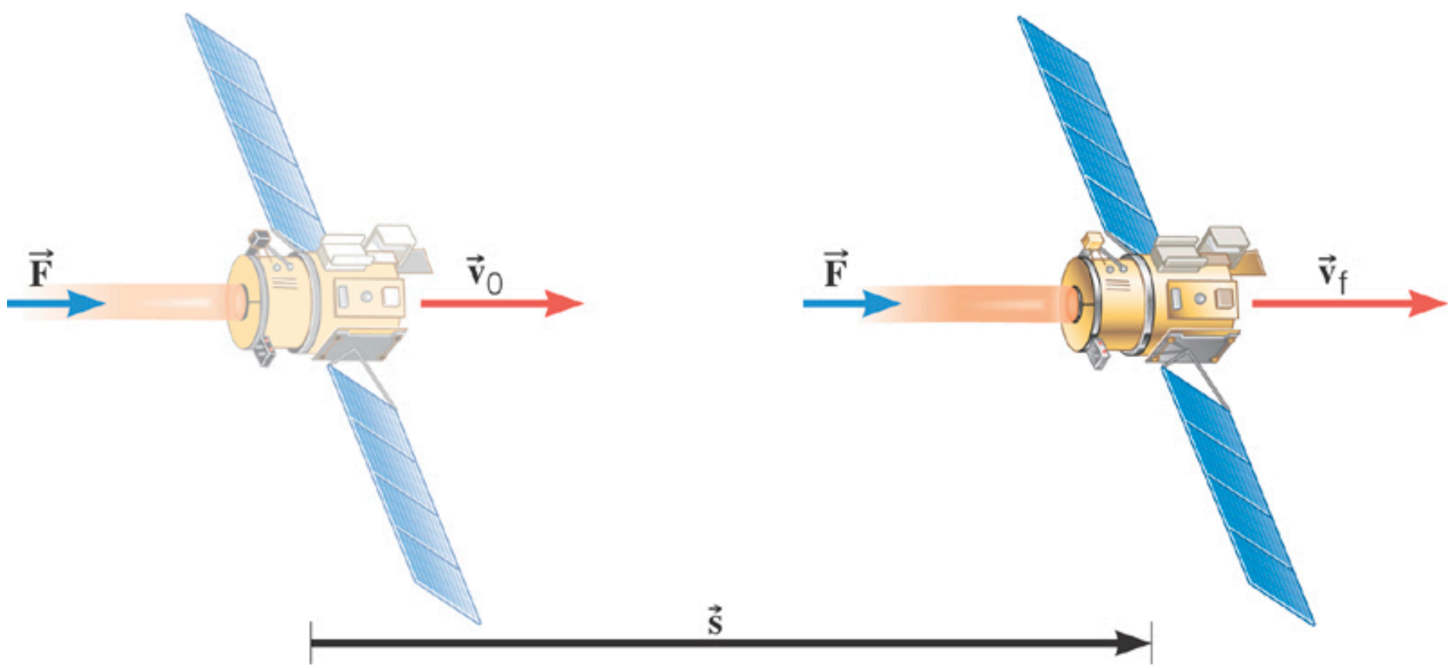
The mass of the space probe is 474-kg and its initial velocity is 275 m/s. If the 56.0-mN force acts on the probe through a displacement of 2.42×10^{9}m, what is its final speed?



6.2 The Work-Energy Theorem and Kinetic Energy

$$W = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2$$

$$W = \left[\left(\sum F\right)\cos\theta\right]s$$



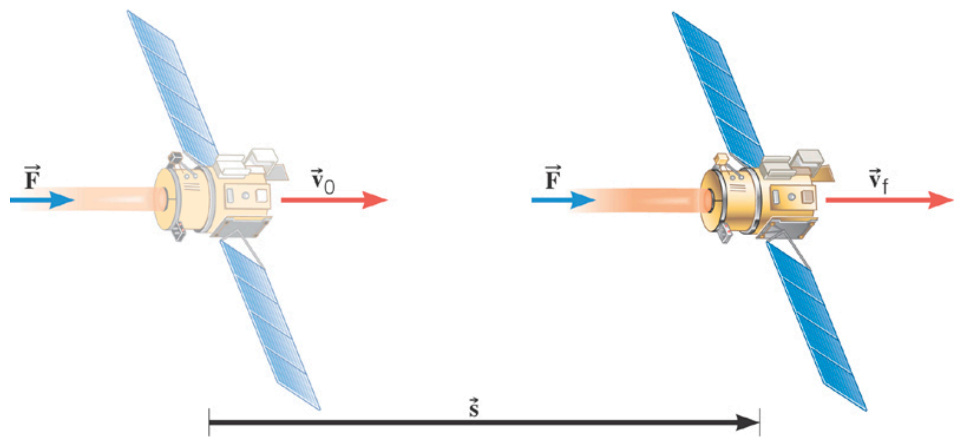
6.2 The Work-Energy Theorem and Kinetic Energy

$$\left[\left(\sum \mathrm{F}\right)\cos\theta\right]s = \tfrac{1}{2}mv_{\mathrm{f}}^2 - \tfrac{1}{2}mv_o^2$$

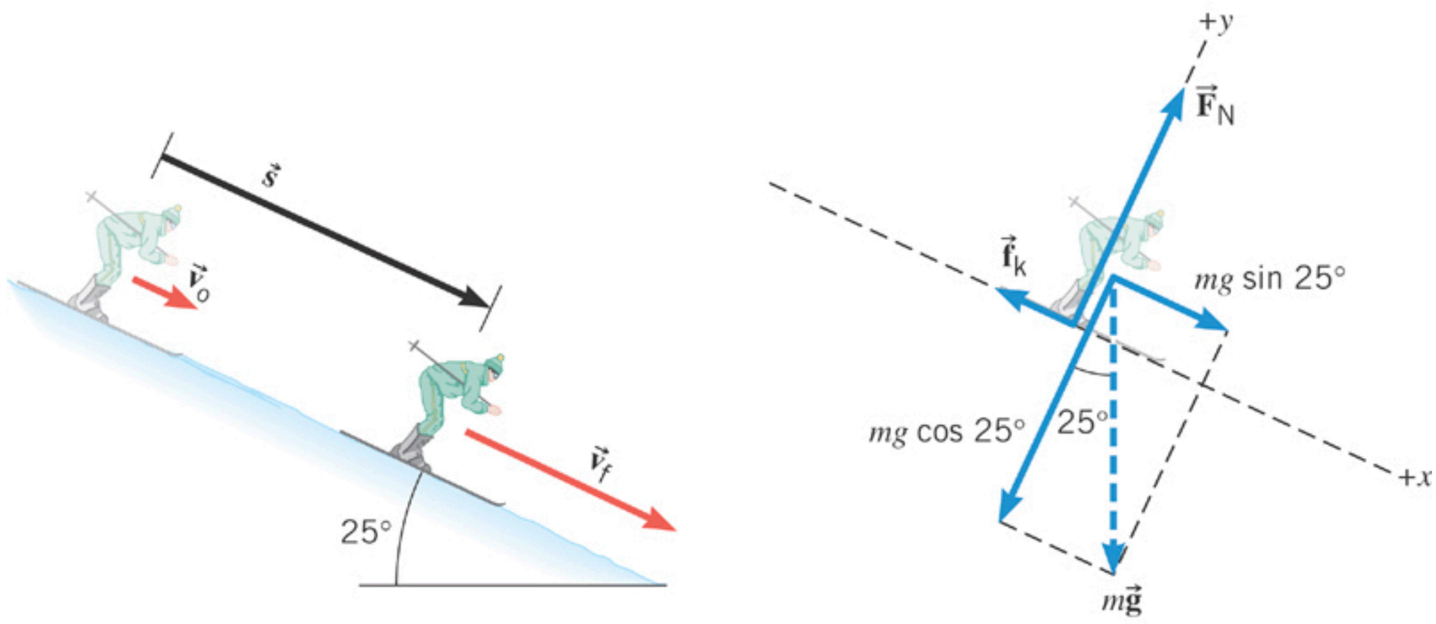
$$\left(5.60\times10^{-2}\mathrm{N}\right)\cos 0^\circ\left(2.42\times10^{9}\,\mathrm{m}\right) =$$

$$= \tfrac{1}{2}\left(474\ \mathrm{kg}\right)v_{\mathrm{f}}^2 - \tfrac{1}{2}\left(474\ \mathrm{kg}\right)\left(275\mathrm{m/s}\right)^2$$

$$v_f = 805\mathrm{m/s}$$



6.2 The Work-Energy Theorem and Kinetic Energy

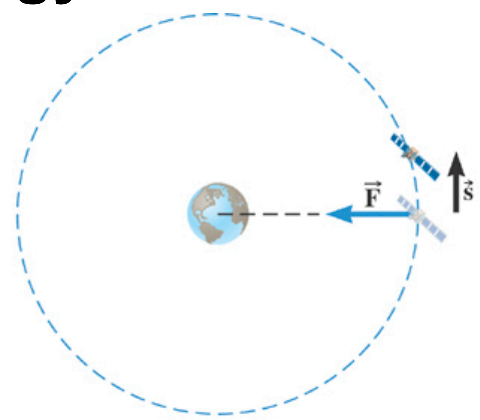


(b) Free-body diagram for the skier

In this case the net force is $\sum F = mg \sin 25^\circ - f_k$

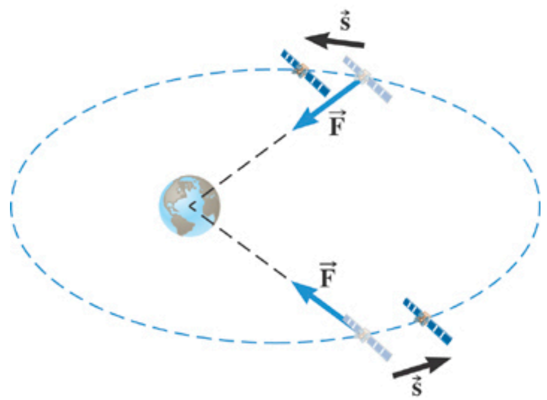
Conceptual Example 6 Work and Kinetic Energy

A satellite is moving about the earth in a circular orbit. Does kinetic energy change during the motion?



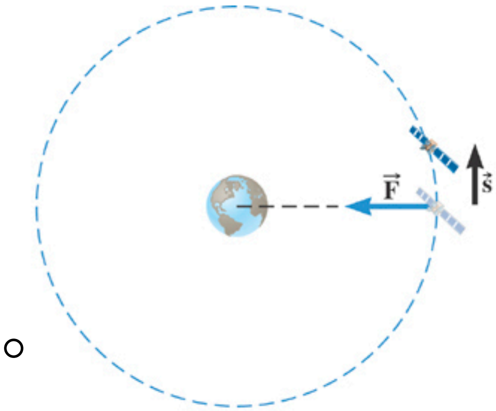
(a)

A satellite is moving about the earth in an elliptical orbit. Does kinetic energy change during the motion?



(b)

Conceptual Example 6 Work and Kinetic Energy



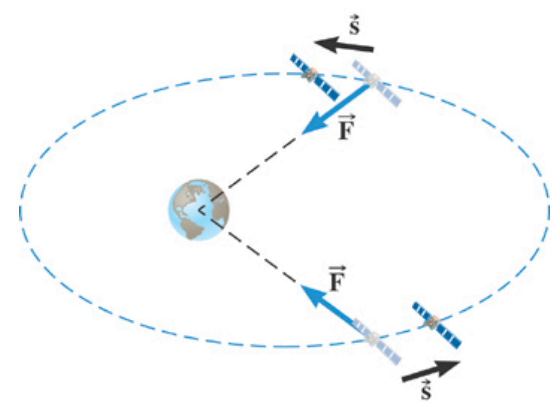
(a)

A satellite is moving about the earth in a circular orbit. Does kinetic energy change during the motion?

$$\text{(work) } W = (F\cos\theta)s, \text{ but between } \vec{\mathbf{F}} \text{ and } \vec{\mathbf{s}},\ \theta = 90^\circ$$

$$= (F \cdot \cos 90^\circ)s = \text{ZERO}$$

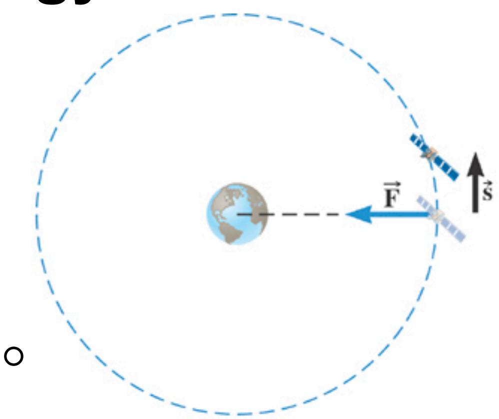
$$\boxed{\begin{aligned} KE_f - KE_o &= W \\ &= 0 \text{ (no change in kinetic energy)} \end{aligned}}$$



(b)

A satellite is moving about the earth in an elliptical orbit. Does kinetic energy change during the motion?

Conceptual Example 6 Work and Kinetic Energy



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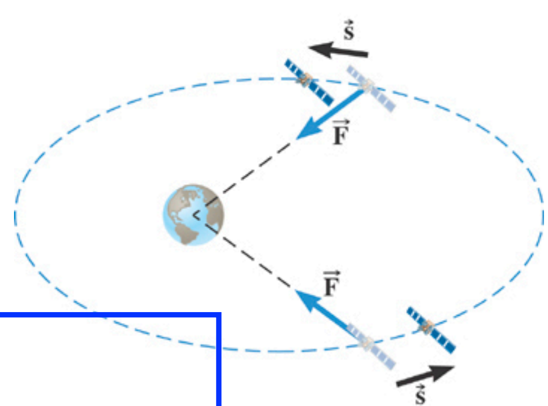
$$= (F\cdot\cos 90^\circ)s = \text{ZERO}$$

$$KE_f - KE_o = W$$

$$= 0 \text{ (no change in kinetic energy)}$$



(b)

A satellite is moving about the earth in an elliptical orbit. Does kinetic energy change during the motion?

$$\text{(work) } W = (F\cos\theta)s, \quad \theta \text{ not always equal to } 90^\circ$$

kinetic energy can increase at places and decrease at others.