Chapter 6

Work and Energy

continued

6.2 The Work-Energy Theorem and Kinetic Energy

Chapters 1–5

Motion equations were been developed, that relate the concepts of velocity, speed, displacement, time, and with a particular emphasis on the acceleration of an object.

Newton discovered three laws that define the concept of a force. The first two laws provide the relationship of forces to the acceleration of an object. The third law describes the nature of forces at the point of contact between two objects.

The effect of forces, including human generated forces, on the motion of an object have been solved in common situations . The forces of gravity, friction, elastic (tension and compression), have been desribed.

6.2 The Work-Energy Theorem and Kinetic Energy

Relate these to Energy

Work: the effect of a force acting on an object making a displacement.

$$\boxed{W = (F\cos\theta)s,}$$

where W is the work done, F, s are the magnitudes of the force and displacement, and θ is the angle between $\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$.

The origin of the force does not affect the calculation of the work done.

Work can be done by: gravity, elastic, friction, explosion, or human forces.

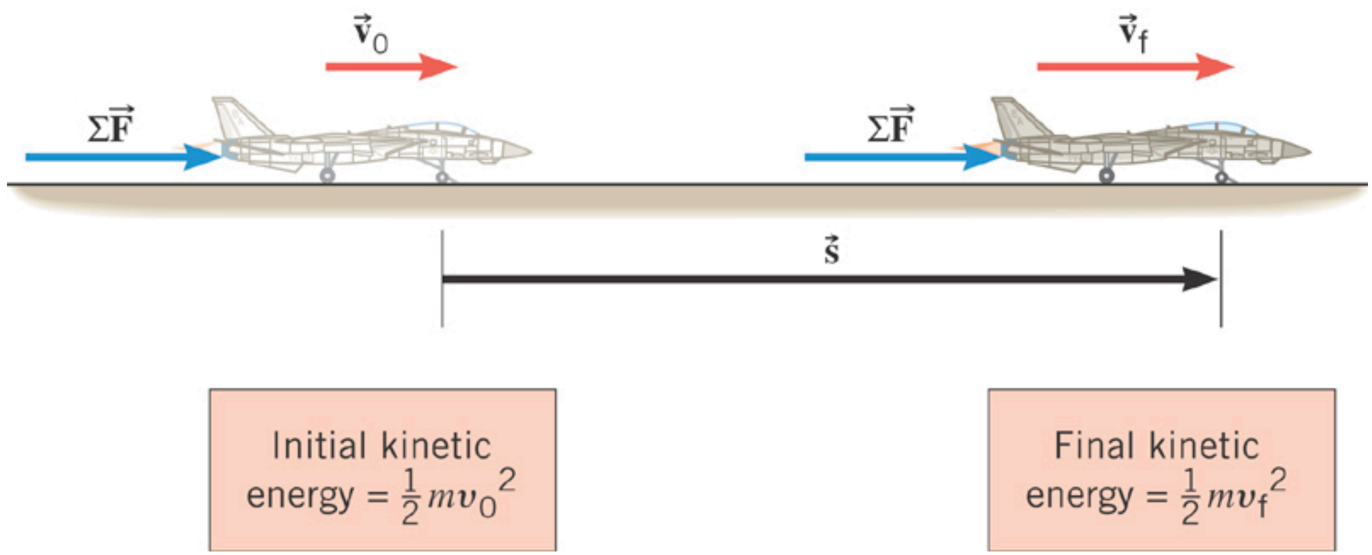
Kinetic energy: property of a mass (m) and the square of its speed (v).

$$\boxed{\mathrm{KE} = \tfrac{1}{2}mv^2}$$

Work-Energy Theorem: Work changes the Kinetic Energy of an object.

$$\boxed{\mathrm{KE_f} = \mathrm{KE_0} + W} \quad \text{or} \quad \boxed{\mathrm{KE_f} - \mathrm{KE_0} = W}$$

6.2 The Work-Energy Theorem and Kinetic Energy



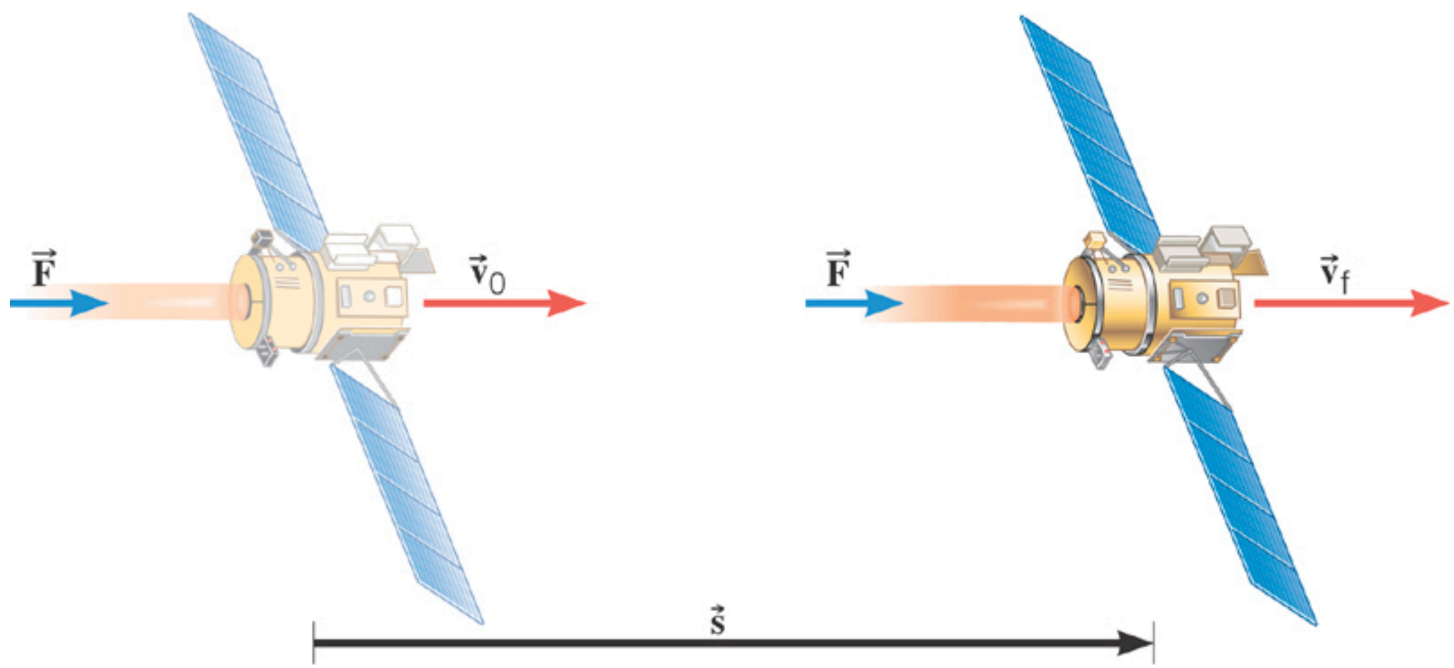
THE WORK-ENERGY THEOREM

When a net external force does work on an object, the kinetic energy of the object changes according to

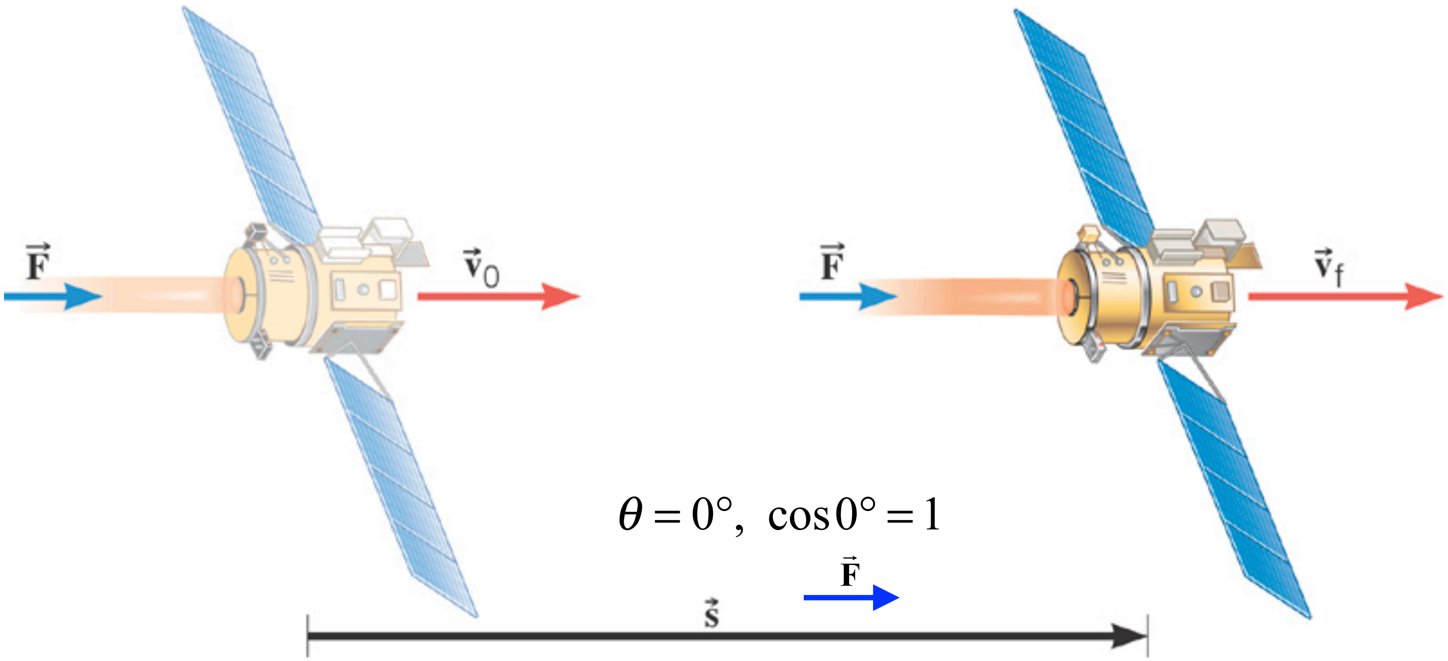
$$W = \mathrm{KE}_f - \mathrm{KE}_0 = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_0^2$$

Example 4 Deep Space 1

The mass of the space probe is 474-kg and its initial velocity is 275 m/s. If the 56.0-mN force acts on the probe through a displacement of 2.42×10^{9}m, what is its final speed?



6.2 The Work-Energy Theorem and Kinetic Energy



$$W = \left[F\cos\theta\right]s = \left(5.60\times10^{-2}\text{N}\right)\left(2.42\times10^{9}\text{m}\right) = 1.36\times10^{8}\,\text{J}$$

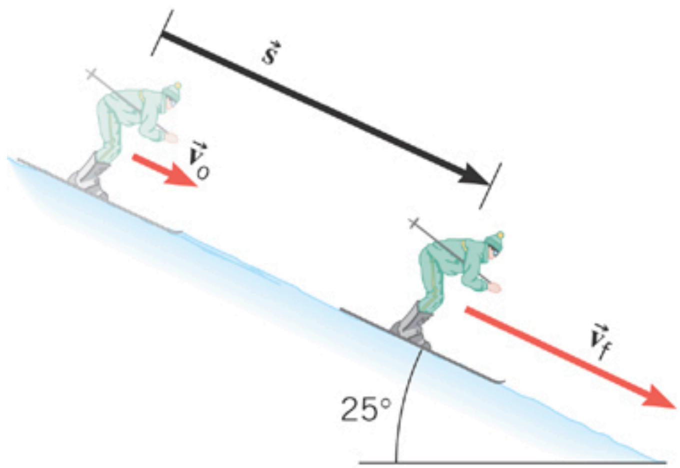
$$W = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2$$

$$v_f^2 = \frac{2W}{m} + v_o^2 = \frac{2.72\times10^{8}\,\text{J}}{474\ \text{kg}} + (275\text{m/s})^2$$

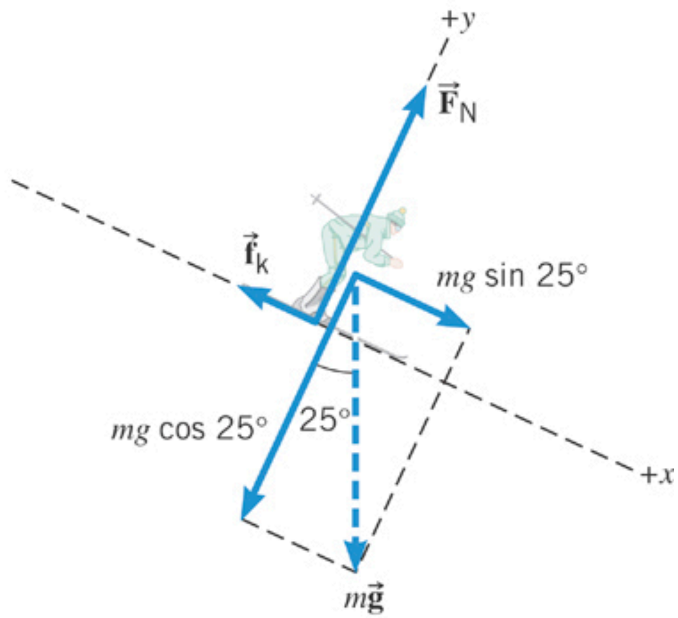
$$v_f = 806\ \text{m/s}$$

6.2 The Work-Energy Theorem and Kinetic Energy

Familiar decomposition of the gravitation force, mg downward



(a)



(b) Free-body diagram for the skier

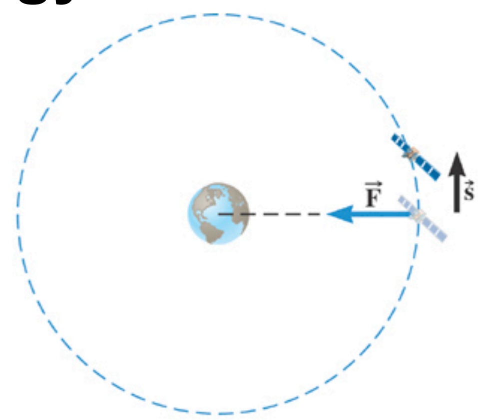
f_k is given as 71 N

Choosing downward as +

In this case the net force is $\sum F = mg\sin 25^{\circ} - f_k$

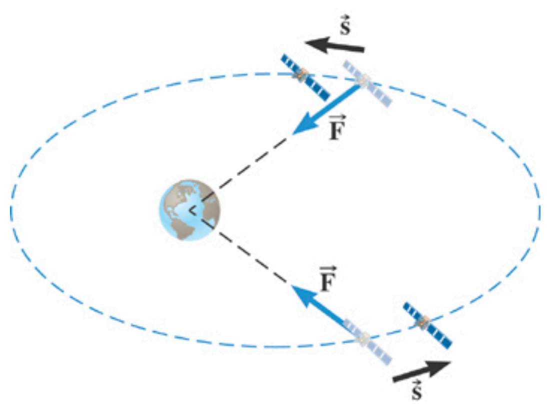
Conceptual Example 6 Work and Kinetic Energy

A satellite is moving about the earth in a circular orbit. Does kinetic energy change during the motion?



(a)

A satellite is moving about the earth in an elliptical orbit. Does kinetic energy change during the motion?



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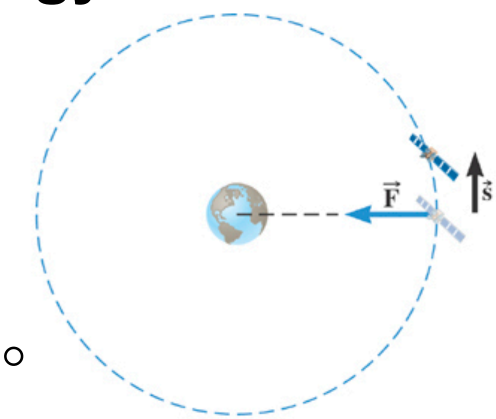
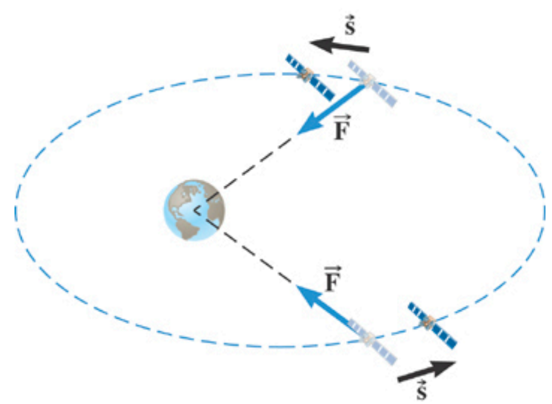
Conceptual Example 6 Work and Kinetic Energy



(*a*)

A satellite is moving about the earth in a circular orbit. Does kinetic energy change during the motion?

$$\text{(work) } W = (F\cos\theta)s, \text{ but between } \vec{\mathbf{F}} \text{ and } \vec{\mathbf{s}},\ \theta = 90^\circ$$
$$= (F \cdot \cos 90^\circ)s = \text{ZERO}$$



(*b*)

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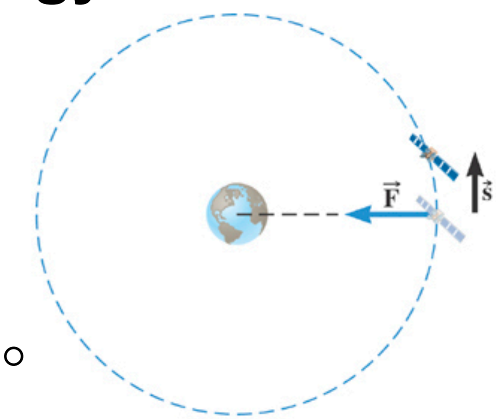
Conceptual Example 6 Work and Kinetic Energy



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$$\boxed{\begin{aligned} KE_f - KE_o &= W \\ &= 0 \text{ (no change in kinetic energy)} \end{aligned}}$$

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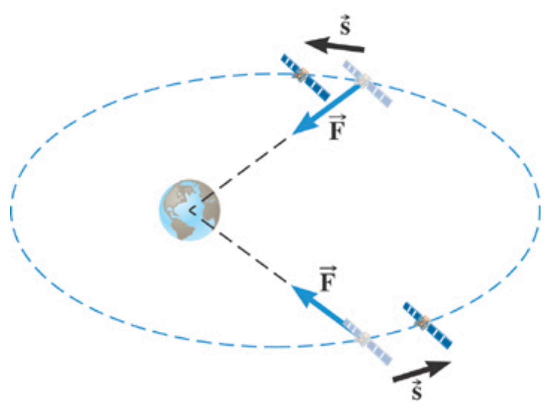
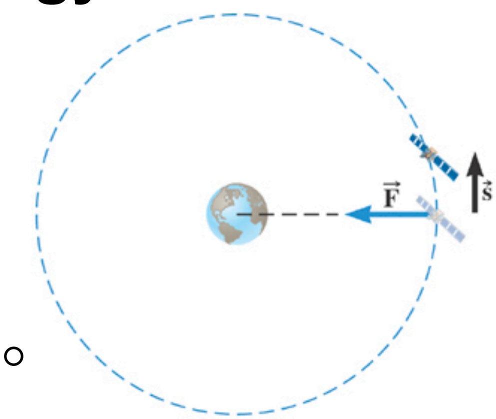


(b)

Conceptual Example 6 Work and Kinetic Energy



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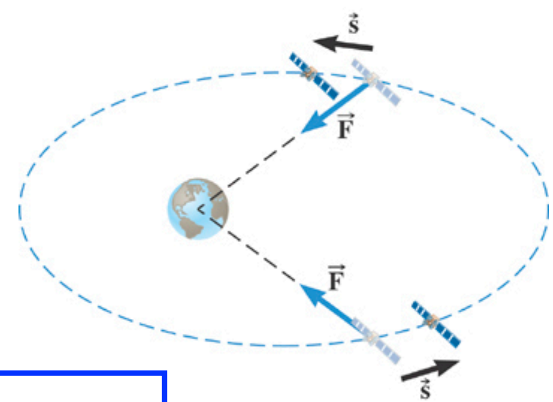


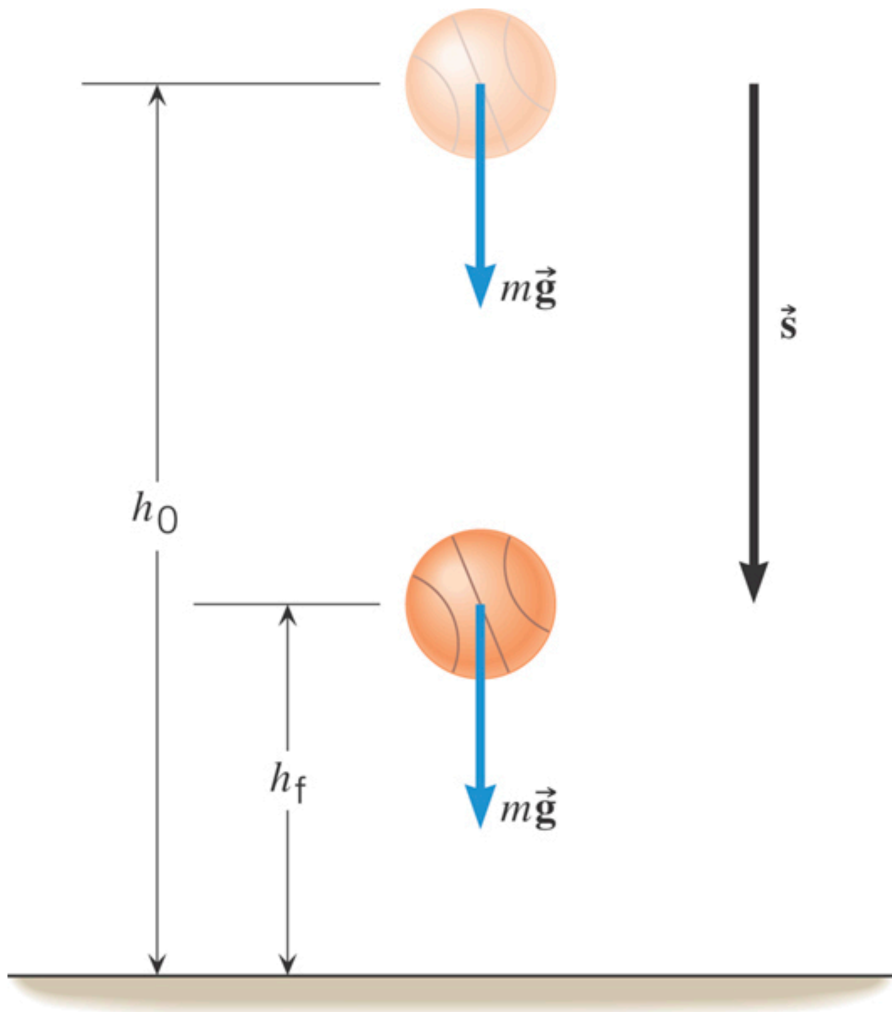
(b)

A satellite is moving about the earth in an elliptical orbit. Does kinetic energy change during the motion?

$$\boxed{\begin{aligned} &\text{(work) } W = (F\cos\theta)s, \quad \theta \text{ not always equal to } 90^\circ \\ &\text{kinetic energy increases at places and decreases at others.} \end{aligned}}$$

This θ is the angle between $\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$.

$$W = \left(F\cos\theta\right)s$$
$$= mgs$$



$s = \text{distance of fall} = (h_0 - h_f)$

$$W_G = mg\left(h_0 - h_f\right)$$

Why use $(h_0 - h_f)$ instead of s? $\Rightarrow$ because we already have KE_f and KE_0

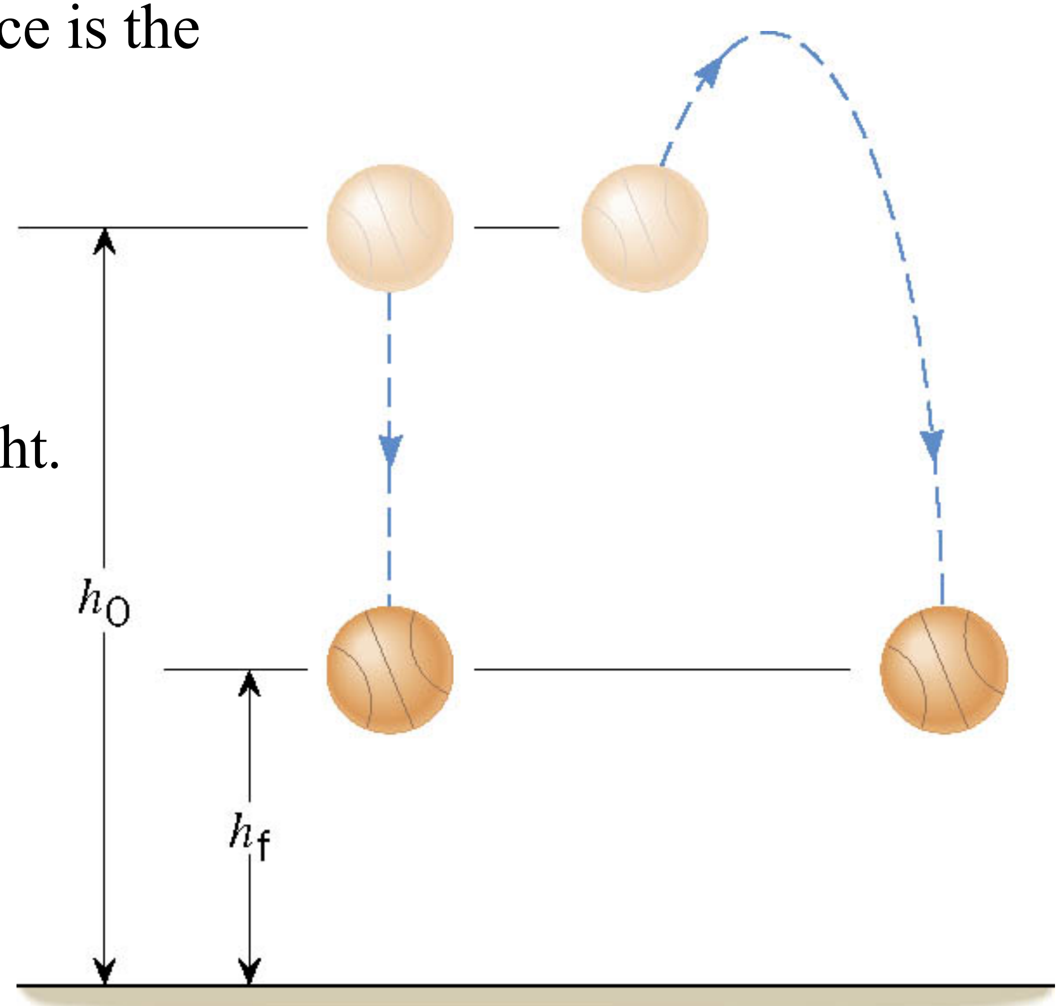
6.3 Gravitational Potential Energy

When returning to the initial height, h_0, the work done by gravity is zero. There is no displacement, $s = 0$.

Work by the gravitational force is the same over the two paths

$$W_G = mg\left(h_0 - h_f\right)$$

Same starting and ending height.



Gravity is unusual!

It is a "conservative force".

DEFINITION OF A CONSERVATIVE FORCE

Version 1 A force is conservative when the work it does on a moving object is independent of the path between the object's initial and final positions.

Version 2 A force is conservative when it does no net work on an object moving around a closed path, starting and finishing at the same point.

Also:

Version 3 A force is conservative when the energy absorbed from a mass by the force can be returned to the mass without loss by that force.

6.4 Conservative Versus Nonconservative Forces

Table 6.2 Some Conservative and Nonconservative Forces

Conservative Forces
Gravitational force (Ch. 4)
Elastic spring force (Ch. 10)
Electric force (Ch. 18, 19)
Nonconservative Forces
Static and kinetic frictional forces
Air resistance
Muscular forces
Explosions
Jet or rocket forces

6.3 Gravitational Potential Energy

Because gravity is a conservative force, when a mass moves upward against the gravitational force, the kinetic energy of the mass decreases, but when the mass falls to its initial height that kinetic energy returns completely to the mass.

When the kinetic energy decreases, where does it go?

DEFINITION OF GRAVITATIONAL POTENTIAL ENERGY

The gravitational potential energy PE is the energy that an object of mass m has by virtue of its position relative to the surface of the earth. That position is measured by the height h of the object relative to an arbitrary zero level:

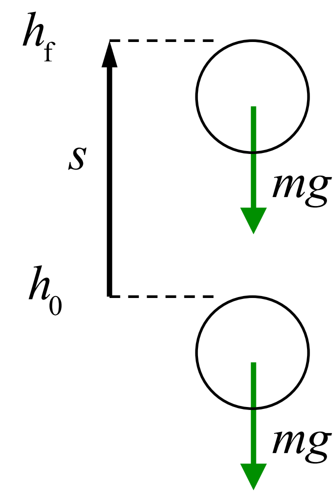
$$\boxed{\mathrm{PE} = mgh} \qquad 1\,\mathrm{N}\cdot\mathrm{m} = 1\ \mathrm{joule}\,(\mathrm{J})$$

(h can be + or –)

6.3 Gravitational Potential Energy

Moving upward

Gravitational work is negative.



$$W_G = \left(F\cos 180^\circ\right)s$$
$$= -mg(h_f - h_0)$$

Gravitational Potential Energy increases.

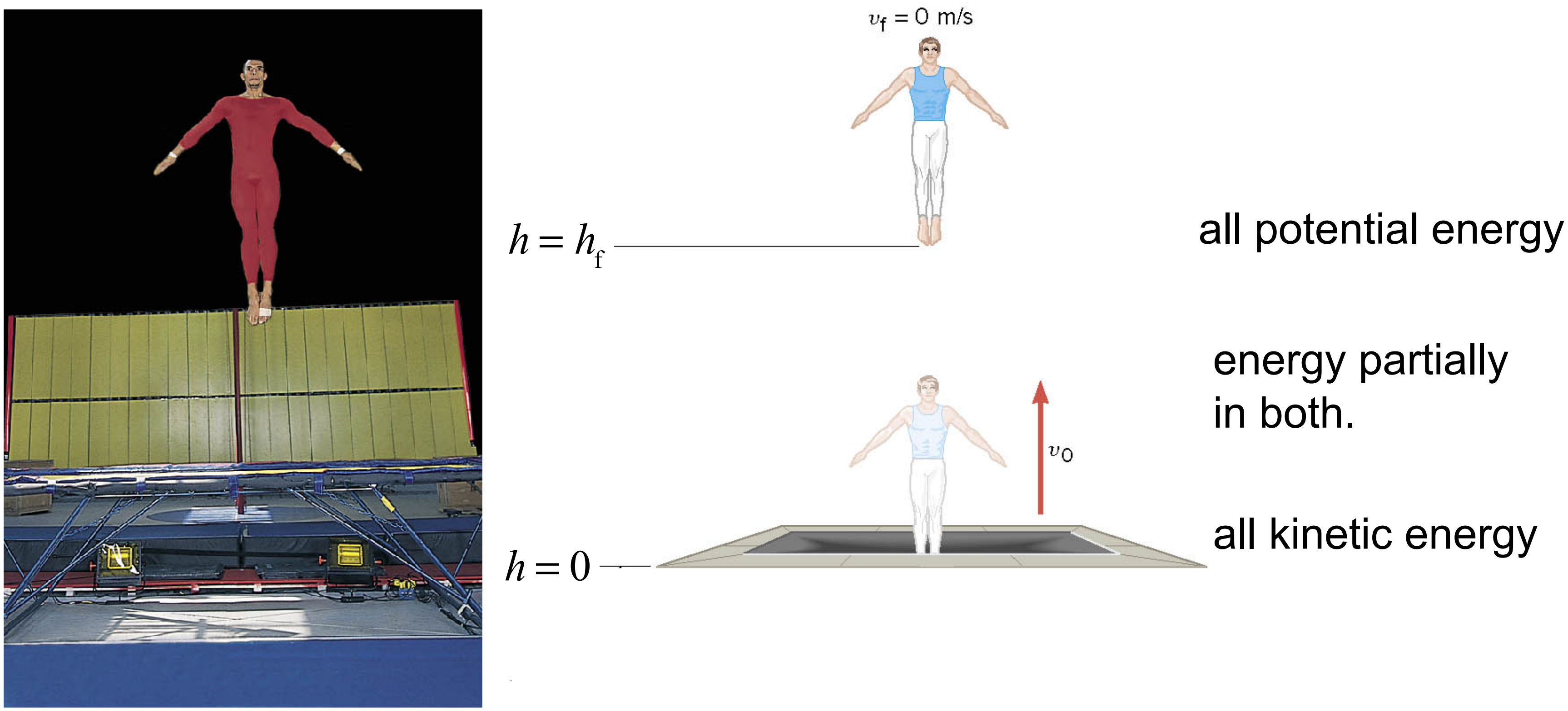
$$\mathrm{PE}_f - \mathrm{PE}_0 = mg\left(h_f - h_0\right)$$
$$= -W_G$$

Work-Energy Theorem becomes:

$$\mathrm{KE}_f - \mathrm{KE}_0 = W_G$$
$$= -(\mathrm{PE}_f - \mathrm{PE}_0)$$
$$\boxed{\mathrm{KE}_f + \mathrm{PE}_f = \mathrm{KE}_0 + \mathrm{PE}_0 \quad \text{Conservation of Energy}}$$

A Gymnast on a Trampoline (only force during flight is gravity)

The gymnast leaves the trampoline at an initial height of 0 m and reaches a maximum height of 3.60 m before falling back down. What was the initial speed of the gymnast?



(a) (b)

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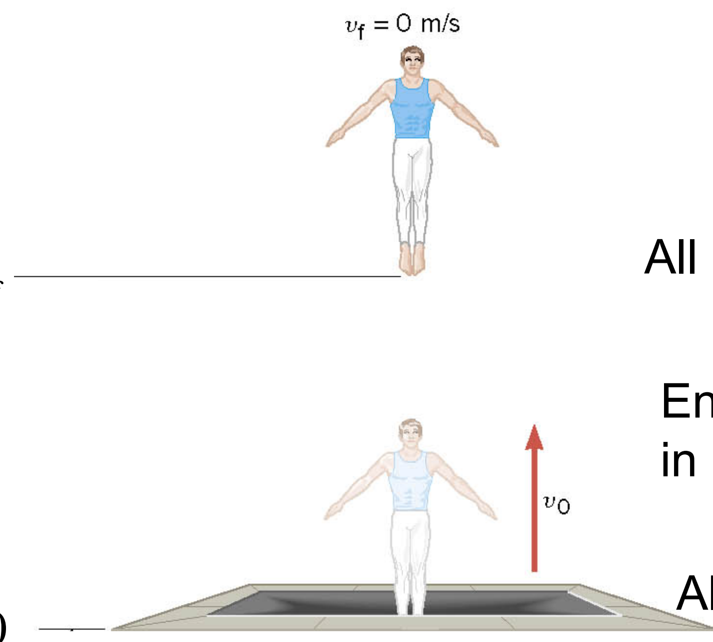


kinetic energy	potential energy		
$KE_f = 0$	$PE_f = mgh_f$	$h = h_f$	All potential energy
			Energy partially in both.
$KE_0 = \frac{1}{2}mv_0^2$	$PE_0 = 0$	$h = 0$	All kinetic energy

Initial Kinetic Energy winds up in Gravitational Potential Energy

$$KE_0 = PE_f \qquad \frac{1}{2}\cancel{m}v_0^2 = \cancel{m}gh_f \Rightarrow v_0 = \sqrt{2gh}$$

$$v_0 = \sqrt{2(9.8\,\text{m/s}^2)(3.6\,\text{m})} = 8.4\,\text{m/s}^2$$

6.3 Gravitational Potential Energy

What if the initial height is above the ground?

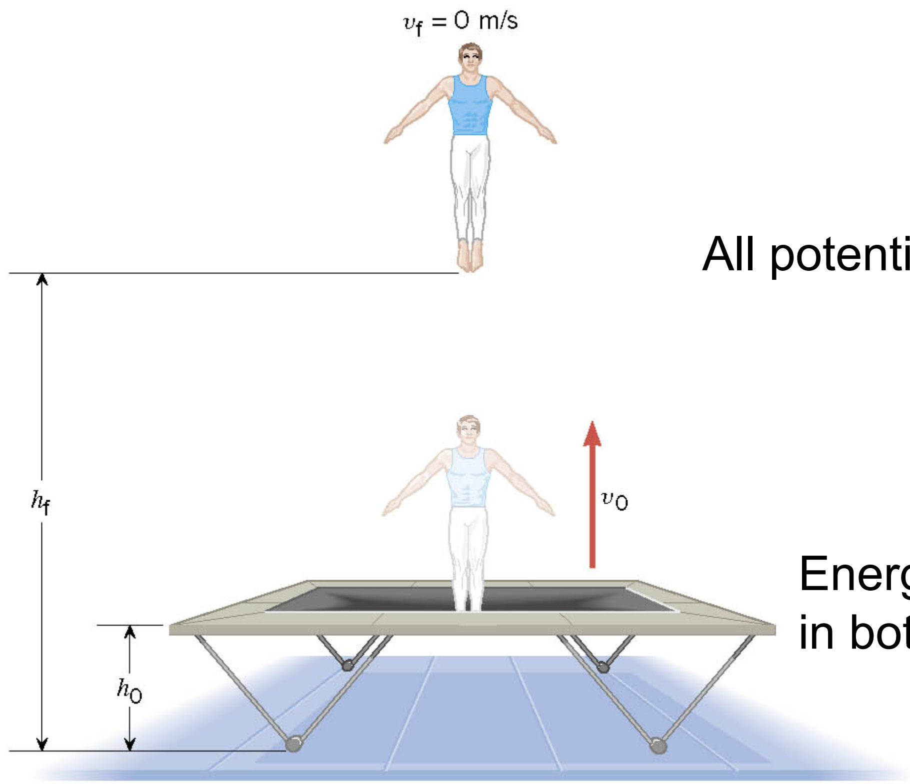
<u>kinetic energy</u>   <u>potential energy</u>

$$KE_f = 0 \qquad PE_f = mgh_f$$

All potential energy

$$KE_0 = \tfrac{1}{2}mv_0^2 \qquad PE_0 = mgh_0$$

Energy partially in both.



(b)

$$KE_0 + PE_0 = KE_f + PE_f$$

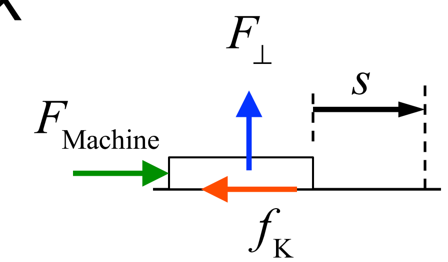
$$\tfrac{1}{2}\cancel{m}v_0^2 + mgh_0 = \cancel{m}gh_f, \qquad h_f - h_0 = (4.8 - 1.2)\,\mathrm{m} = 3.6\,\mathrm{m}$$

$$v_0 = \sqrt{2g\left(h_f - h_0\right)} = \sqrt{2(9.8\,\mathrm{m/s^2})(3.6\,\mathrm{m})} = 8.4\,\mathrm{m/s^2}$$

6.4 Conservative Versus Nonconservative Forces

An example of a nonconservative force is the kinetic frictional force.

Sanding machine in the Quiz, but now the work done by the frictional force.



$$W = \left(F\cos\theta\right)s = f_{\text{k}}\cos 180^{\circ} s = -f_{\text{k}}s$$

$\vec{\mathbf{f}}_{\text{k}}$ and $\vec{\mathbf{s}}$ are in opposite directions

The work done by the *kinetic* frictional force is always negative. Thus, it is impossible for the work it does on an object that moves around a closed path to be zero.

The concept of potential energy is not defined for a nonconservative force.

6.4 Conservative Versus Nonconservative Forces

In normal situations both conservative and nonconservative forces act simultaneously on an object, so the work done by the net external force can be written as

$$W = W_c + W_{nc}$$

$W_C = W_G$

or other conservative force

Work-Energy Theorem becomes:

$$\mathrm{KE_f} - \mathrm{KE_0} = W_\mathrm{G} + W_{NC}$$
$$= -(\mathrm{PE_f} - \mathrm{PE_0}) + W_{NC}$$

$$\boxed{\mathrm{KE_f} + \mathrm{PE_f} = \mathrm{KE_0} + \mathrm{PE_0} + W_\mathrm{NC}}$$

non-conservative forces add or remove energy

Also

$$\boxed{\begin{aligned}\left(\mathrm{KE_f} - \mathrm{KE_0}\right) + \left(\mathrm{PE_f} - \mathrm{PE_0}\right) &= W_\mathrm{NC} \\ \Delta\mathrm{KE} + \Delta\mathrm{PE} &= W_\mathrm{NC}\end{aligned}}$$

energy changes will disagree if non-conservative forces are doing work

6.5 The Conservation of Mechanical Energy

$$\boxed{\mathrm{KE_f} + \mathrm{PE_f} = \mathrm{KE_0} + \mathrm{PE_0} + W_{\mathrm{NC}}}$$

non-conservative forces add or remove energy

$$\left(\mathrm{KE_f} + \mathrm{PE_f}\right) - \left(\mathrm{KE_0} + \mathrm{PE_0}\right) = W_{\mathrm{NC}}$$

$$\text{Let total energy, } \mathrm{E} = \mathrm{KE} + \mathrm{PE}$$

$$\mathrm{E_f} - \mathrm{E_0} = W_{\mathrm{NC}}$$

non-conservative forces add or remove energy

If the net work on an object by nonconservative forces is zero, then its total energy does not change:

$$\boxed{\mathrm{E_f} - \mathrm{E_0} = 0 \quad \text{or} \quad \mathrm{E_f} = \mathrm{E_0}}$$

with only conservative forces total energy does not change.