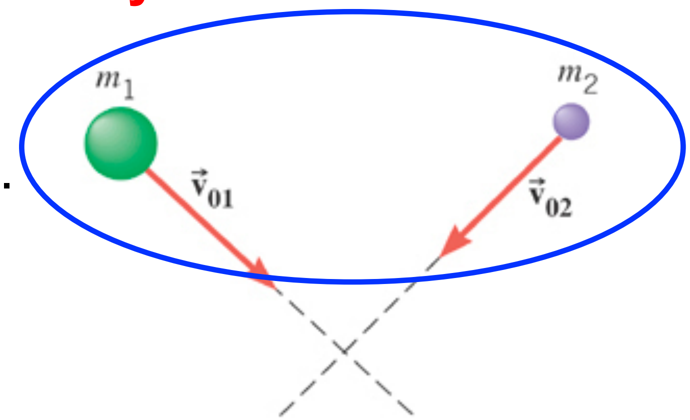
Chapter 7

Impulse and Momentum

continued

7.2 *The Principle of Conservation of Linear Momentum*

System of two masses



(*a*) Before collision

External forces **–** Forces exerted on the objects by agents external to the system. Net force changes the velocity (and momentum) of a mass.

$\boxed{\text{Newton's } 2^{nd} \text{ Law}}$ $\quad \vec{\mathbf{W}}_1 \;\&\; \vec{\mathbf{W}}_2$

Internal forces **–** Forces that objects **within the system** exert on each other. These forces have equal magnitudes and opposite directions.

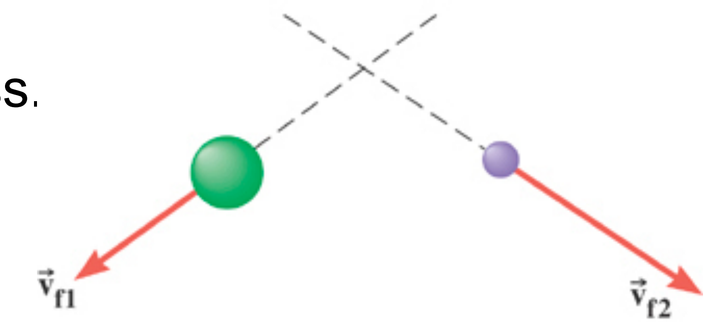
$\boxed{\text{Newton's } 3^{rd} \text{ Law}}$

Also $\vec{\mathbf{W}}_1 \;\&\; \vec{\mathbf{W}}_2$

(*b*) During collision

External forces **–** Forces exerted on the objects by agents external to the system. Net force changes the velocity (and momentum) of a mass.

$\boxed{\text{Newton's } 2^{nd} \text{ Law}}$ $\quad \vec{\mathbf{W}}_1 \;\&\; \vec{\mathbf{W}}_2$



(*c*) After collision

7.2 The Principle of Conservation of Linear Momentum

$\ell.c.$ p

$$\vec{\mathbf{p}}_{\mathbf{o}1} = m_1 \vec{\mathbf{v}}_{\mathrm{o}1}$$ initial momentum of mass 1

Capital P

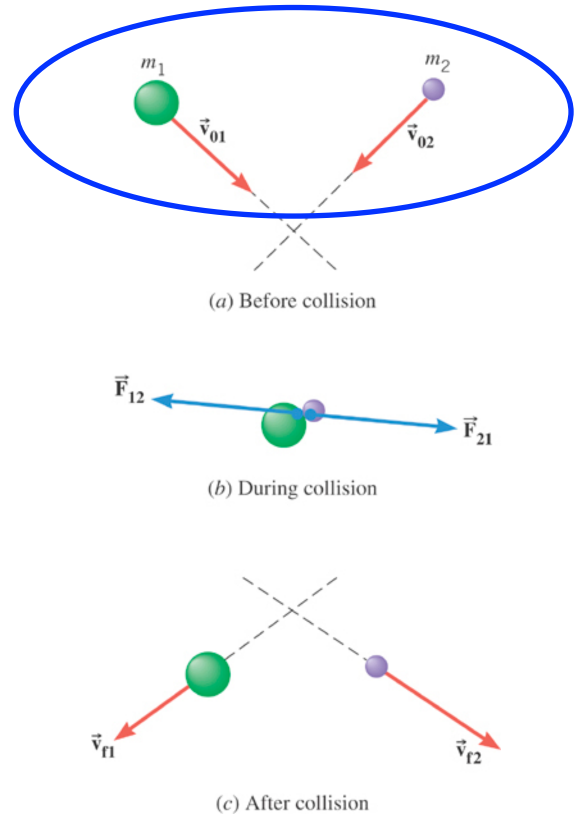
$$\vec{\mathbf{P}}_{\mathbf{o}} = m_1 v_{\mathrm{o}1} + m_2 v_{\mathrm{o}2} + \ldots$$ initial momentum sum

$$\vec{\mathbf{P}}_{\mathbf{f}} = m_1 v_{\mathrm{f}1} + m_2 v_{\mathrm{f}2} + \ldots$$ final momentum sum

Net External Force — Changes the momentum sum

$$\left(\sum \vec{\overline{\mathrm{F}}}_{\mathrm{external}}\right)\Delta t = \vec{\mathbf{P}}_{\mathbf{f}} - \vec{\mathbf{P}}_{\mathbf{o}}$$

System of two masses



Most Important case

Net External Force = 0 $\quad 0 = \vec{\mathbf{P}}_{\mathbf{f}} - \vec{\mathbf{P}}_{\mathbf{o}}$ No change in momentum sum

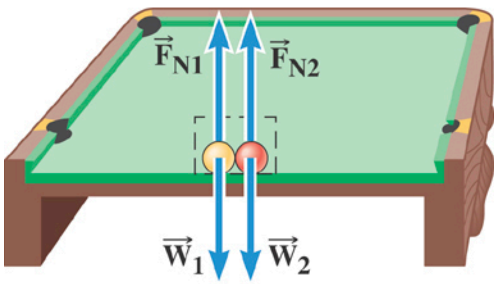
$$\boxed{\vec{\mathbf{P}}_{\mathbf{f}} = \vec{\mathbf{P}}_{\mathbf{o}}}$$ Conservation of Linear Momentum Sum

7.2 The Principle of Conservation of Linear Momentum

PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

The total linear momentum of an isolated system is constant (conserved). An isolated system is one for which the sum of the average external forces acting on the system is zero.

In the top picture, the net external force on the system is zero.



Sum of Weight force and Normal Force on each mass is zero, so no net external force.

If the cue of a pool player hits one of the balls, in that collision the momentum sum changes from zero to non-zero.

The force of the cue on the ball is a net external force.

7.2 The Principle of Conservation of Linear Momentum

Why is adding momentum vectors of different masses together useful at all?

System of two masses

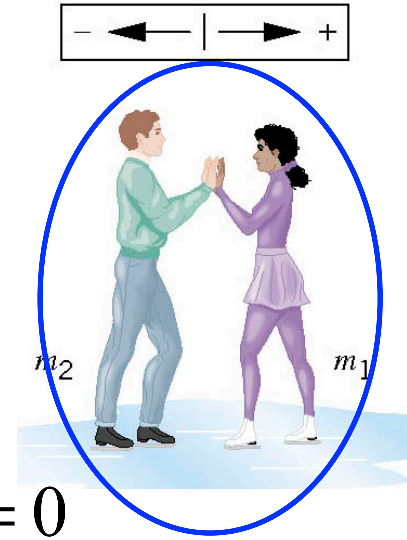
$$\vec{\mathbf{P}}_{\mathbf{o}} = m_1\vec{\mathbf{v}}_{o1} + m_2\vec{\mathbf{v}}_{o2} + \ldots$$ initial momentum sum

$$\vec{\mathbf{P}}_{\mathbf{f}} = m_1\vec{\mathbf{v}}_{f1} + m_2\vec{\mathbf{v}}_{f2} + \ldots$$ final momentum sum

Net External Force on system of two skaters is zero.

$$\vec{\mathbf{P}}_{\mathrm{f}} = \vec{\mathbf{P}}_{\mathbf{o}}$$

$$\vec{\mathbf{P}}_{\mathrm{f}} = 0$$



$$\vec{\mathbf{P}}_{\mathbf{o}} = 0$$

Before

Still not so clear why adding momentum vectors is useful. But,

$$\vec{\mathbf{P}}_{\mathbf{f}} = m_1\vec{\mathbf{v}}_{f1} + m_2\vec{\mathbf{v}}_{f2} = 0$$

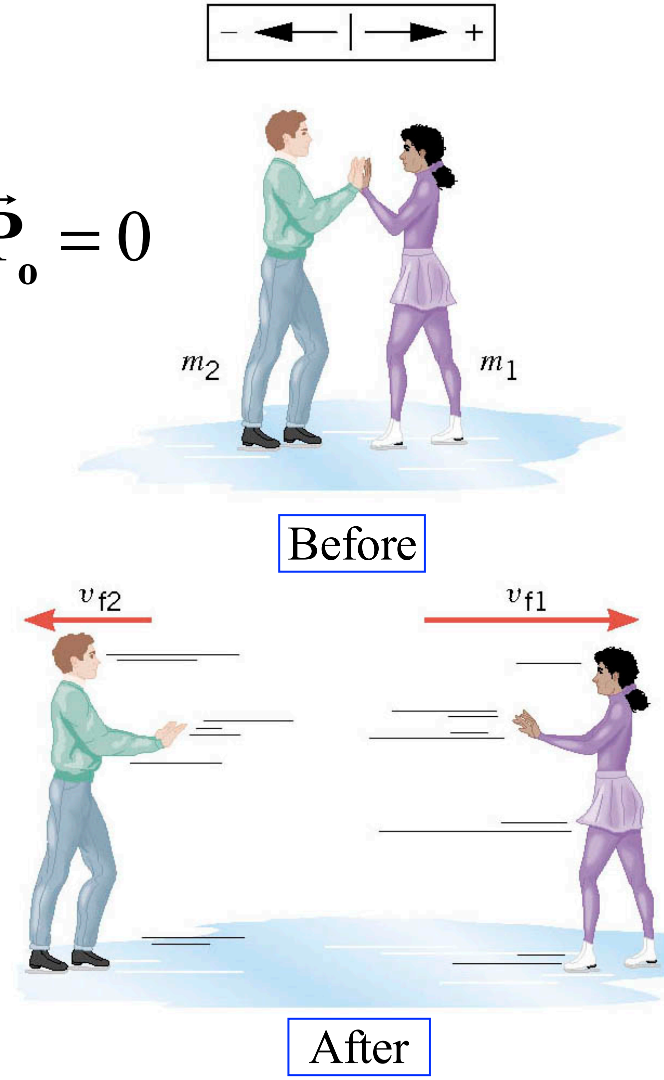
Momentum vector of mass 2 $\quad m_2\vec{\mathbf{v}}_{f2} = -m_1\vec{\mathbf{v}}_{f1}\quad$ is opposite to Momentum vector of mass 1

Example 6 Ice Skaters

Starting from rest, two skaters push off against each other on ice where friction is negligible.

One is a 54-kg woman and one is a 88-kg man. The woman moves away with a speed of +2.5 m/s. Find the recoil velocity of the man.

$$\vec{\mathbf{P}}_{\mathbf{o}} = 0$$



$$\vec{\mathbf{P}}_{\mathbf{f}} = \vec{\mathbf{P}}_{\mathbf{o}}$$

$$m_1\vec{\mathbf{v}}_{f1} + m_2\vec{\mathbf{v}}_{f2} = 0 \quad \Rightarrow \quad m_2\vec{\mathbf{v}}_{f2} = -m_1\vec{\mathbf{v}}_{f1}$$

$$\vec{\mathbf{v}}_{f2} = -\frac{m_1}{m_2}\vec{\mathbf{v}}_{f1} = -\frac{54}{88}(+2.5\text{ m/s}) = -1.5\text{ m/s}$$

7.2 *The Principle of Conservation of Linear Momentum*

Applying the Principle of Conservation of Linear Momentum

1. Decide which objects are included in the system.

2. Relative to the system, identify the internal and external forces.

3. Verify that the system is isolated.

4. Set the final momentum of the system equal to its initial momentum. Remember that momentum is a vector.

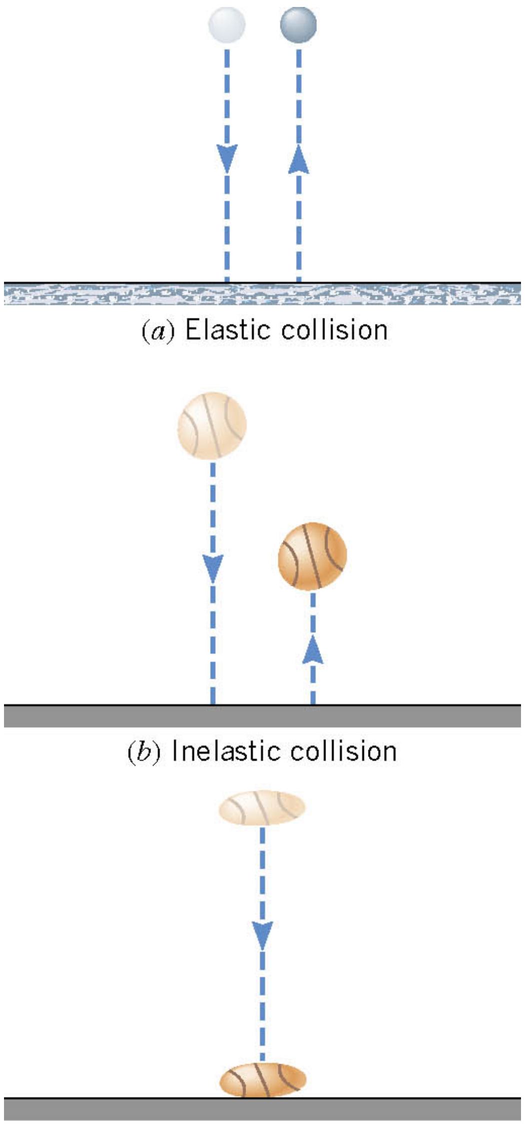
7.3 Collisions in One Dimension

The total linear momentum is conserved when two objects collide, provided they constitute an isolated system.
Isolated system is the ball **and** the earth.

Energy considerations in collisions

Elastic collision -- One in which the total kinetic energy of the system after the collision is equal to the total kinetic energy before the collision.

Inelastic collision -- One in which the total kinetic energy of the system after the collision is *not* equal to the total kinetic energy before the collision; if the objects stick together after colliding, the collision is said to be completely inelastic.



(*a*) Elastic collision

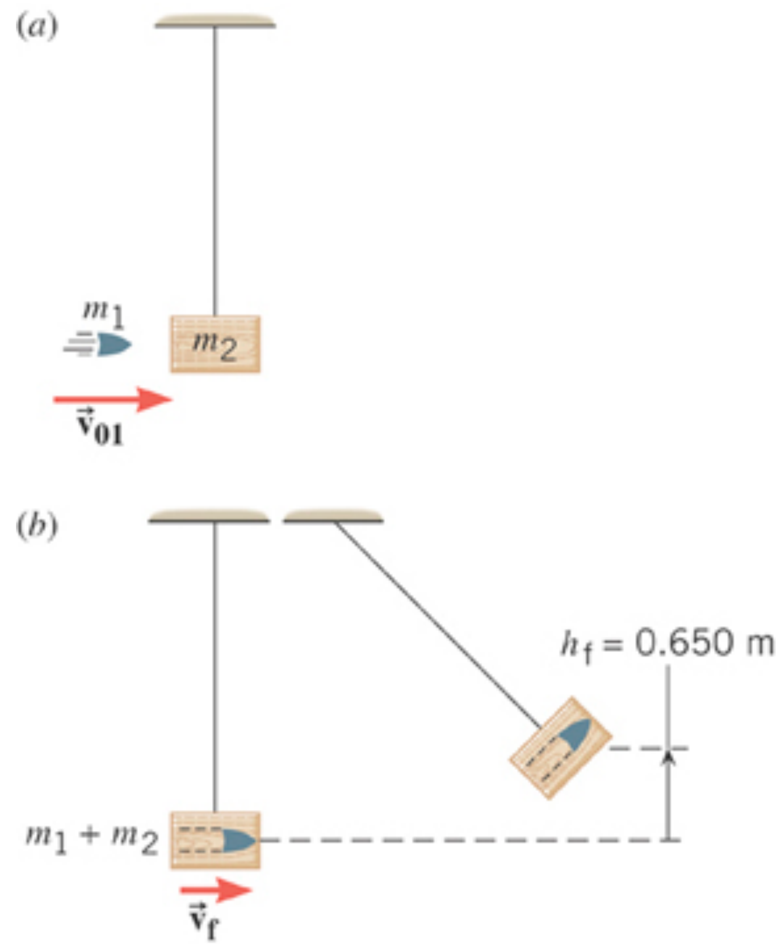
(*b*) Inelastic collision

(*c*) Completely inelastic collision

Example 8 A Ballistic Pendulim

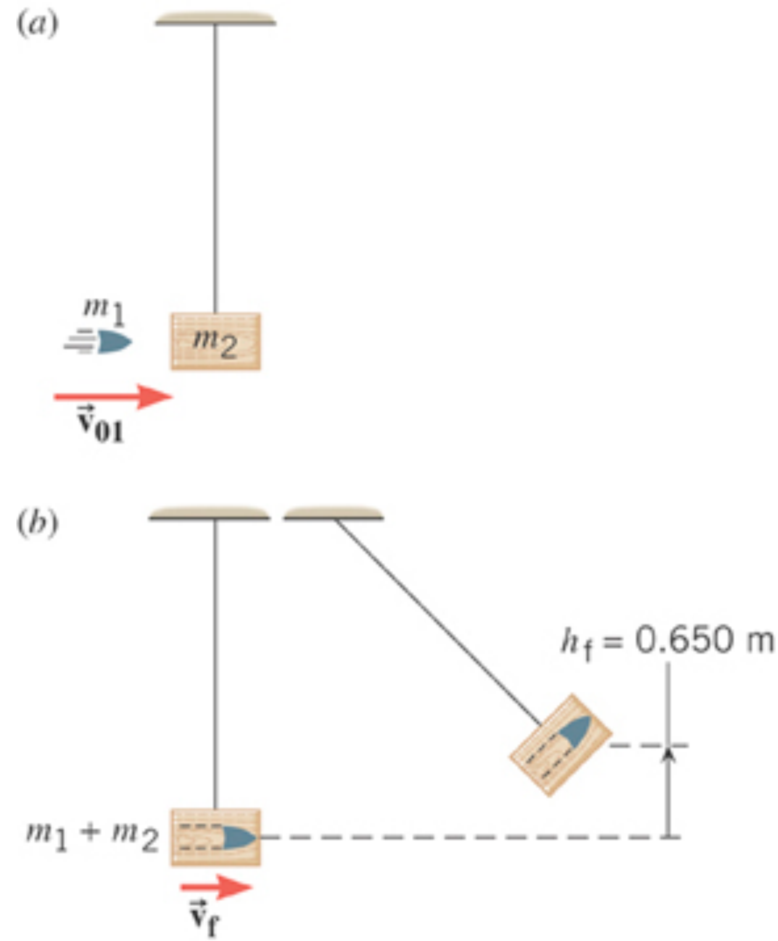
The mass of the block of wood is 2.50-kg and the mass of the bullet is 0.0100-kg. The block swings to a maximum height of 0.650 m above the initial position.

Find the initial speed of the bullet.



7.3 Collisions in One Dimension

In the collision of bullet and block, tension in string and weight are external forces but the net external force is zero. Therefore, momentum is conserved.



Because the bullet is stopped in the block by friction, energy is not conserved in collision.

But, after the collision, only gravity (a conservative force) does work. Therefore, energy is conserved.

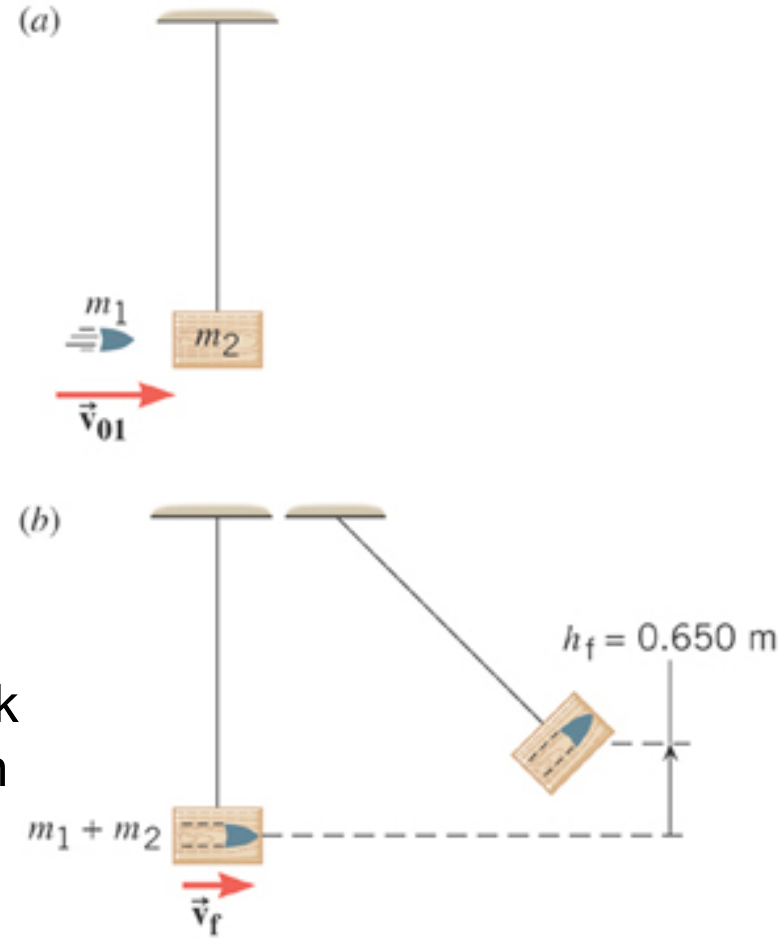
7.3 Collisions in One Dimension

Apply conservation of momentum to the collision:

$$m_1 v_{f1} + m_2 v_{f2} = m_1 v_{o1} + m_2 v_{o2}$$

$$\left(m_1 + m_2\right) v_f = m_1 v_{o1}$$

$$v_{o1} = \frac{\left(m_1 + m_2\right)}{m_1} v_f$$ need v_f



To determine the speed of the bullet + block after collision use conservation of energy in the swing. collision v_f = swing v_0

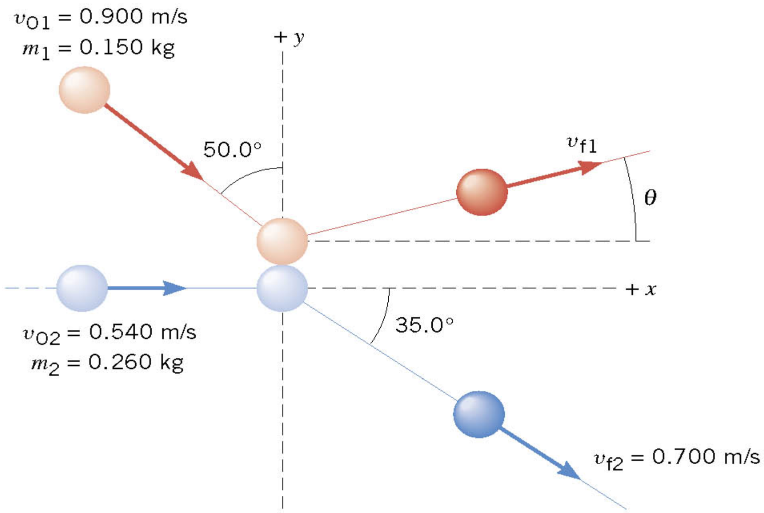
$$\mathrm{KE}_0 + \mathrm{PE}_0 = \mathrm{PE}_f + \mathrm{KE}_f$$

$$\tfrac{1}{2} m v_0^2 + 0 = mgh + 0$$

$$v_0 = \sqrt{2gh}$$

$$v_{o1} = \frac{\left(m_1 + m_2\right)}{m_1} \sqrt{2gh} = +896\,\mathrm{m/s}$$

7.4 Collisions in Two Dimensions



Momentum conserved in each of the two dimensions, x and y.

$$m_1 = 0.150 \text{ kg}, \quad m_2 = 0.260 \text{ kg}$$

x-components: $m_1 v_{f1} + m_2 v_{f2} = m_1 v_{o1} + m_2 v_{o2}$;

$$v_{o1} = 0.900 \sin 50° \text{ m/s} \text{ , } v_{o2} = 0.540 \text{ m/s}, \ v_{f2} = 0.700 \cos 35° \text{ m/s}$$

y-components: $m_1 v_{f1} + m_2 v_{f2} = m_1 v_{o1} + m_2 v_{o2}$;

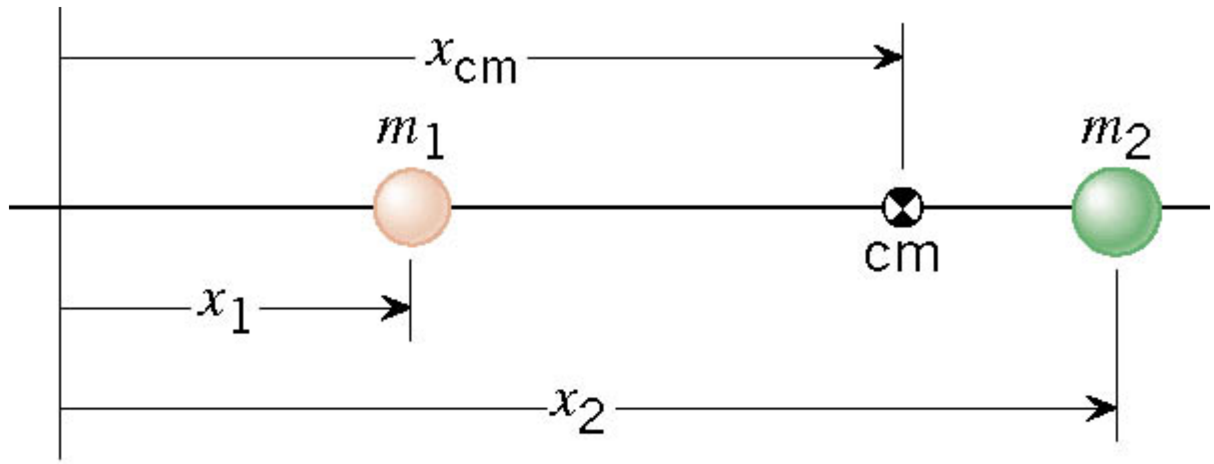
$$v_{o1} = 0.900 \cos 50° \text{ m/s} \quad v_{o2} = 0, \quad v_{f2} = 0.700 \sin 35° \text{ m/s}$$

$$\text{final } x : v_{1x} = +0.63 \text{ m/s} \qquad \text{final } y : v_{1y} = +0.12 \text{ m/s}$$

$$v_1 = \sqrt{v_{1x}^2 + v_{1y}^2} = +0.64 \text{ m/s}; \quad \theta = \tan^{-1}\left(v_{1y} / v_{1x}\right) = 11°$$

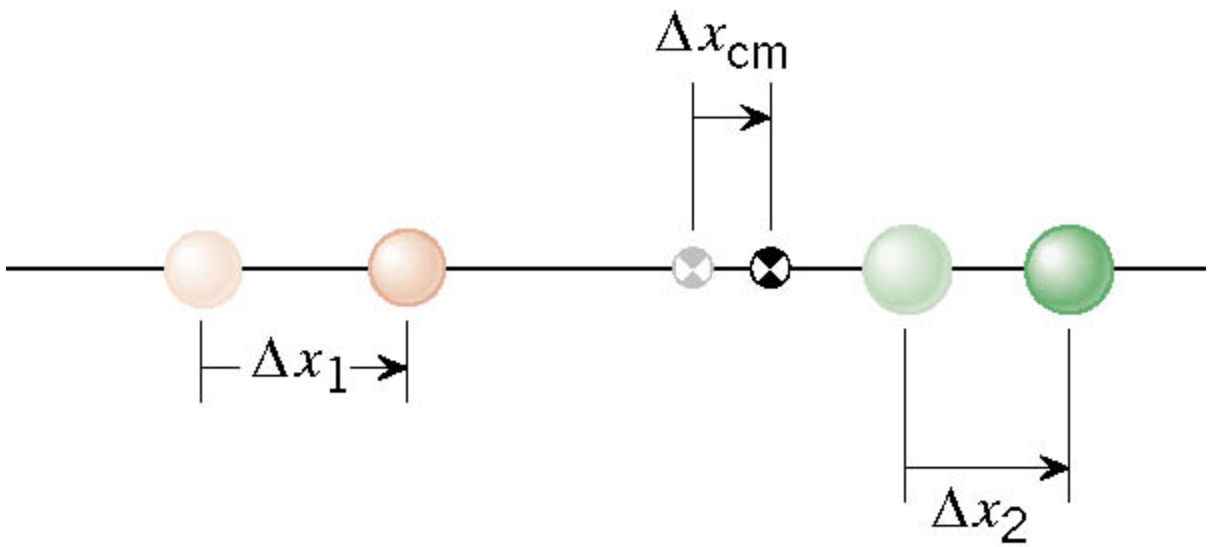
7.5 Center of Mass

The center of mass is a point that represents the average location for the total mass of a system.



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

7.5 Center of Mass



$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2} \Rightarrow v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

7.5 Center of Mass

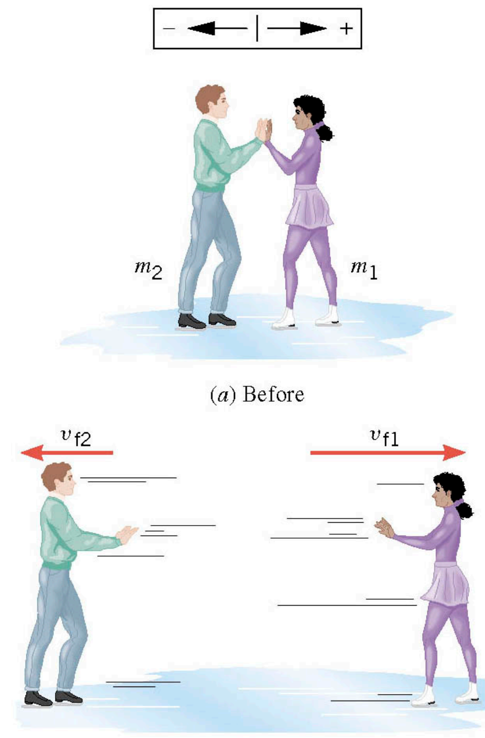
$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

In an isolated system, the total linear momentum does not change, therefore the velocity of the center of mass does not change.

7.5 Center of Mass

BEFORE

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0$$



(a) Before

(b) After

AFTER

$$v_{cm} = \frac{(88\text{ kg})(-1.5\text{m/s}) + (54\text{ kg})(+2.5\text{m/s})}{88\text{ kg} + 54\text{ kg}} = 0.002 \approx 0$$