Quick review of Ch. 6 & 7

Quiz to follow

Energy and energy conservation

Work: $W = Fs\cos\theta$

Kinetic Energy: $KE = \frac{1}{2}mv^2$

Work changes kinetic energy: $W = KE_\mathrm{f} - KE_0 = \frac{1}{2}mv_\mathrm{f}^2 - \frac{1}{2}mv_0^2$

Conservative forces $\Rightarrow$ Potential Energy

Gravitational Potential Energy: $PE = mgh$

Total Energy: $E = KE + PE$

Work by non-conservative forces (friction, humans, explosions)

changes total energy: $W_\mathrm{NC} = \left(KE_\mathrm{f} - KE_0\right) + \left(PE_\mathrm{f} - PE_0\right)$

If $W_\mathrm{NC} = 0$, there is total energy conservation:

$$E_\mathrm{f} = E_0 \quad \Rightarrow \quad KE_\mathrm{f} + PE_\mathrm{f} = KE_0 + PE_0$$

Average power = Work/time = (Energy change)/time = $F\,\overline{v}$

Momentum and momentum conservation

Impulse: $\vec{\mathbf{J}} = \vec{\mathbf{F}}t$

Momentum: $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$

Net average impulse changes momentum:

$$\sum \bar{\vec{\mathbf{F}}}\Delta t = \vec{\mathbf{p}}_{\mathrm{f}} - \vec{\mathbf{p}}_0 = m\vec{\mathbf{v}}_{\mathrm{f}} - m\vec{\mathbf{v}}_0$$

Momentum of 2 masses in collision: $\vec{\mathbf{P}} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2$

No net external force, momentum is conserved:

$$\vec{\mathbf{P}}_{\mathrm{f}} = \vec{\mathbf{P}}_0 \quad \Rightarrow \quad m_1\vec{\mathbf{v}}_{1\mathrm{f}} + m_2\vec{\mathbf{v}}_{2\mathrm{f}} = m_1\vec{\mathbf{v}}_{1\mathrm{o}} + m_2\vec{\mathbf{v}}_{2\mathrm{o}}$$

Center of mass position: $x_{\mathrm{cm}} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$, and velocity $v_{\mathrm{cm}} = \dfrac{m_1v_1 + m_2v_2}{m_1 + m_2}$

Conservation of momentum $\Rightarrow$ v_{cm} remains constant

Chapter 8

Rotational Kinematics

8.1 Rotational Motion and Angular Displacement

Why are there 360 degrees in a circle?
Why are there 60 minutes in an hour?
Why are there 60 seconds in a minute?

Because the Greeks, who invented these units were enamored with numbers that are divisible by most whole numbers, 12 or below (except 7 and 11).

Strange, because later it was the Greeks who discovered that the ratio of the radius to the circumference of a circle was a number known as 2π.

A new unit, radians, is really useful for angles.

8.1 Rotational Motion and Angular Displacement

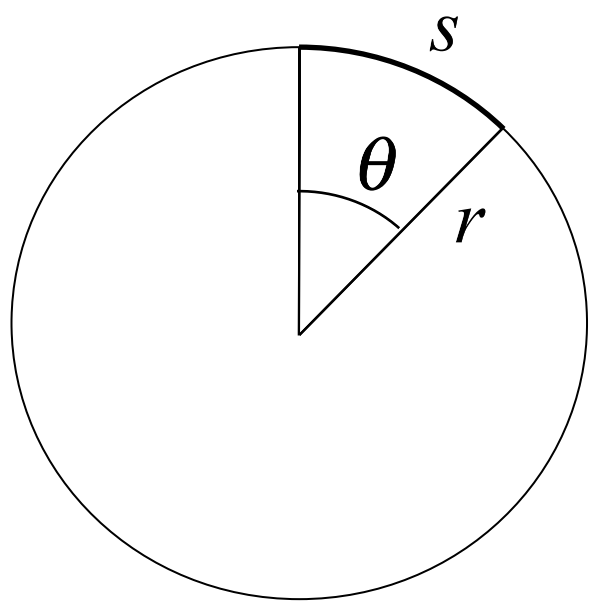
A new unit, radians, is really useful for angles.

Radian measure

$$\theta(\text{radians}) = \frac{s \text{ (arc length)}}{r \text{ (radius)}}$$

$$s = r\theta$$

(s in same units as r)



Full circle

$$\theta = \frac{s}{r} = \frac{2\pi \cancel{r}}{\cancel{r}}$$

$$= 2\pi \text{ (radians)}$$

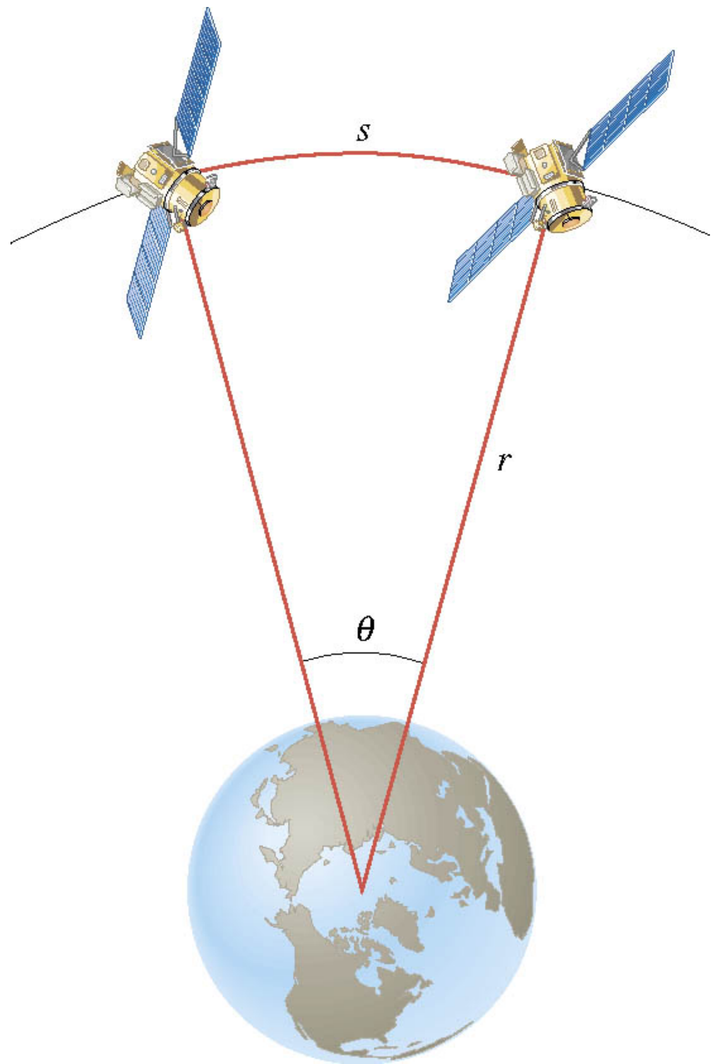
Conversion of degree to radian measure

$$\theta(\text{rad}) = \theta(\text{deg.})\left(\frac{2\pi}{360}\frac{\text{rad}}{\text{deg.}}\right)$$

$$\left(\frac{2\pi}{360}\frac{\text{rad}}{\text{deg.}}\right) = 1$$

Example 1 Adjacent Synchronous Satellites

Synchronous satellites are put into an orbit whose radius is 4.23×10^{7}m.



If the angular separation of the two satellites is 2.00 degrees, find the arc length that separates them.

Convert degree to radian measure

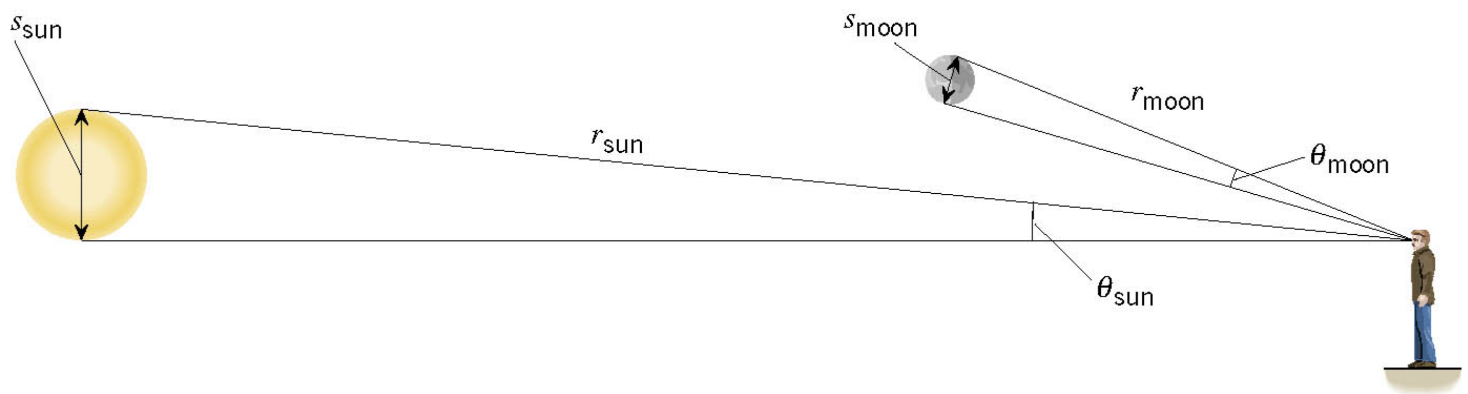
$$2.00\,\mathrm{deg}\left(\frac{2\pi\ \mathrm{rad}}{360\,\mathrm{deg}}\right)=0.0349\ \mathrm{rad}$$

Determine arc length

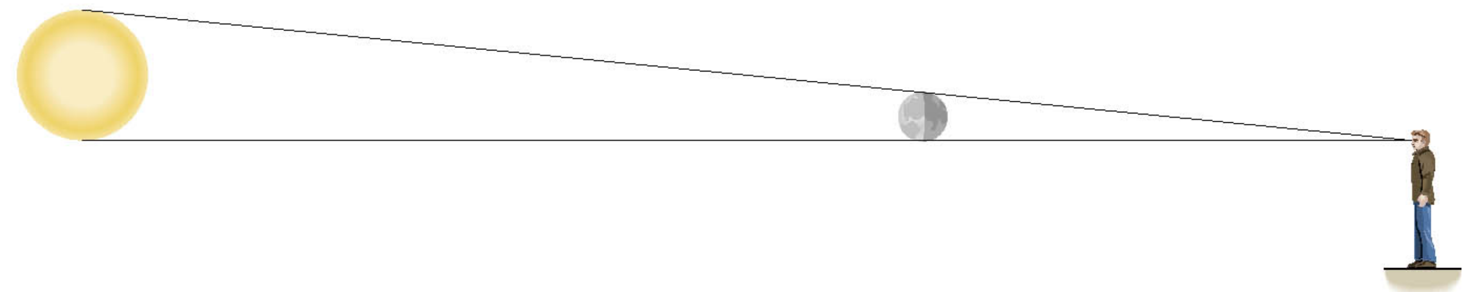
$$s=r\theta=\left(4.23\times10^{7}\,\mathrm{m}\right)\left(0.0349\ \mathrm{rad}\right)$$

$$=1.48\times10^{6}\,\mathrm{m}\ \ (920\ \mathrm{miles})$$

8.1 Rotational Motion and Angular Displacement



(a)



(b)

For an observer on the earth, an eclipse can occur because angles subtended by the sun and the moon are the same.

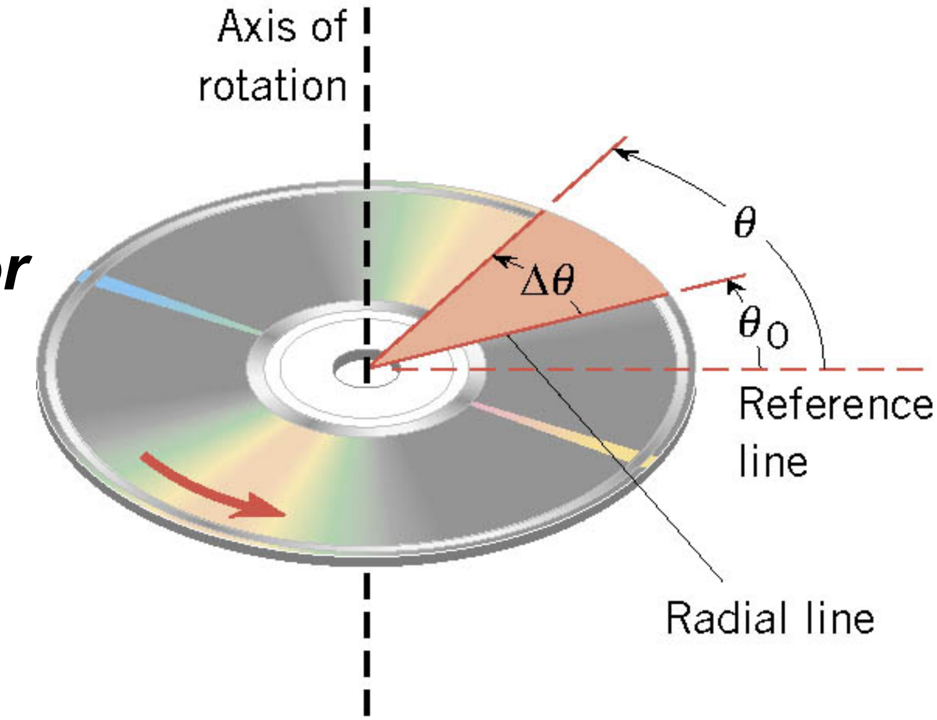
$$\theta = \frac{s_{\text{Sun}}}{r_{\text{Sun}}} \approx \frac{s_{\text{Moon}}}{r_{\text{Moon}}} \approx 9.3 \text{ mrad}$$

8.1 Rotational Motion and Angular Displacement

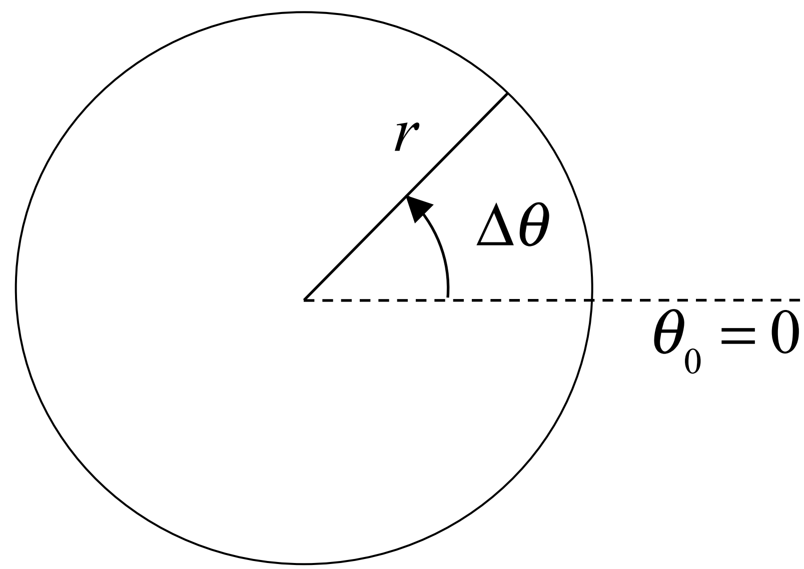
The angle through which the object rotates is called the ***angular displacement vector***

$$\Delta\theta = \theta - \theta_o$$

SI unit of angular displacement, radian (rad)



Simplified using $\theta_o = 0$, and $\Delta\theta = \theta$, angular displacement vector.



Vector

Counter-clockwise is + displacement Clockwise is – displacement

DEFINITION OF AVERAGE ANGULAR VELOCITY

$$\text{Average angular velocity} = \frac{\text{Angular displacement}}{\text{Elapsed time}}$$

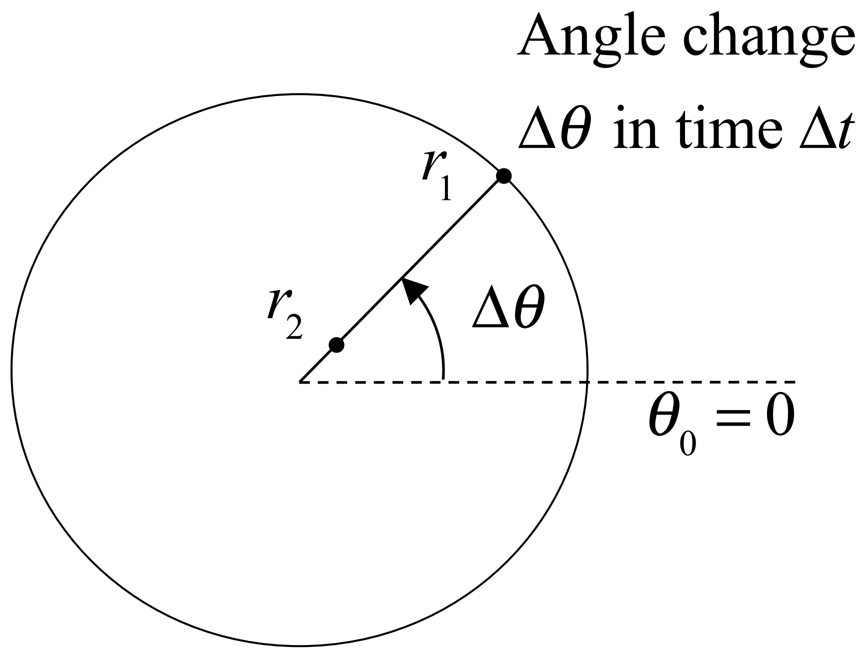
$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} \qquad \text{where } \Delta t = t - t_o$$

SI Unit of Angular Velocity: radian per second (rad/s)

$\Delta\theta$ is the same at all radii.

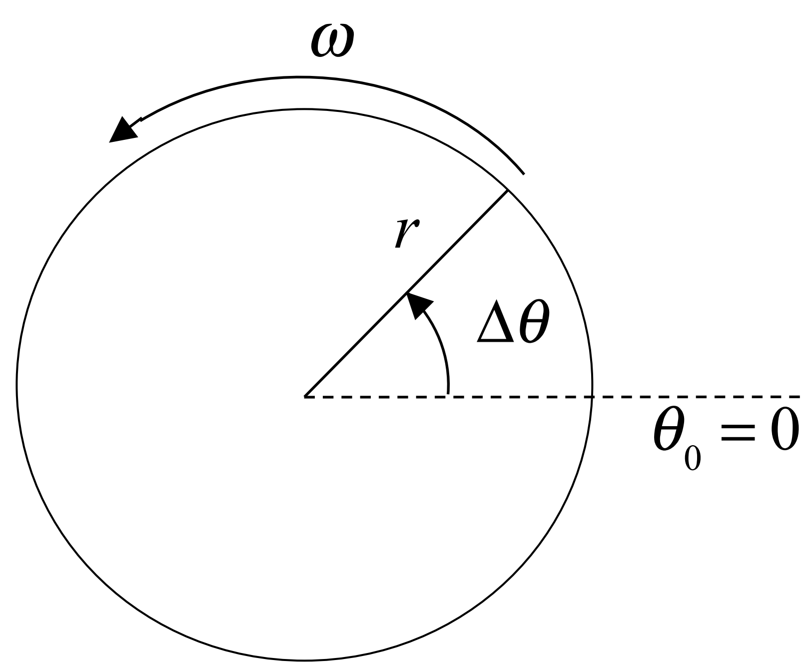
Δt is the same at all radii.

$$\boxed{\omega = \frac{\Delta\theta}{\Delta t} \text{ is the same at all radii.}}$$



8.2 Angular Velocity and Angular Acceleration

Case 1: Constant angular velocity, ω.



$$\omega = \frac{\Delta\theta}{\Delta t} \qquad \Delta\theta = \omega\,\Delta t$$

Example: A disk rotates with a constant angular velocity of +1 rad/s. What is the angular displacement of the disk in 13 seconds? How many rotations has the disk made in that time?

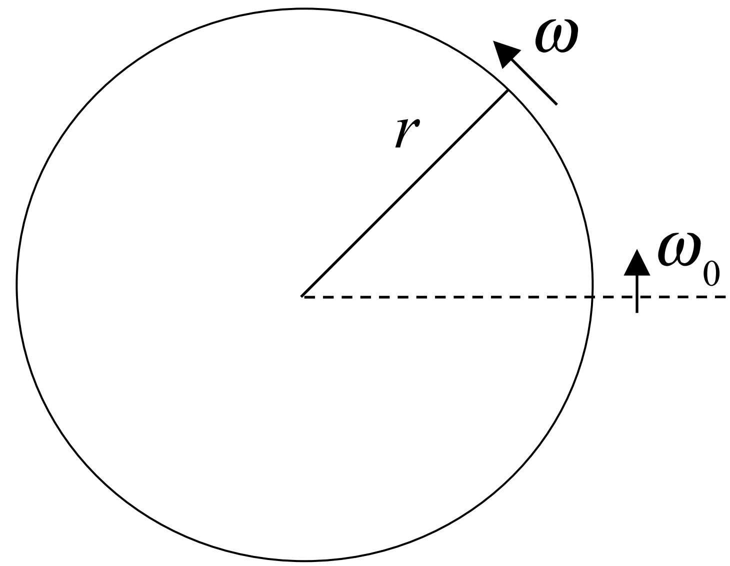
$$\Delta\theta = \omega\,\Delta t = (+1 \text{ rad/s})(13 \text{ s}) = +13 \text{ rad}$$

$$2\pi \text{ radians} = 1 \text{ rotation} \Rightarrow 2\pi \text{ rad/rot.}$$

$$n_{rot} = \frac{\Delta\theta}{2\pi \text{ rad/rot.}} = \frac{13 \text{ rad}}{6.3 \text{ rad/rot}} = 2.1 \text{ rot.}$$

8.2 Angular Velocity and Angular Acceleration

Case 2: Angular velocity, ω, changes in time.



Instantaneous angular velocity at time t.

$$\omega = \lim_{\Delta t = 0} \frac{\Delta \theta}{\Delta t}$$

DEFINITION OF AVERAGE ANGULAR ACCELERATION

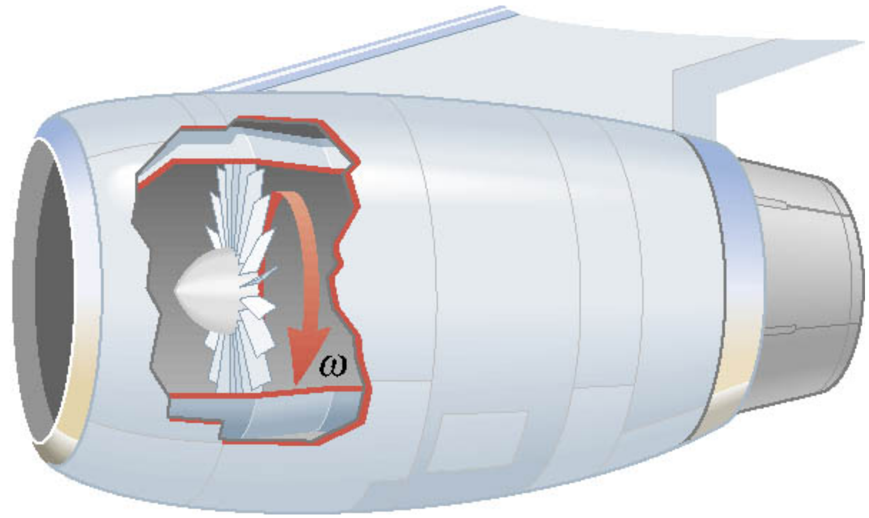
$$\text{Average angular acceleration} = \frac{\text{Change in angular velocity}}{\text{Elapsed time}}$$

$$\overline{\alpha} = \frac{\omega - \omega_o}{t - t_o} = \frac{\Delta \omega}{\Delta t}$$

SI Unit of Angular acceleration: radian per second squared (rad/s^2)

Example 4 A Jet Revving Its Engines

As seen from the front of the engine, the fan blades are rotating with an angular speed of –110 rad/s. As the plane takes off, the angular velocity of the blades reaches –330 rad/s in a time of 14 s.



Rotation is clockwise (negative)

Find the angular acceleration, assuming it to be constant.

$$\bar{\alpha} = \frac{(-330\,\text{rad/s}) - (-110\,\text{rad/s})}{14\ \text{s}} = -16\,\text{rad/s}^2$$

8.3 *The Equations of Rotational Kinematics*

The equations of rotational kinematics for constant angular acceleration:

ANGULAR ACCELERATION

ANGULAR VELOCITY

$$\omega = \omega_o + \alpha t$$

$$\theta = \tfrac{1}{2}\left(\omega_o + \omega\right)t$$

TIME

ANGULAR DISPLACEMENT

$$\omega^2 = \omega_o^2 + 2\alpha\theta$$

$$\theta = \omega_o t + \tfrac{1}{2}\alpha t^2$$

8.3 The Equations of Rotational Kinematics

Table 8.2 Symbols Used in Rotational and Linear Kinematics

Rotational Motion	Quantity	Linear Motion
θ	Displacement	x
ω_0	Initial velocity	v_0
ω	Final velocity	v
α	Acceleration	a
t	Time	t

Table 8.1 The Equations of Kinematics for Rotational and Linear Motion

Rotational Motion (α = constant)		Linear Motion (a = constant)	
$\omega = \omega_0 + \alpha t$	(8.4)	$v = v_0 + at$	(2.4)
$\theta = \frac{1}{2}(\omega_0 + \omega)t$	(8.6)	$x = \frac{1}{2}(v_0 + v)t$	(2.7)
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	(8.7)	$x = v_0 t + \frac{1}{2}at^2$	(2.8)
$\omega^2 = \omega_0^2 + 2\alpha\theta$	(8.8)	$v^2 = v_0^2 + 2ax$	(2.9)

Reasoning Strategy

1. Make a drawing.

2. Decide which directions are to be called positive (+) and negative (–).

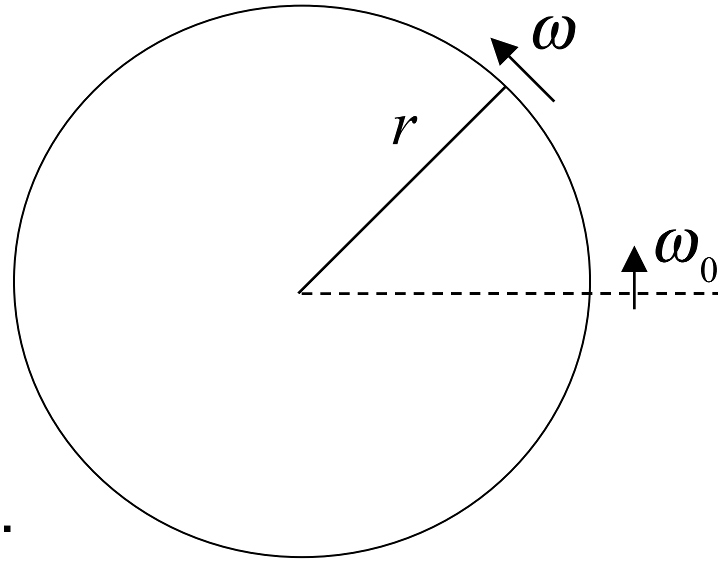
3. Write down the values that are given for any of the five kinematic variables.

4. Verify that the information contains values for at least three of the five kinematic variables. Select the appropriate equation.

5. When the motion is divided into segments, remember that the final angular velocity of one segment is the initial angular velocity for the next.

6. Keep in mind that there may be two possible answers to a kinematics problem.

Example: A disk has an initial angular velocity of +375 rad/s. The disk accelerates and reaches a greater angular velocity after rotating through an angular displacement of +44.0 rad.



If the angular acceleration has a constant value of +1740 rad/s^2, find the final angular velocity of the disk.

$$\text{Given: } \omega_0 = +375 \text{ rad/s}, \theta = +44 \text{ rad}, \alpha = 1740 \text{ rad/s}^2$$

Want: final angular velocity, ω.

No time!

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$= (375 \text{ rad/s})^2 + 2(1740 \text{ rad/s}^2)(+44 \text{ rad})$$

$$\omega = 542 \text{ rad/s}$$

8.4 Angular Variables and Tangential Variables

$\omega =$ angular velocity is the same at all radii

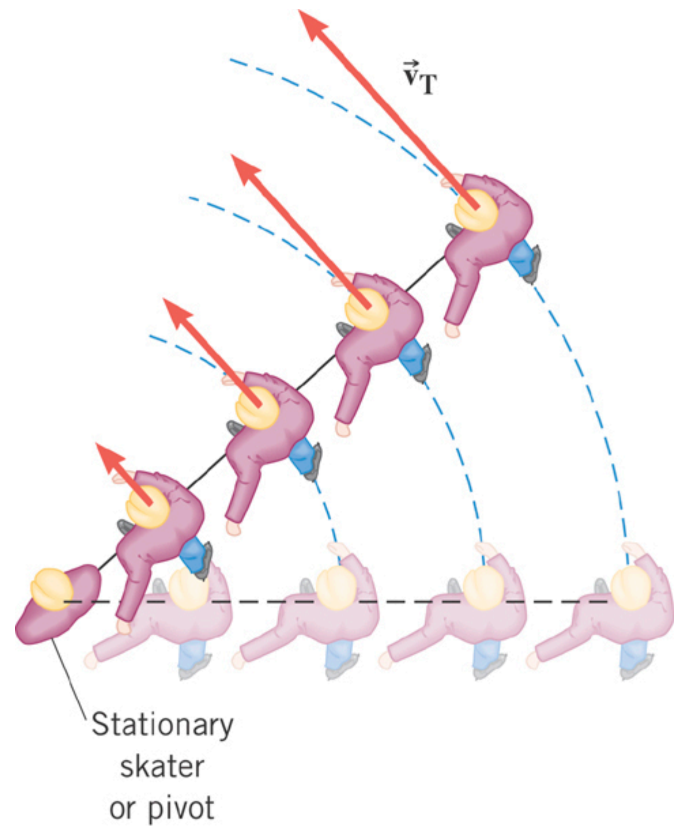
$\alpha_T =$ angular acceleration is the same at all radii

$\vec{\mathbf{v}}_{\mathbf{T}} =$ tangential velocity is different at each radius

$\mathbf{a}_T =$ tangential acceleration is different at each radius

$$\mathbf{v}_T = \omega r \qquad \mathbf{a}_T = \alpha r$$

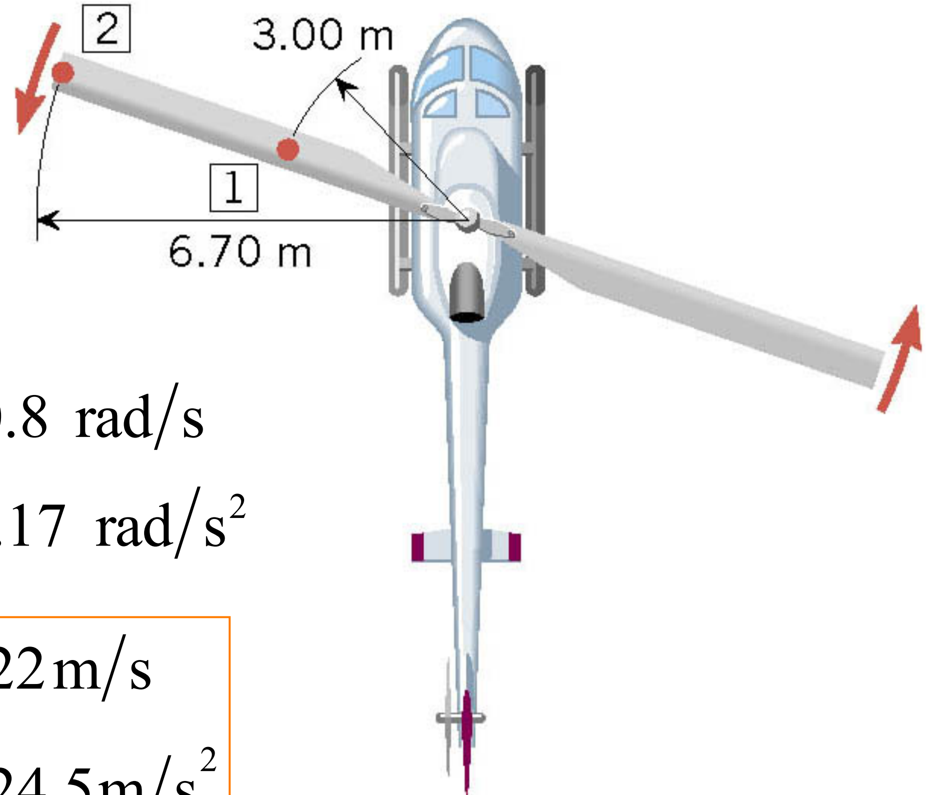
$\mathbf{v}_T$ (m/s)	$\mathbf{a}_T$ (m/s^2)
ω (rad/s)	α (rad/s^2)
r (m)	r (m)



Example 6 A Helicopter Blade

A helicopter blade has an angular speed of 6.50 rev/s and an angular acceleration of 1.30 rev/s^2.

For point 1 on the blade, find the magnitude of (a) the tangential speed and (b) the tangential acceleration.



Convert revolutions to radians

$$\omega = \left(6.50\ \text{rev/s}\right)\left(2\pi\ \text{rad/rev}\right) = 40.8\ \text{rad/s}$$

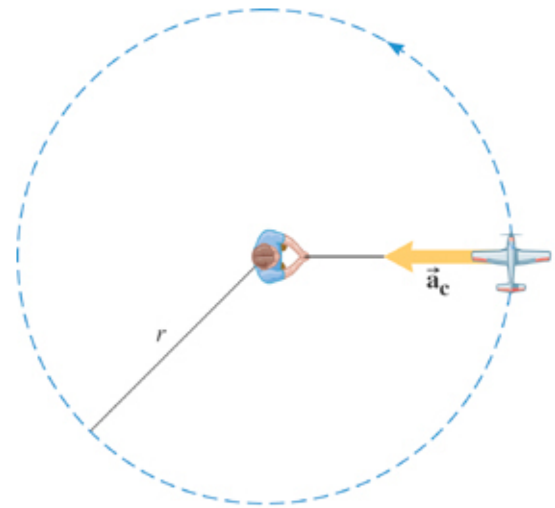
$$\alpha = \left(1.30\ \text{rev/s}^2\right)\left(2\pi\ \text{rad/rev}\right) = 8.17\ \text{rad/s}^2$$

$$\boxed{\begin{aligned} v_T &= \omega r = \left(40.8\,\text{rad/s}\right)\left(3.00\ \text{m}\right) = 122\,\text{m/s} \\ a_T &= \alpha r = \left(8.17\,\text{rad/s}^2\right)\left(3.00\ \text{m}\right) = 24.5\,\text{m/s}^2 \end{aligned}}$$

8.5 Centripetal Acceleration and Tangential Acceleration

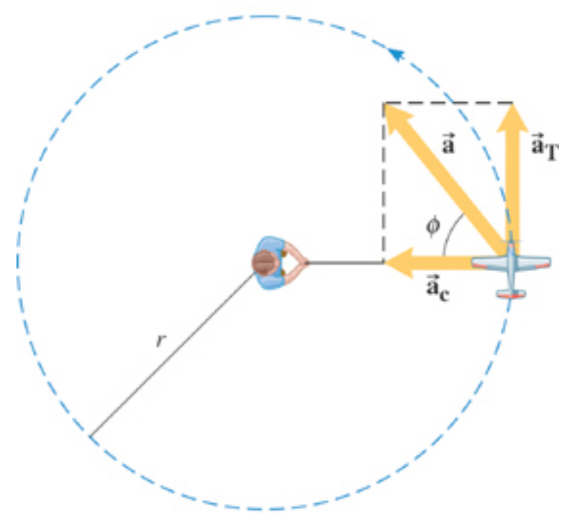
$$a_c = \frac{v_T^2}{r} \quad \text{(Chapter 5)}$$

$$= \frac{(\omega r)^2}{r} = \omega^2 r \quad (\omega \text{ in rad/s constant})$$



(a) Uniform circular motion

$$a_T = \alpha r$$



(b) Nonuniform circular motion

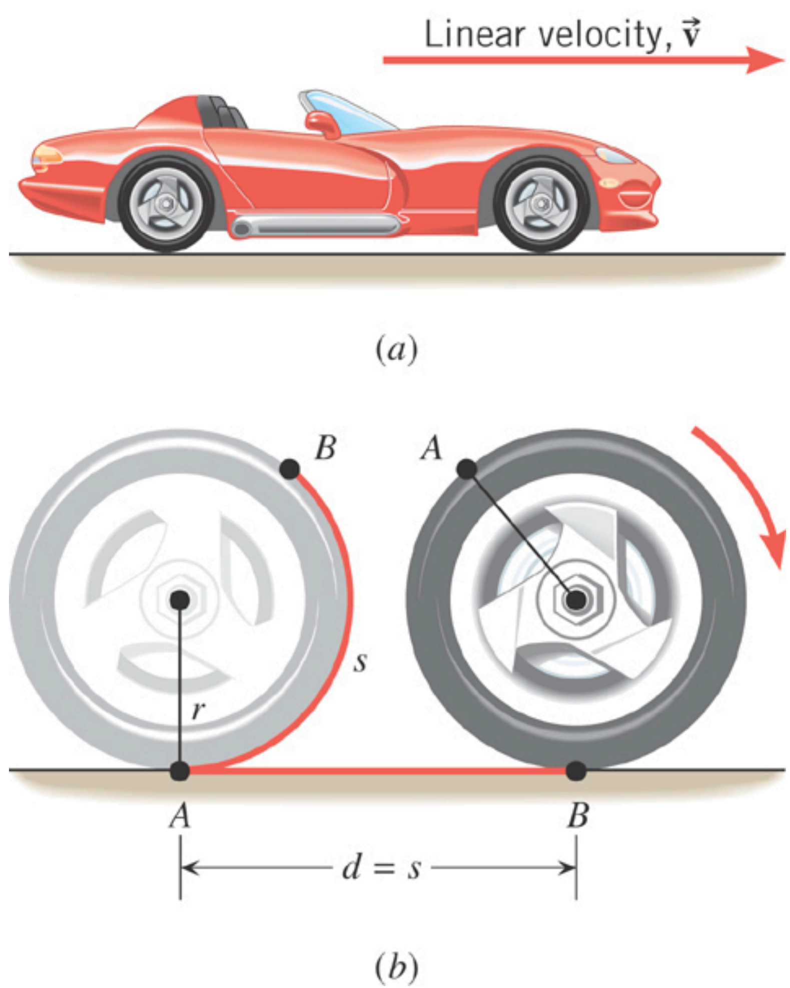
$$a_{total} = \sqrt{a_c^2 + \alpha^2 r^2}$$

8.6 Rolling Motion

The tangential speed of a point on the outer edge of the tire is equal to the speed of the car over the ground.

$$v = v_T = \omega r$$

$$a = a_T = \alpha r$$



8.7 The Vector Nature of Angular Variables

Right-Hand Rule: Grasp the axis of rotation with your right hand, so that your fingers circle the axis in the same sense as the rotation.

Your extended thumb points along the axis in the direction of the angular velocity.