Chapter 7

Impulse and Momentum

Chaper 6 Review: Work and Energy – Forces and Displacements

Effect of forces acting over a displacement

Work

$$W = (F\cos\theta)s$$

Kinetic Energy

$$\mathrm{KE} = \tfrac{1}{2}mv^2$$

Work changes the
Kinetic Energy of a mass

Work - Energy Theorem (true always)

$$W = \mathrm{KE_f} - \mathrm{KE_0}$$

Conservative Force

Gravity

Potential Energy

$$\mathrm{PE} = mgh$$

Non-Conservative Forces doing work

W_{NC} Humans, Friction, Explosions

Work - Energy Theorem (still true always)

$$W_{\mathrm{NC}} = \left(\mathrm{KE_f} - \mathrm{KE_0}\right) + \left(\mathrm{PE_f} - \mathrm{PE_0}\right)$$

All of these quantities are scalars.

7.1 The Impulse-Momentum Theorem

Chapter 7 is about the COLLISION of two masses.

Both masses are needed to understand their interaction.

Newton's 3rd Law plays a very important part.

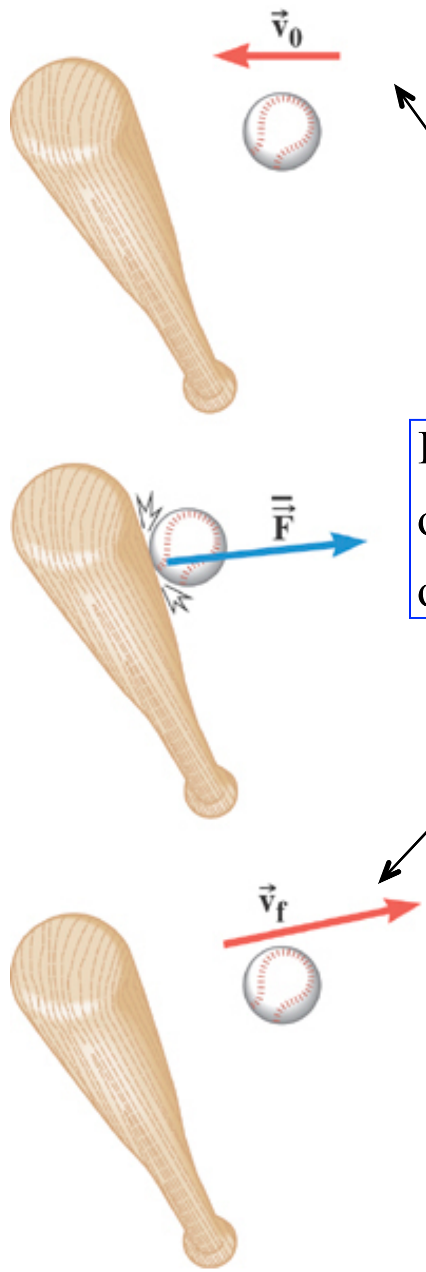
Collisions involve two new concepts: Impulse and Momentum.

Impulse concept leads to the Momentum definition.

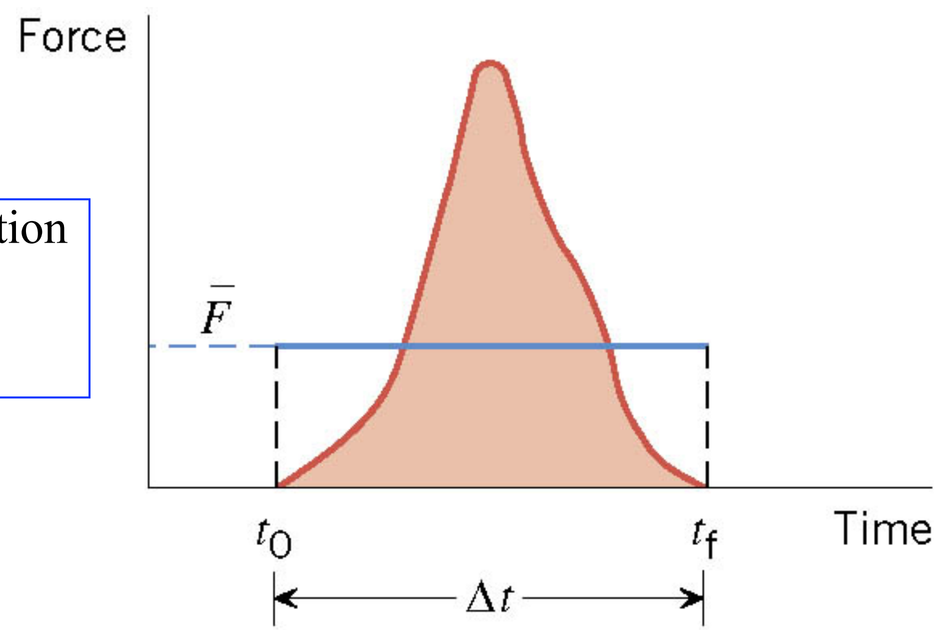
Also applied to two (or more) masses blown apart by an explosion.

7.1 The Impulse-Momentum Theorem

What is the effect of force acting over a short time?

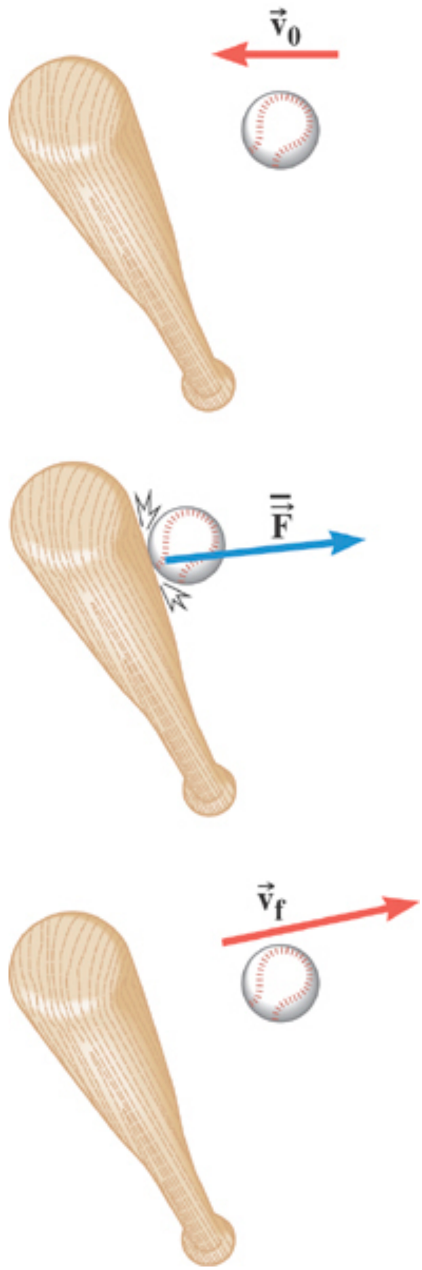


Force changes the direction of the velocity vector of the baseball.



The bat/ball force is not constant and the mass makes a very short displacement while it acts.

7.1 The Impulse-Momentum Theorem



$\bar{\vec{F}}$ acts on the Baseball

$m, \vec{v}$, and $\bar{\vec{a}}$ are of the Baseball

$$\sum \bar{\vec{F}} = m\bar{\vec{a}}$$

$$\bar{\vec{a}} = \frac{\vec{v}_f - \vec{v}_o}{\Delta t}$$

$$\sum \bar{\vec{F}} = \frac{m\vec{v}_f - m\vec{v}_o}{\Delta t}$$

Momentum

$$\vec{p} = m\vec{v}$$

$$\underbrace{\left(\sum \bar{\vec{F}}\right)}_{\text{on BALL}} \Delta t = \underbrace{m\vec{v}_f - m\vec{v}_o}_{\text{of BALL}}$$

Impulse

$$\left(\sum \bar{\vec{F}}\right)\Delta t$$

Newton's 3rd Law gives force on Bat

$$\left(\sum \overline{\vec{F}}\right)_{\text{on BAT}} = -\left(\sum \overline{\vec{F}}\right)_{\text{on BALL}}$$

DEFINITION OF IMPUSLE

The impulse of a force is the product of the average force and the time interval during which the force acts:

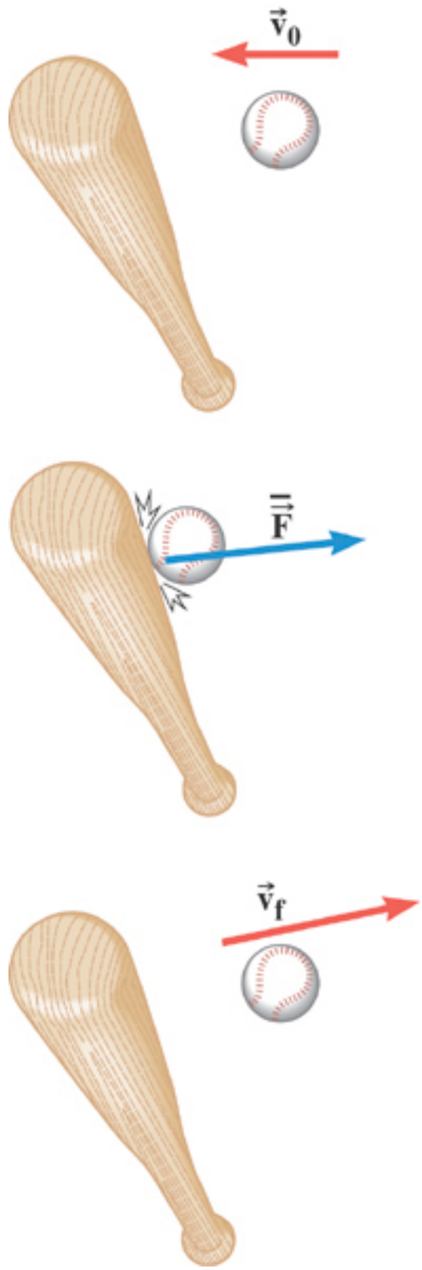
$$\vec{\mathbf{J}} = \overline{\vec{\mathbf{F}}}\Delta t$$

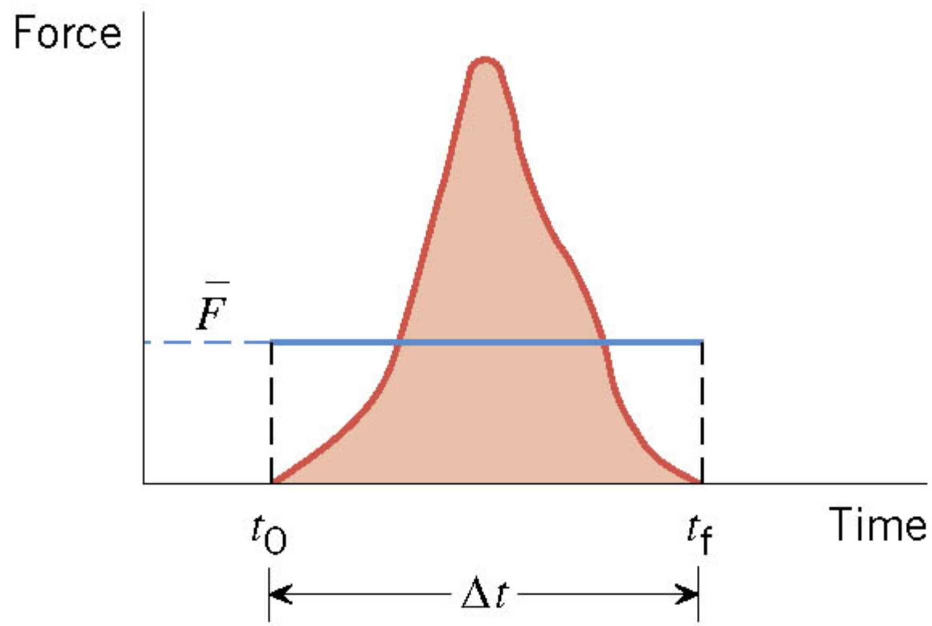
$\overline{\vec{\mathbf{F}}} = \text{average force vector}$

Impulse is a vector quantity and has the same direction as the average force.

$$\text{newton} \cdot \text{seconds } (\text{N} \cdot \text{s})$$

7.1 The Impulse-Momentum Theorem





(b)

$$\vec{J} = \overline{\vec{F}}\Delta t$$

DEFINITION OF LINEAR MOMENTUM

The linear momentum of an object is the product of the object's mass times its velocity:

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

Linear momentum is a vector quantity and has the same direction as the velocity.

$$\text{kilogram} \cdot \text{meter/second } (\text{kg} \cdot \text{m/s})$$

IMPULSE-MOMENTUM THEOREM

When a net force acts on an object, the impulse of this force is equal to the change in the momentum of the object

$$\underbrace{\left(\sum \vec{\mathbf{F}}\right)\Delta t}_{\text{impulse}} = \underbrace{m\vec{\mathbf{v}}_{\mathbf{f}}}_{\text{final momentum}} \overbrace{-}^{\text{minus}} \underbrace{m\vec{\mathbf{v}}_{\mathbf{o}}}_{\text{initial momentum}}$$

Time averaged force acting on the mass.	Changes the momentum of the mass.

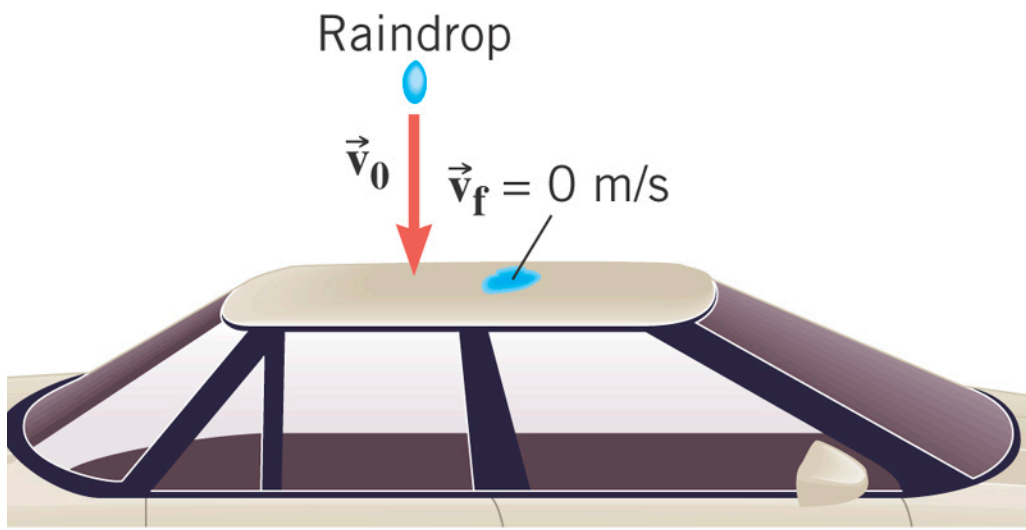
Example 2 A Rain Storm

Rain comes down with a velocity of -15 m/s and hits the roof of a car. The mass of rain per second that strikes the roof of the car is 0.060 kg/s. Assuming that rain comes to rest upon striking the car, find the average force exerted by the rain on the roof.

$$\left(\sum \vec{\mathbf{F}}\right)\Delta t = m\vec{\mathbf{v}}_{\mathbf{f}} - m\vec{\mathbf{v}}_{\mathbf{o}}$$

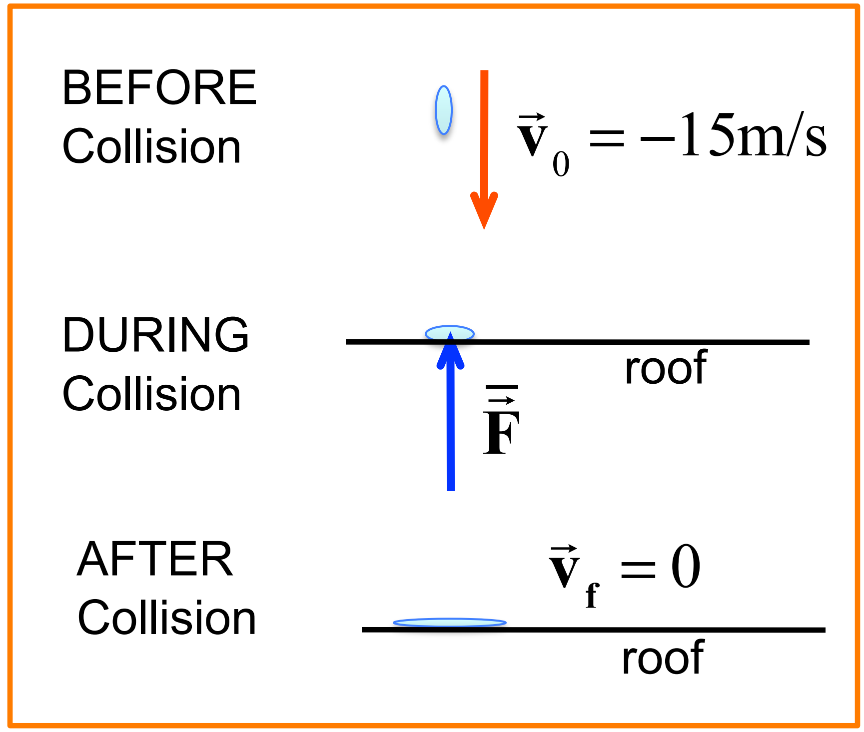
Using this, you will determine the average force on the raindrops.

But, using Newton's 3rd law you can get the average force on the roof .



7.1 *The Impulse-Momentum Theorem*

Neglecting the raindrop's weight, the average net force on the raindrops caused by the collisions with the roof is obtained.



Impulse of roof on raindrops	Changes momentum of the raindrops

$$\bar{\vec{F}}\Delta t = m\vec{v}_f - m\vec{v}_o$$

$$\boxed{\vec{v}_f = 0}$$

$$\bar{\vec{F}} = -\left(\frac{m}{\Delta t}\right)\vec{v}_o$$

$$\boxed{\text{mass of rain per second } \left(\frac{m}{\Delta t}\right) = 0.060 \text{ kg/s}}$$

$$\bar{\vec{F}} = -\left(0.060\,\text{kg/s}\right)\left(-15\,\text{m/s}\right)$$

$$= +0.90 \text{ N}$$

By Newton's 3rd Law average force of raindrops on the roof is

$$\bar{\vec{F}} = -0.90\,\text{N}$$

7.2 *The Principle of Conservation of Linear Momentum*

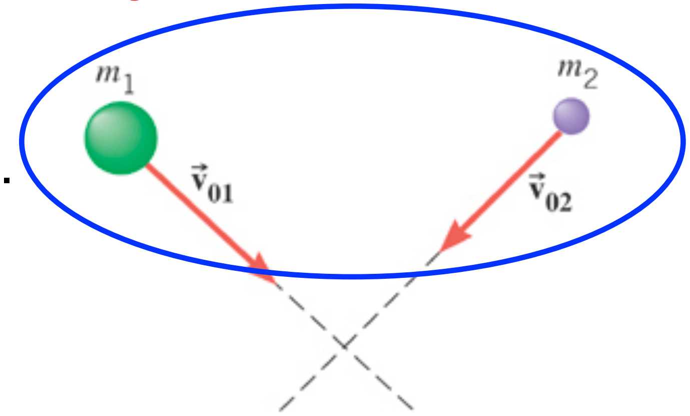
WORK-ENERGY THEOREM ⇔CONSERVATION OF ENERGY

IMPULSE-MOMENTUM THEOREM ⇔???

Apply the impulse-momentum theorem to the midair collision between two objects while falling due to gravity.

7.2 *The Principle of Conservation of Linear Momentum*

System of two masses



(*a*) Before collision

External forces **–** Forces exerted on the objects by agents external to the system. Net force changes the velocity (and momentum) of a mass.

$\boxed{\text{Newton's } 2^{\text{nd}} \text{ Law}}$ $\quad \vec{\mathbf{W}}_1 \;\&\; \vec{\mathbf{W}}_2$

Internal forces **–** Forces that objects **within the system** exert on each other. These forces have equal magnitudes and opposite directions.

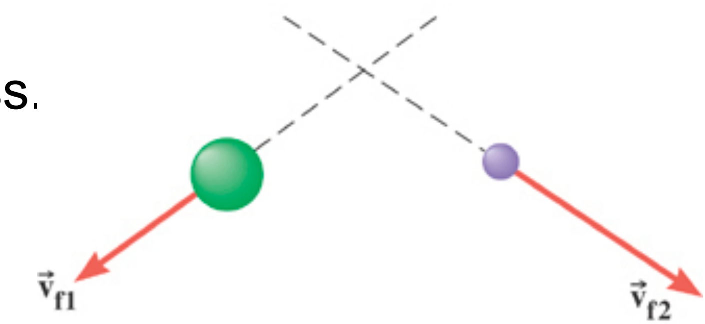
$\boxed{\text{Newton's } 3^{\text{rd}} \text{ Law}}$

Also $\vec{\mathbf{W}}_1 \;\&\; \vec{\mathbf{W}}_2$

(*b*) During collision

External forces **–** Forces exerted on the objects by agents external to the system. Net force changes the velocity (and momentum) of a mass.

$\boxed{\text{Newton's } 2^{\text{nd}} \text{ Law}}$ $\quad \vec{\mathbf{W}}_1 \;\&\; \vec{\mathbf{W}}_2$



(*c*) After collision

7.2 The Principle of Conservation of Linear Momentum

$$\left(\sum \vec{\mathbf{F}}\right)\Delta t = m\vec{\mathbf{v}}_{\mathbf{f}} - m\vec{\mathbf{v}}_{\mathbf{o}}$$

OBJECT 1

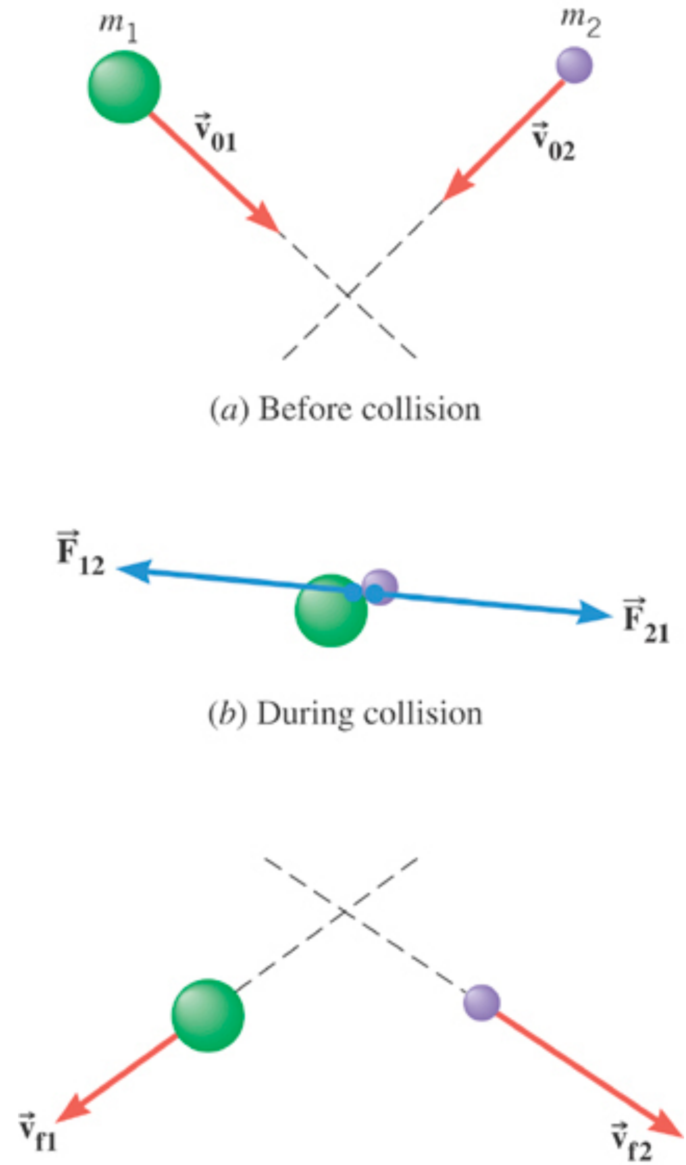
$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{F}}_{12}\right)\Delta t = m_1\vec{\mathbf{v}}_{\mathbf{f}1} - m_1\vec{\mathbf{v}}_{\mathbf{o}1}$$

Force on mass 1 generated by mass 2

OBJECT 2

$$\left(\vec{\mathbf{W}}_2 + \vec{\mathbf{F}}_{21}\right)\Delta t = m_2\vec{\mathbf{v}}_{\mathbf{f}2} - m_2\vec{\mathbf{v}}_{\mathbf{o}2}$$

Force on mass 2 generated by mass 1



(*a*) Before collision

(*b*) During collision

(*c*) After collision

7.2 The Principle of Conservation of Linear Momentum

$$\left(\vec{\mathbf{W}}_1 + \bar{\vec{\mathbf{F}}}_{12}\right)\Delta t = m_1\vec{\mathbf{v}}_{\mathbf{f}1} - m_1\vec{\mathbf{v}}_{\mathbf{o}1}$$

$$+$$

$$\left(\vec{\mathbf{W}}_2 + \bar{\vec{\mathbf{F}}}_{21}\right)\Delta t = m_2\vec{\mathbf{v}}_{\mathbf{f}2} - m_2\vec{\mathbf{v}}_{\mathbf{o}2}$$

For the effect of all the impulses on the system of two masses, add the equations together.

$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{W}}_2 + \bar{\vec{\mathbf{F}}}_{12} + \bar{\vec{\mathbf{F}}}_{21}\right)\Delta t = \left(m_1\vec{\mathbf{v}}_{\mathbf{f}1} + m_2\vec{\mathbf{v}}_{\mathbf{f}2}\right) - \left(m_1\vec{\mathbf{v}}_{\mathbf{o}1} + m_2\vec{\mathbf{v}}_{\mathbf{o}2}\right)$$

$$\bar{\vec{\mathbf{F}}}_{12} = -\bar{\vec{\mathbf{F}}}_{21}$$

The impulses due to Internal forces will cancel

$$\vec{\mathbf{P}}_{\mathbf{f}}$$

Final momentum of System

$$\vec{\mathbf{P}}_{\mathbf{o}}$$

Final momentum of System

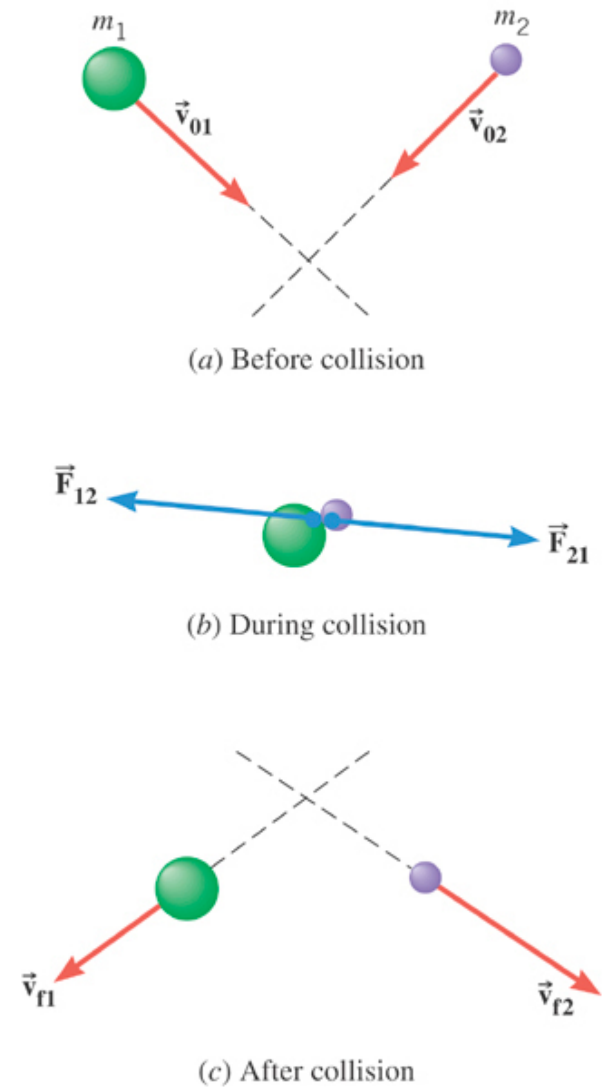
7.2 *The Principle of Conservation of Linear Momentum*

Leaving

$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{W}}_2\right)\Delta t = \vec{\mathbf{P}}_\mathbf{f} - \vec{\mathbf{P}}_\mathbf{o}$$

Sum of average external forces.

Changes momentum



7.2 The Principle of Conservation of Linear Momentum

$$(\text{sum of average external forces})\Delta t = \vec{\mathbf{P}}_{\mathbf{f}} - \vec{\mathbf{P}}_{\mathbf{o}}$$

If the sum of the external forces is zero, then

$$0 = \vec{\mathbf{P}}_{\mathbf{f}} - \vec{\mathbf{P}}_{\mathbf{o}}$$

$$\boxed{\vec{\mathbf{P}}_{\mathbf{f}} = \vec{\mathbf{P}}_{\mathbf{o}}}$$

PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

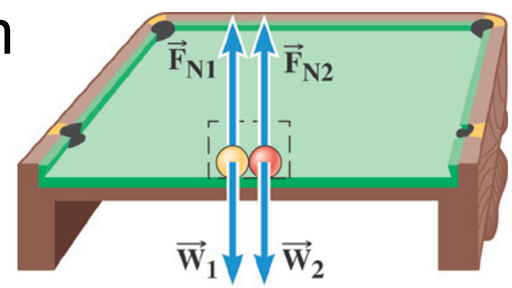
The total linear momentum of an isolated system is constant (conserved). An isolated system is one for which the sum of the average external forces acting on the system is zero.

Most Important example

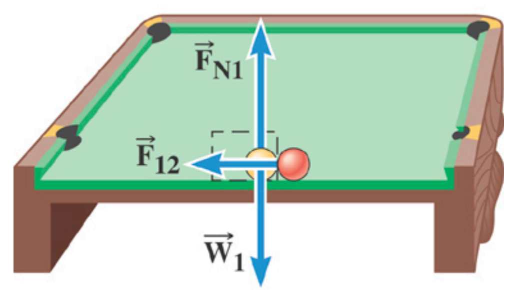
If there are NO external forces acting (gravity is not affecting objects), then the momentum of the system is conserved.

Conceptual Example 4 Is the Total Momentum Conserved?

Imagine two balls colliding on a billiard table that is friction-free. Use the momentum conservation principle in answering the following questions. (a) Is the total momentum of the two-ball system the same before and after the collision? (b) Answer part (a) for a system that contains only one of the two colliding balls.



(*a*)



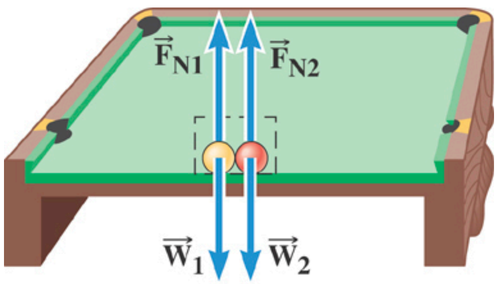
(*b*)

7.2 The Principle of Conservation of Linear Momentum

PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

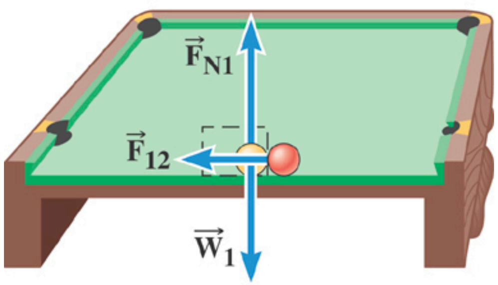
The total linear momentum of an isolated system is constant (conserved). An isolated system is one for which the sum of the average external forces acting on the system is zero.

In the top picture the net external force on the system is zero.



(a)

In the bottom picture the net external force on the system is not zero.

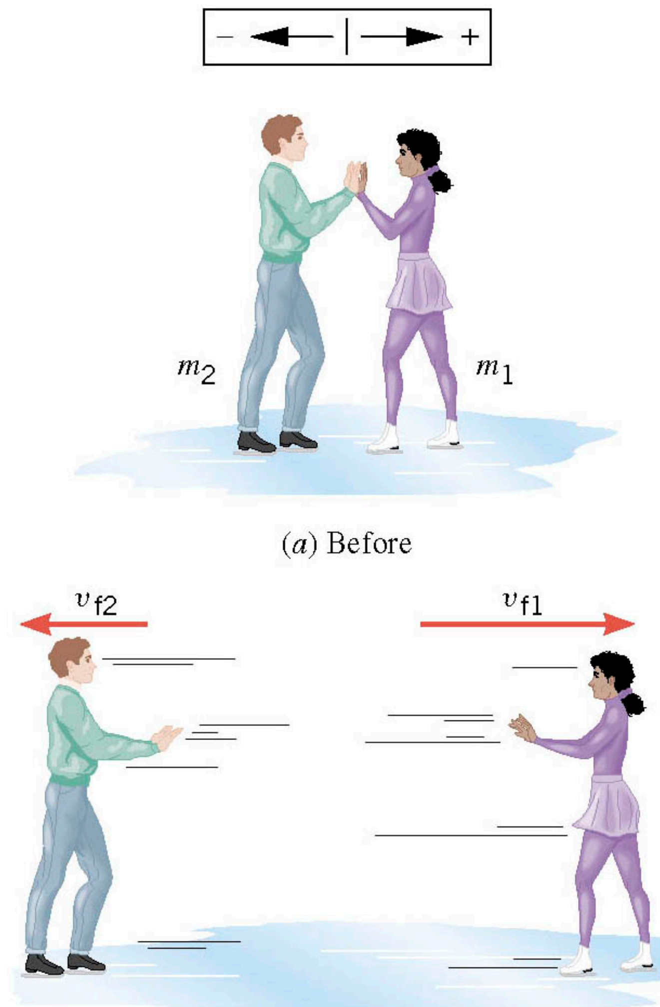


(b)

Example 6 Ice Skaters

Starting from rest, two skaters push off against each other on ice where friction is negligible.

One is a 54-kg woman and one is a 88-kg man. The woman moves away with a speed of +2.5 m/s. Find the recoil velocity of the man.



(*a*) Before

(*b*) After

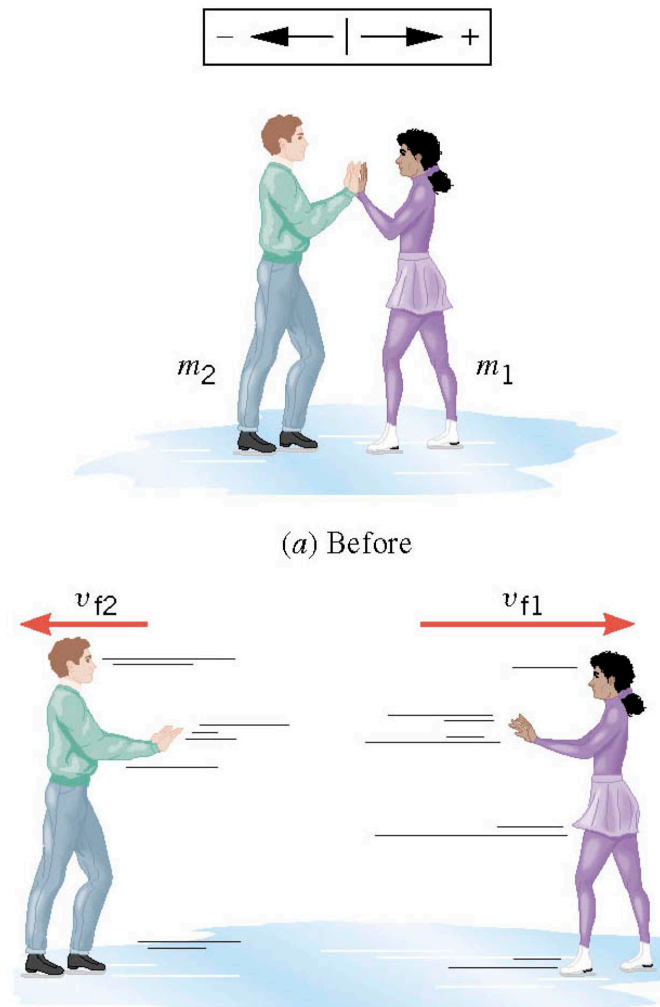
7.2 The Principle of Conservation of Linear Momentum

$$\vec{\mathbf{P}}_{\mathbf{f}} = \vec{\mathbf{P}}_{\mathbf{o}}$$

$$m_1 v_{f1} + m_2 v_{f2} = 0$$

$$v_{f2} = -\frac{m_1 v_{f1}}{m_2}$$

$$v_{f2} = -\frac{(54\ \text{kg})(+2.5\,\text{m/s})}{88\ \text{kg}} = -1.5\,\text{m/s}$$



(*a*) Before

(*b*) After

Applying the Principle of Conservation of Linear Momentum

1. Decide which objects are included in the system.

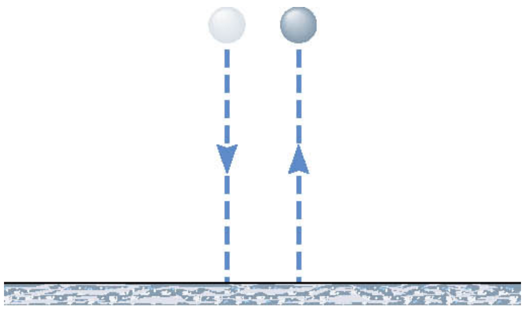
2. Relative to the system, identify the internal and external forces.

3. Verify that the system is isolated.

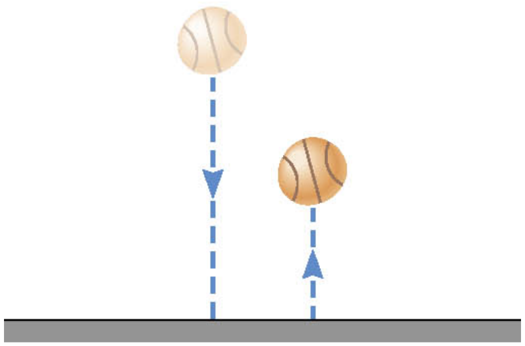
4. Set the final momentum of the system equal to its initial momentum. Remember that momentum is a vector.

7.3 Collisions in One Dimension

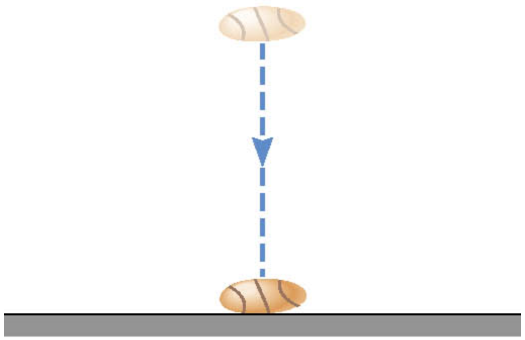
The total linear momentum is conserved when two objects collide, provided they constitute an isolated system.


(a) Elastic collision

Elastic collision -- One in which the total kinetic energy of the system after the collision is equal to the total kinetic energy before the collision.


(b) Inelastic collision

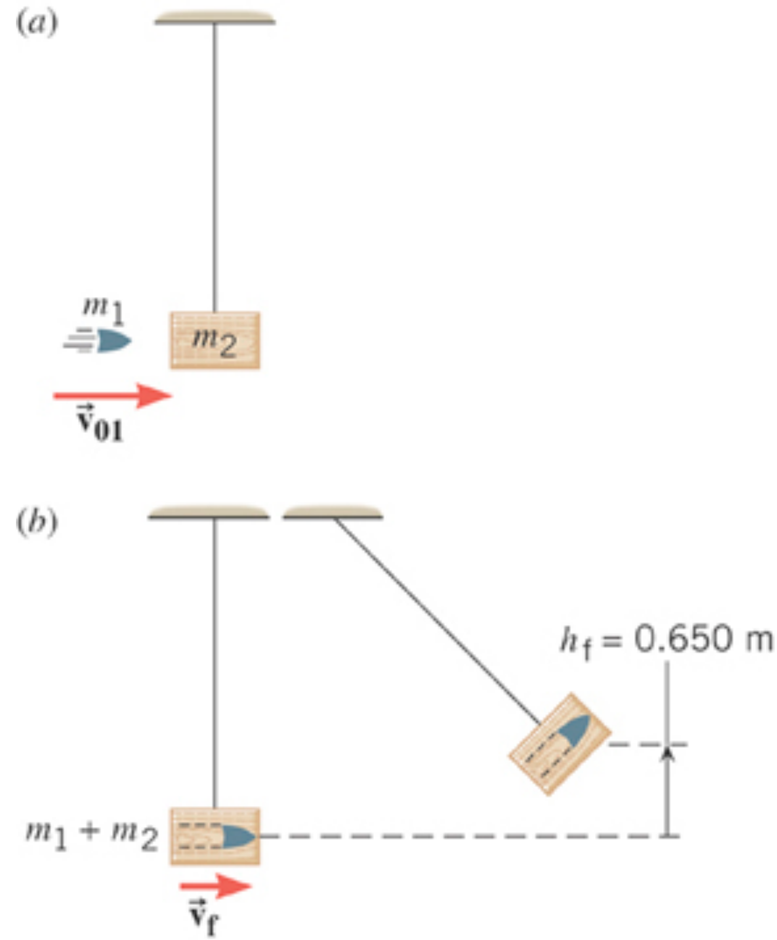
Inelastic collision -- One in which the total kinetic energy of the system after the collision is *not* equal to the total kinetic energy before the collision; if the objects stick together after colliding, the collision is said to be completely inelastic.


(c) Completely inelastic collision

Example 8 A Ballistic Pendulim

The mass of the block of wood is 2.50-kg and the mass of the bullet is 0.0100-kg. The block swings to a maximum height of 0.650 m above the initial position.

Find the initial speed of the bullet.



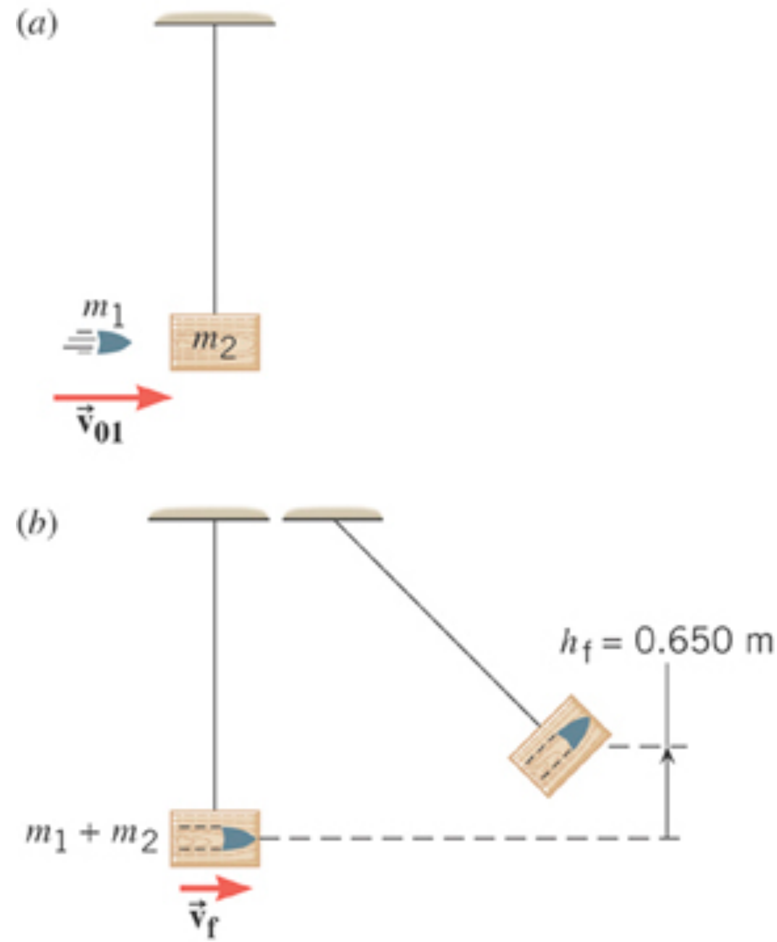
7.3 Collisions in One Dimension

Apply conservation of momentum to the collision:

$$m_1 v_{f1} + m_2 v_{f2} = m_1 v_{o1} + m_2 v_{o2}$$

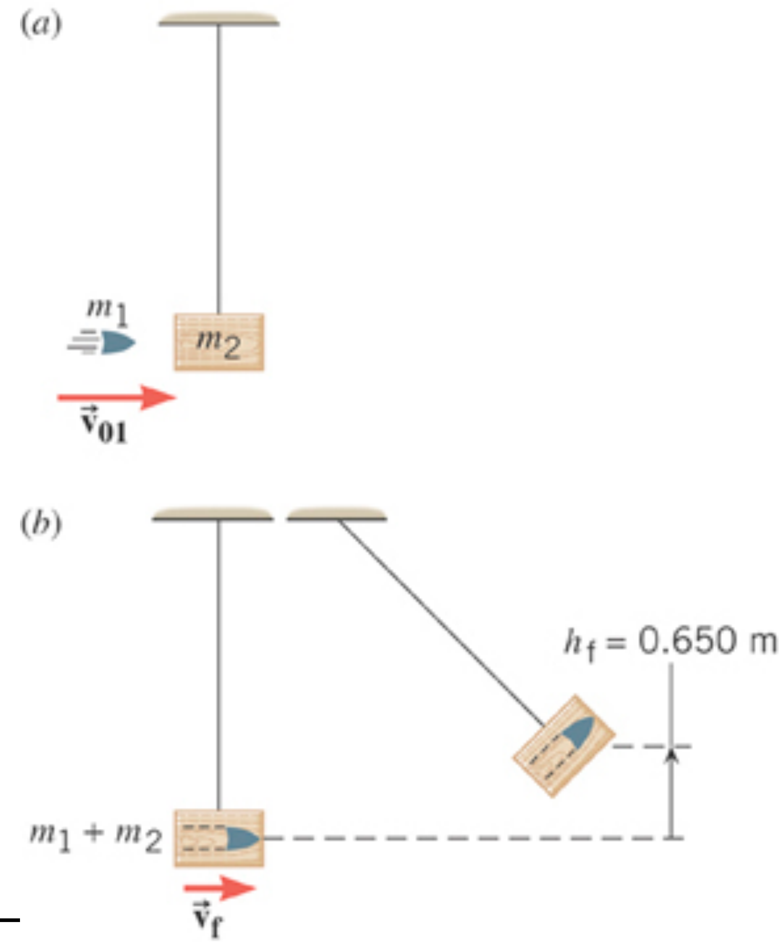
$$\left(m_1 + m_2\right) v_f = m_1 v_{o1}$$

$$v_{o1} = \frac{\left(m_1 + m_2\right) v_f}{m_1}$$



7.3 Collisions in One Dimension

Applying conservation of energy to the swinging motion:



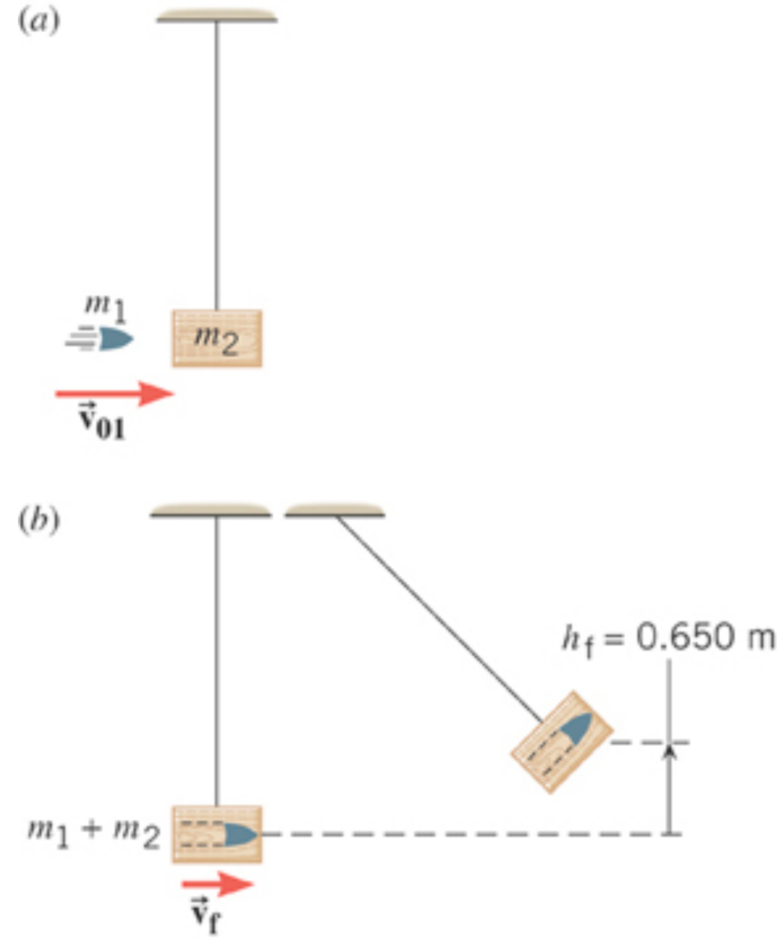
$$mgh = \tfrac{1}{2}mv^2$$

$$(m_1 + m_2)gh_f = \tfrac{1}{2}(m_1 + m_2)v_f^2$$

$$gh_f = \tfrac{1}{2}v_f^2$$

$$v_f = \sqrt{2gh_f} = \sqrt{2(9.80\,\text{m/s}^2)(0.650\text{ m})}$$

7.3 Collisions in One Dimension

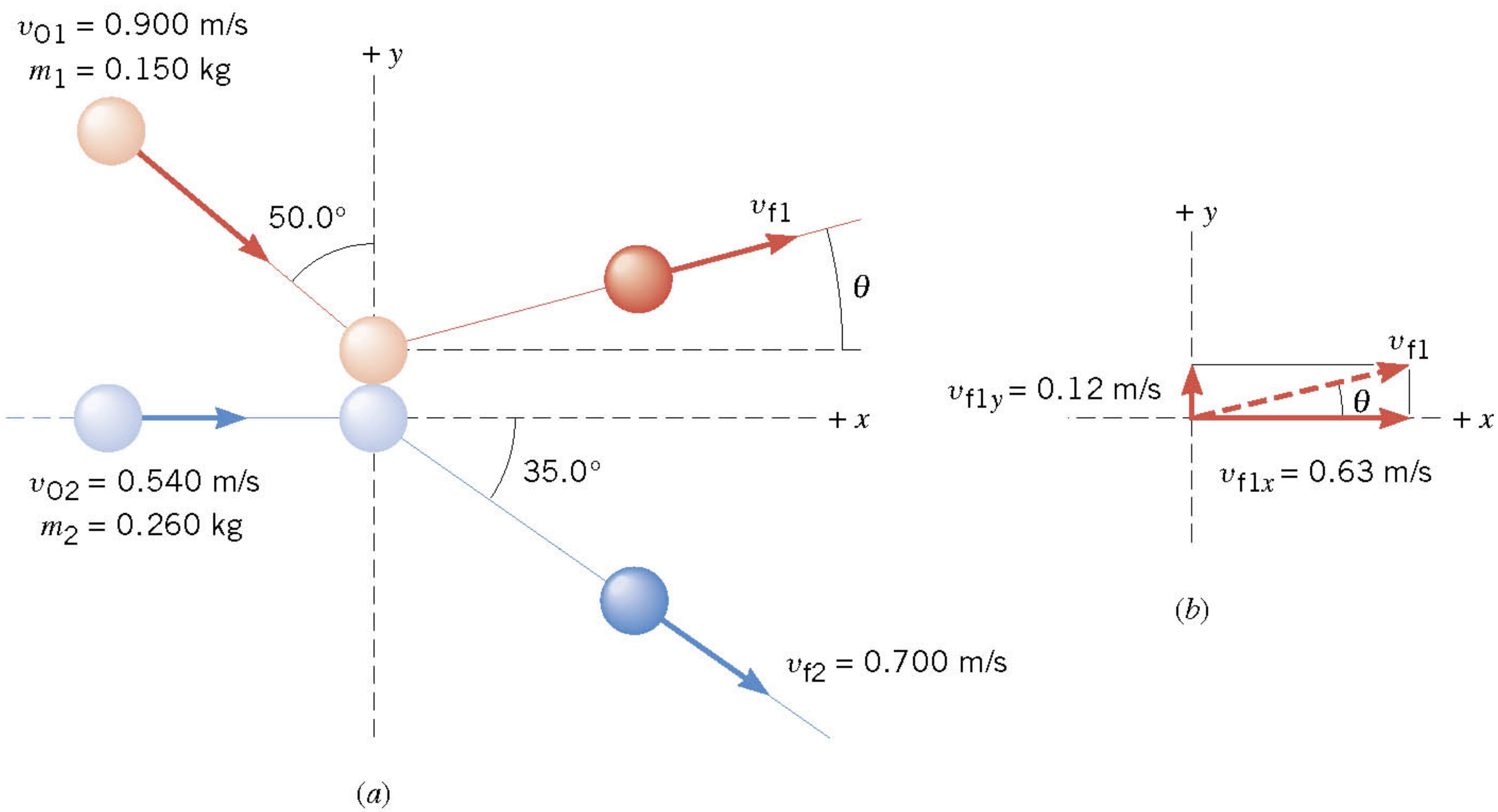


$$v_f = \sqrt{2\left(9.80\,\mathrm{m/s^2}\right)\left(0.650\ \mathrm{m}\right)}$$

$$v_{o1} = \frac{\left(m_1 + m_2\right)v_f}{m_1}$$

$$v_{o1} = \left(\frac{0.0100\ \mathrm{kg} + 2.50\ \mathrm{kg}}{0.0100\ \mathrm{kg}}\right)\sqrt{2\left(9.80\,\mathrm{m/s^2}\right)\left(0.650\ \mathrm{m}\right)} = +896\,\mathrm{m/s}$$

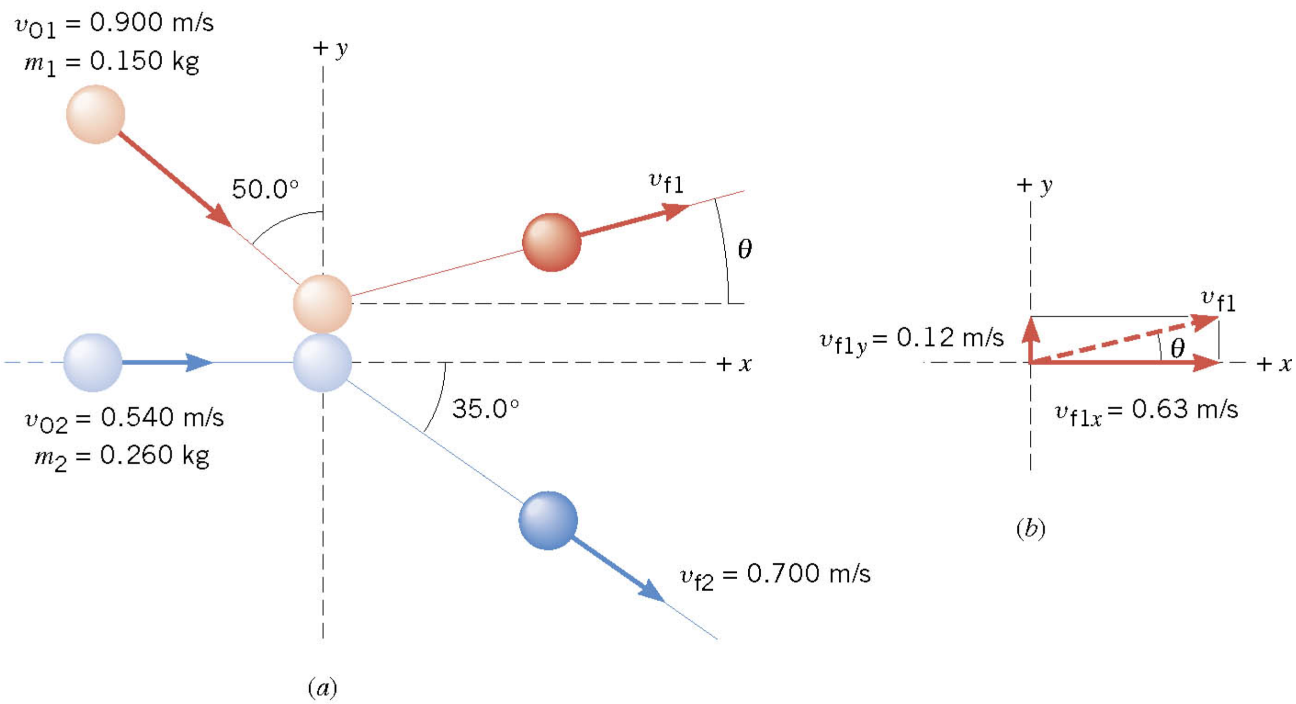
A Collision in Two Dimensions



7.4 Collisions in Two Dimensions

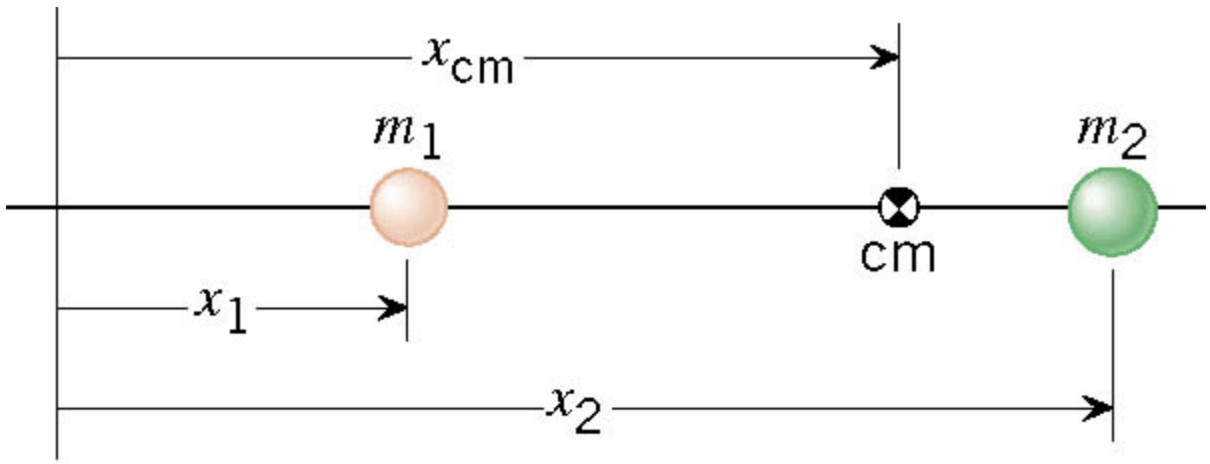
$$m_1 v_{f1x} + m_2 v_{f2x} = m_1 v_{o1x} + m_2 v_{o2x}$$

$$m_1 v_{f1y} + m_2 v_{f2y} = m_1 v_{o1y} + m_2 v_{o2y}$$



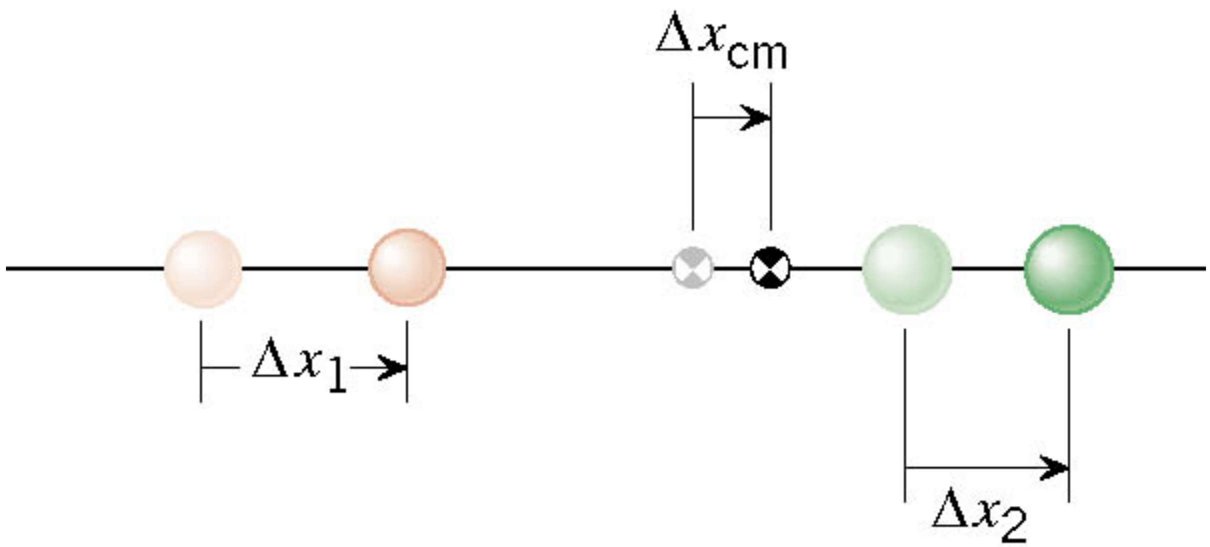
7.5 Center of Mass

The center of mass is a point that represents the average location for the total mass of a system.



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

7.5 Center of Mass



$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2} \quad \Rightarrow \quad v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

7.5 Center of Mass

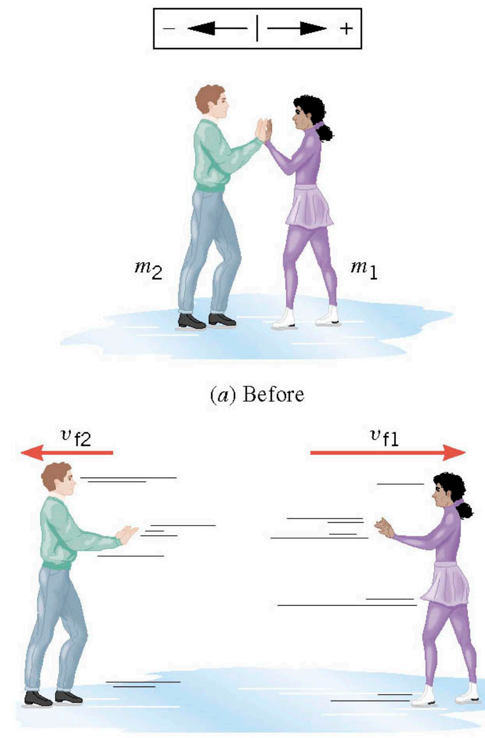
$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

In an isolated system, the total linear momentum does not change, therefore the velocity of the center of mass does not change.

7.5 Center of Mass

BEFORE

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0$$



(a) Before

(b) After

AFTER

$$v_{cm} = \frac{(88\text{ kg})(-1.5\,\text{m/s}) + (54\text{ kg})(+2.5\,\text{m/s})}{88\text{ kg} + 54\text{ kg}} = 0.002 \approx 0$$