Chapter 9

Rotational Dynamics

continued

9.4 Newton's Second Law for Rotational Motion About a Fixed Axis

ROTATIONAL ANALOG OF NEWTON'S SECOND LAW FOR A RIGID BODY ROTATING ABOUT A FIXED AXIS

$$\text{Net external torque} = \begin{pmatrix} \text{Moment of} \\ \text{inertia} \end{pmatrix} \times \begin{pmatrix} \text{Angular} \\ \text{acceleration} \end{pmatrix}$$

$$\boxed{\sum \tau = I\,\alpha}$$

$$\boxed{I = \sum \left(m r^2 \right)}$$

Point masses

Requirement: Angular acceleration must be expressed in radians/s^2.

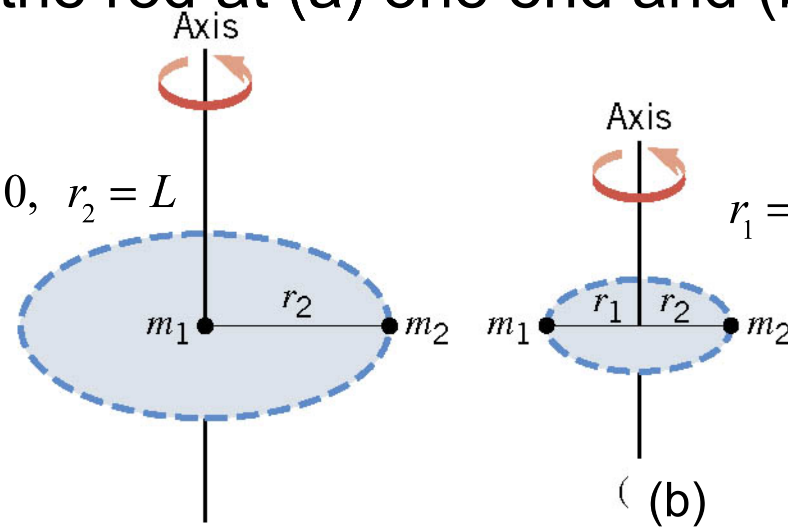
Moment of Inertia depends on axis of rotation.

Two particles each with mass, m, and are fixed at the ends of a thin rigid rod. The length of the rod is L. Find the moment of inertia when this object rotates relative to an axis that is perpendicular to the rod at (a) one end and (b) the center.

$m_1 = m_2 = m$

$r_1 = 0, \quad r_2 = L$

$r_1 = \frac{L}{2}, \quad r_2 = \frac{L}{2}$

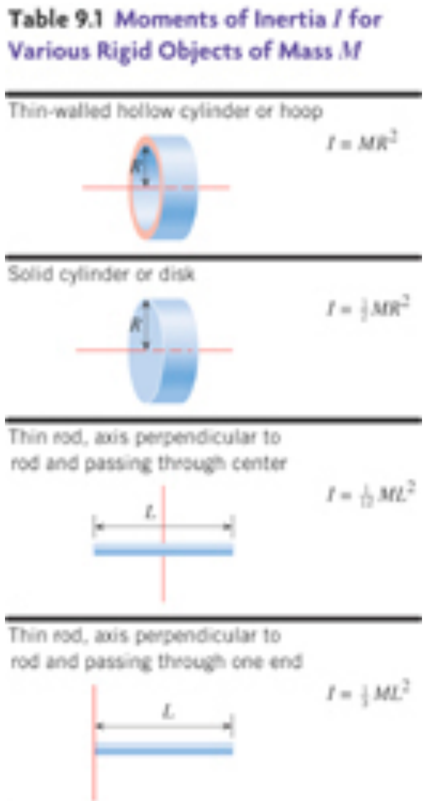


(a)

(a) $$I = \sum\left(mr^2\right) = m_1 r_1^2 + m_2 r_2^2$$
$$= m(0)^2 + m(L)^2$$
$$= mL^2$$

(b) $$I = \sum\left(mr^2\right) = m_1 r_1^2 + m_2 r_2^2$$
$$= m\left(L/2\right)^2 + m\left(L/2\right)^2$$
$$= \tfrac{1}{2} mL^2$$

9.4 Newton's Second Law for Rotational Motion About a Fixed Axis

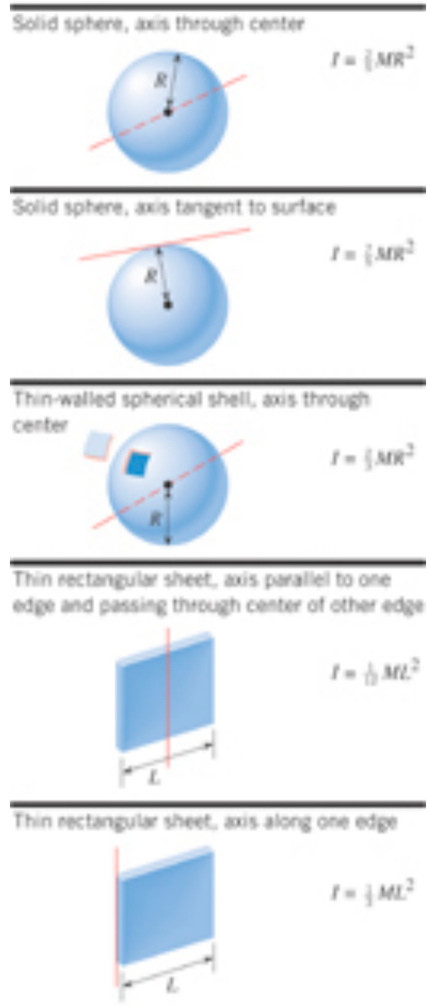


$$I = MR^2$$

$$I = \tfrac{1}{2}MR^2$$

$$I = \tfrac{1}{12}MR^2$$

$$I = \tfrac{1}{3}MR^2$$



$$I = \tfrac{2}{5}MR^2$$

$$I = \tfrac{7}{5}MR^2$$

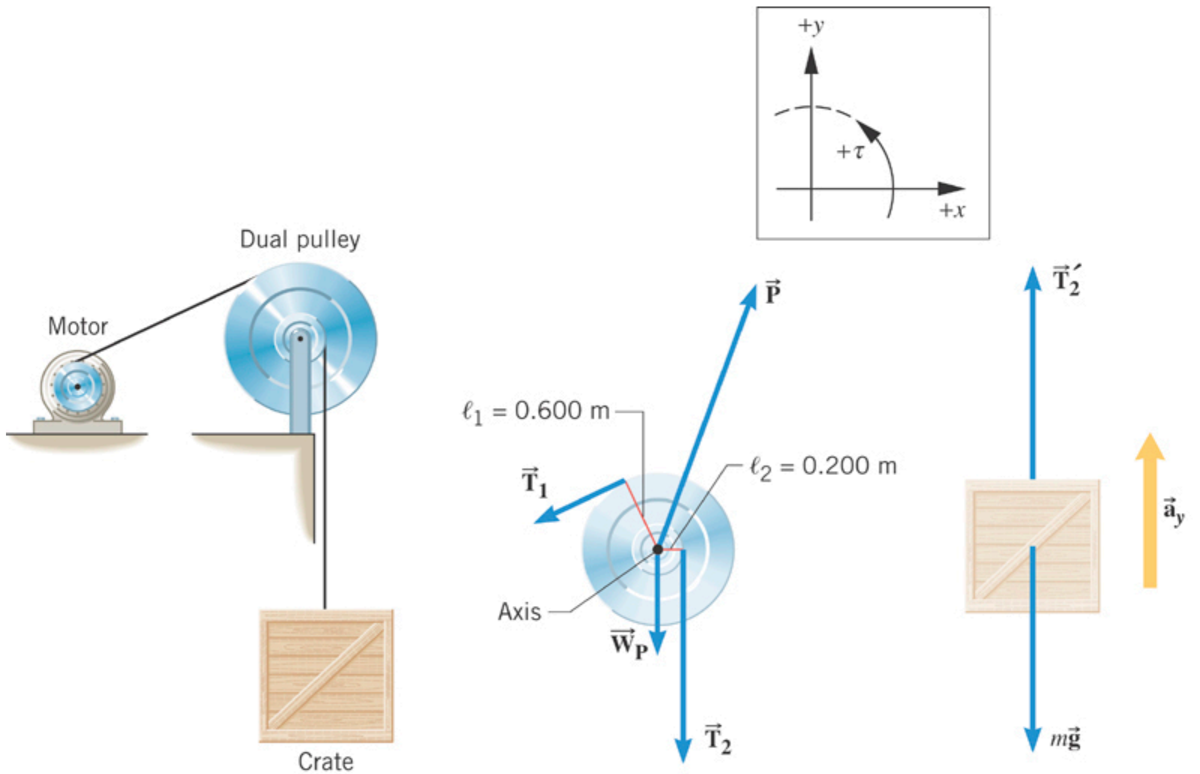
$$I = \tfrac{2}{3}MR^2$$

$$I = \tfrac{1}{12}MR^2$$

$$I = \tfrac{1}{3}MR^2$$

Example 12 Hoisting a Crate

The combined moment of inertia of the dual pulley is 50.0 kg·m^2. The crate weighs 4420 N. A tension of 2150 N is maintained in the cable attached to the motor. Find the angular acceleration of the dual pulley.



(*a*) (*b*) Free-body diagram of pulley (*c*) Free-body diagram of crate

9.4 Newton's Second Law for Rotational Motion About a Fixed Axis

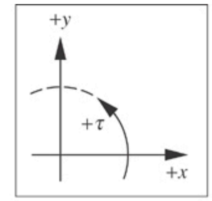
2nd law for linear motion of crate

$$\sum F_y = T_2 - mg = ma_y \qquad a_y = \ell_2 \alpha$$

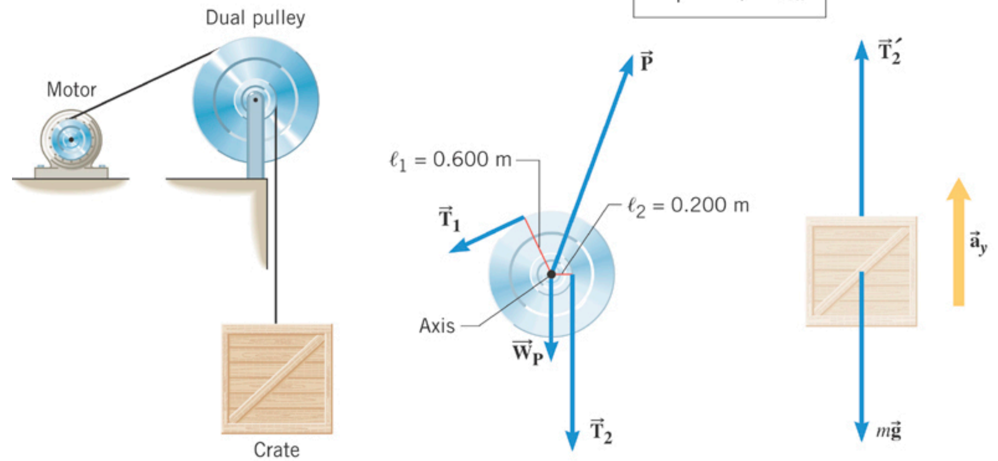
$$\boxed{\begin{aligned} I &= 46\,\text{kg}\cdot\text{m}^2 \\ mg &= 4420\ \text{N} \\ T_1 &= 2150\ \text{N} \end{aligned}}$$

2nd law for rotation of the pulley

$$\sum \tau = T_1 \ell_1 - T_2 \ell_2 = I\alpha$$



$$\boxed{\begin{aligned} T_2 - mg &= m\ell_2 \alpha \\ T_1\ell_1 - T_2\ell_2 &= I\alpha \\ T_2 &= m\ell_2\alpha + mg \\ T_2 &= \frac{T_1\ell_1 - I\alpha}{\ell_2} \\ m\ell_2^2\alpha + I\alpha &= T_1\ell_1 - mg\ell_2 \\ \alpha &= \frac{T_1\ell_1 - mg\ell_2}{m\ell_2^2 + I} = 6.3\ \text{rad/s}^2 \end{aligned}}$$



(a) (b) Free-body diagram of pulley (c) Free-body diagram of crate

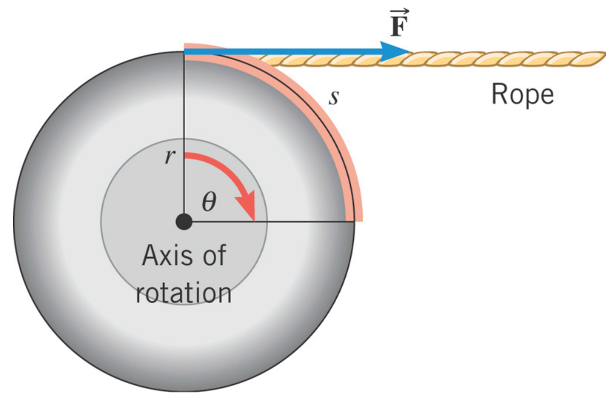
$$\boxed{\begin{aligned} \vec{T}_2' &= +T_2 \\ \vec{T}_2 &= -T_2 \end{aligned}}$$

$$\boxed{T_2 = m\ell_2\alpha + mg = \big(451(.6)(6.3) + 4420\big)\text{N} = 6125\ \text{N}}$$

9.5 Rotational Work and Energy

Work rotating a mass:

$$W = Fs \qquad s = r\theta$$
$$= Fr\theta \qquad Fr = \tau$$
$$= \tau\theta$$



DEFINITION OF ROTATIONAL WORK

The rotational work done by a constant torque in turning an object through an angle is

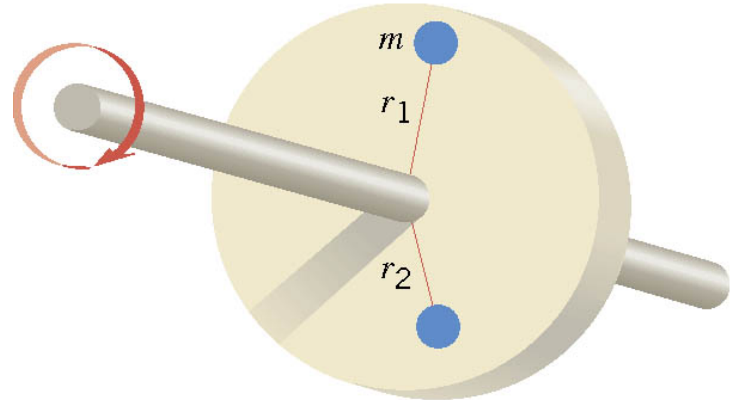
$$\boxed{W_R = \tau\theta}$$

Requirement: The angle must be expressed in radians.

SI Unit of Rotational Work: joule (J)

9.5 Rotational Work and Energy

Kinetic Energy of a rotating one point mass



$$KE = \tfrac{1}{2} m v_T^2 \qquad v_T = r\omega$$

$$= \tfrac{1}{2} m r^2 \omega^2$$

Kinetic Energy of many rotating point masses

$$KE = \sum \left(\tfrac{1}{2} m r^2 \omega^2\right) = \tfrac{1}{2}\left(\sum m r^2\right)\omega^2 = \tfrac{1}{2} I \omega^2$$

DEFINITION OF ROTATIONAL KINETIC ENERGY

The rotational kinetic energy of a rigid rotating object is

$$\boxed{KE_R = \tfrac{1}{2} I \omega^2}$$

Requirement: The angular speed must be expressed in rad/s.

SI Unit of Rotational Kinetic Energy: joule (J)

Clicker Question 9.3 Rotational Energy

A bowling ball is rolling without slipping at constant speed toward the pins on a lane. What percentage of the ball's total kinetic energy is translational kinetic energy?

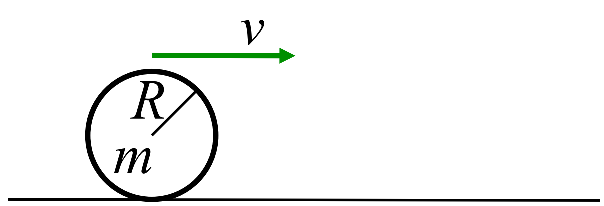
a) 50%

b) 71%

c) 46%

d) 29%

e) 33%



Hint: Express both KE types with $\omega^2 R^2$

$$KE = \tfrac{1}{2} m v^2, \text{ with } v = \omega R$$

$$KE_{rot} = \tfrac{1}{2} I \omega^2, \text{ with } I = \tfrac{2}{5} m R^2$$

$$\text{fraction} = \frac{KE}{KE + KE_{rot}}$$

Clicker Question 9.3 Rotational Energy

A bowling ball is rolling without slipping at constant speed toward the pins on a lane. What percentage of the ball's total kinetic energy is translational kinetic energy?

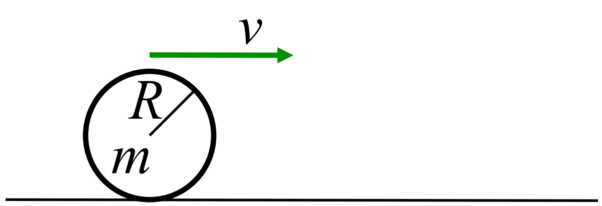
a) 50%

b) $\boxed{71\%}$

c) 46%

d) 29%

e) 33%



$$KE = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\omega^2R^2, \text{ because } v = \omega R$$

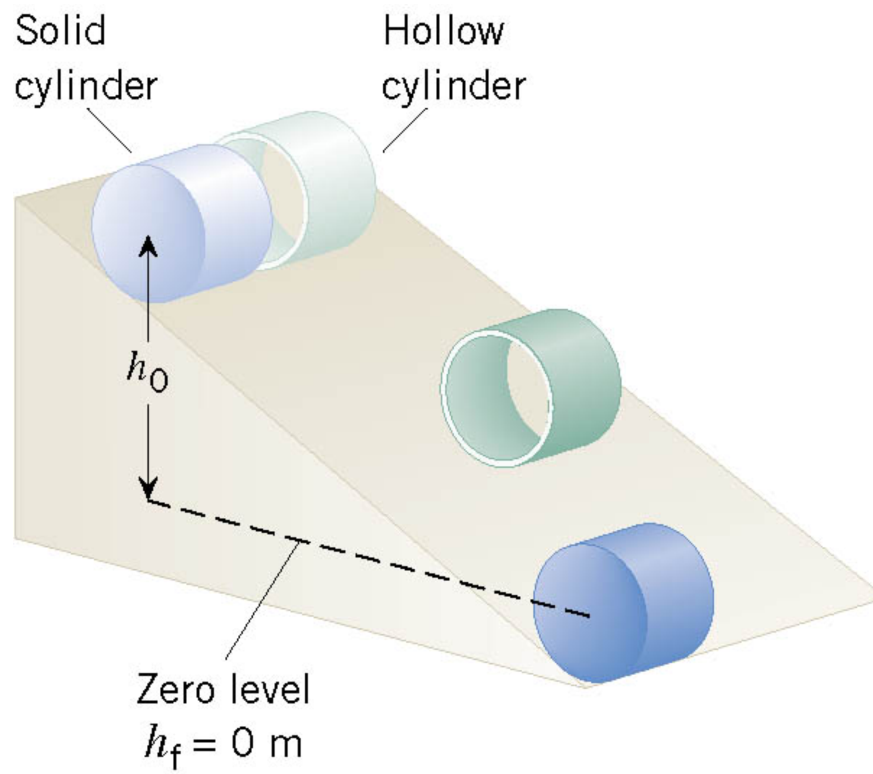
$$KE_{rot} = \tfrac{1}{2}I\omega^2 = \tfrac{1}{5}m\omega^2R^2, \text{ because } I = \tfrac{2}{5}mR^2$$

$$\frac{KE}{KE + KE_{rot}} = \frac{\frac{1}{2}}{\frac{1}{5}+\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{2}{10}+\frac{5}{10}} = \frac{5}{7} = 71\%$$

Example 13 Rolling Cylinders

A thin-walled hollow cylinder (mass = m_h, radius = r_h) and a solid cylinder (mass = m_s, radius = r_s) start from rest at the top of an incline.

Determine which cylinder has the greatest translational speed upon reaching the bottom.



9.5 *Rotational Work and Energy*

Total Energy = Kinetic Energy + Rotational Energy + Potential Energy

$$E = \tfrac{1}{2} m v^2 + \tfrac{1}{2} I \omega^2 + mgh$$

ENERGY CONSERVATION $E_f = E_0$

$$\tfrac{1}{2} m v_f^2 + \tfrac{1}{2} I \omega_f^2 + mgh_f = \tfrac{1}{2} m v_0^2 + \tfrac{1}{2} I \omega_0^2 + mgh_0$$

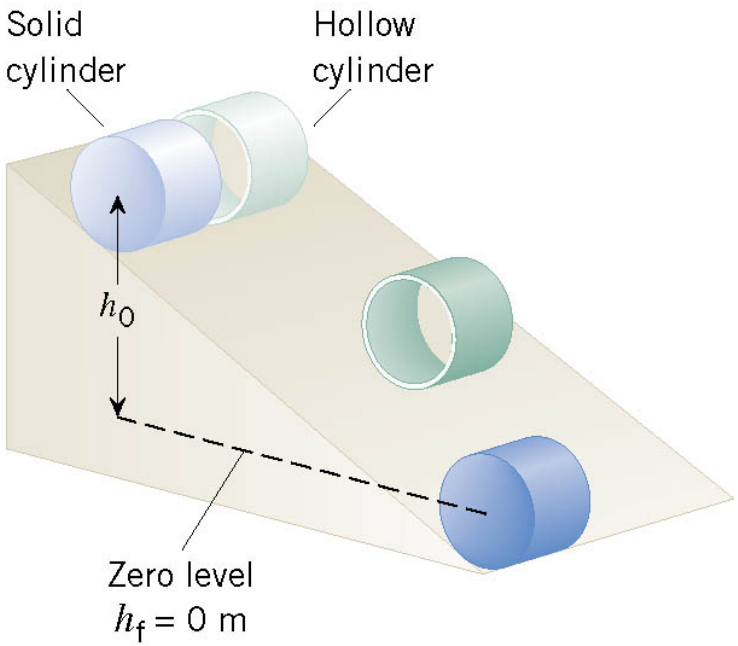
$$\boxed{\omega_f = v_f / r}$$

$$\tfrac{1}{2} m v_f^2 + \tfrac{1}{2} I v_f^2 / r^2 = mgh_0$$

$$v_f^2 (m + I/r^2) = 2mgh_0$$

$$\boxed{v_f = \sqrt{\frac{2mgh_o}{m + I/r^2}}}$$

Same mass for cylinder and hoop



The cylinder with the smaller moment of inertia will have a greater final translational speed.

DEFINITION OF ANGULAR MOMENTUM

The angular momentum *L* of a body rotating about a fixed axis is the product of the body's moment of inertia and its angular velocity with respect to that axis:

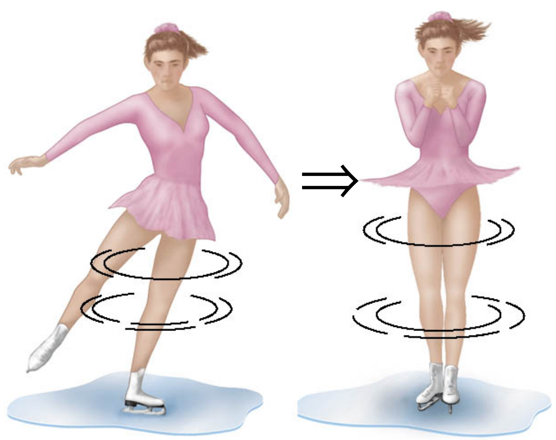
$$\boxed{L = I\omega}$$

Requirement: The angular speed must be expressed in rad/s.

SI Unit of Angular Momentum: $kg \cdot m^2/s$

PRINCIPLE OF CONSERVATION OF ANGULAR MOMENTUM

The angular momentum of a system remains constant (is conserved) if the net external torque acting on the system is zero.



Angular momentum, L

$$L_i = I_i\omega_i; \quad L_f = I_f\omega_f$$

No external torque

$\Rightarrow$ Angular momentum conserved

$$L_f = L_i$$

$$I_f\omega_f = I_i\omega_i$$

$$\omega_f = \frac{I_i}{I_f}\omega_i; \quad \frac{I_i}{I_f} > 1$$

$\omega_f > \omega_i$ (angular speed increases)

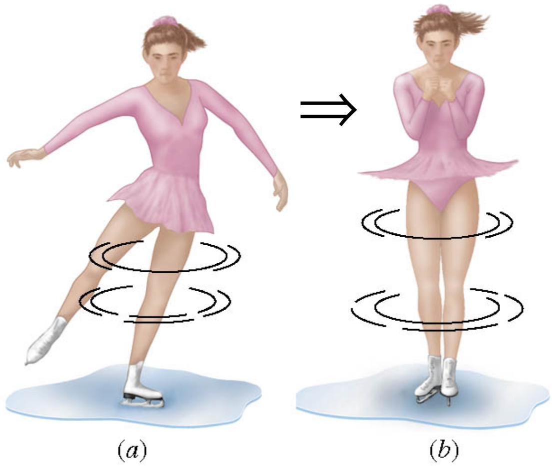
Moment of Inertia decreases

$$I = \sum mr^2, \ r_f < r_i$$

$$I_f < I_i$$

$$\frac{I_i}{I_f} > 1$$

9.6 Angular Momentum



From Angular Momentum Conservation

$$\omega_f = \left(I_i / I_f\right)\omega_i$$

because $I_i / I_f > 1$

Angular velocity increases

Is Energy conserved?

$$KE_f = \tfrac{1}{2} I_f \omega_f^2$$

$$= \tfrac{1}{2} I_f \left(I_i / I_f\right)^2 \omega_i^2$$

$$= \left(I_i / I_f\right)\left(\tfrac{1}{2} I_i \omega_i^2\right) \qquad KE_i = \tfrac{1}{2} I_i \omega_i^2;$$

$$= \left(I_i / I_f\right) KE_i \quad \Rightarrow \quad \text{Kinetic Energy increases}$$

Energy is not conserved because pulling in the arms does (NC) work on their mass and increases the kinetic energy of rotation

Rotational Dynamics Summary

linear	rotational
x	θ
v	ω
a	α
m	$I = mr^2$ (point m)
F	$\tau = Fr\sin\theta$
p	$L = I\omega$

linear	rotational
$\vec{\mathbf{F}} = m\vec{\mathbf{a}}$	$\vec{\tau} = I\vec{\alpha}$
$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$	$\vec{L} = I\vec{\omega}$
$W = Fs\cos\theta$	$W_{rot} = \tau\theta$
$KE = \frac{1}{2}mv^2$	$KE_{rot} = \frac{1}{2}I\omega^2$
$W \Rightarrow \Delta\mathrm{KE}$	$W_{rot} \Rightarrow \Delta\mathrm{KE}_{rot}$
$\vec{\mathbf{F}}\Delta t \Rightarrow \Delta\vec{\mathbf{p}}$	$\vec{\tau}\Delta t \Rightarrow \Delta\vec{L}$

Gravitational

$$PE_G = mgh$$

Conservation laws

Conserved:

$$\boxed{\text{If } W_{\mathrm{NC}} = 0,\quad KE + PE}$$

$$\boxed{\text{If } \mathbf{F}_{ext} = 0,\quad \mathbf{P} = \sum \mathbf{p}}$$

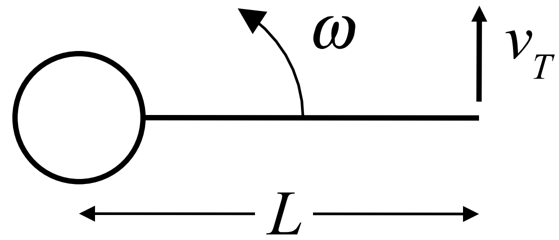
$$\boxed{\text{If } \tau_{ext} = 0,\quad L = I\omega}$$

8.37 Length of nylon rotates around one end. Rotation angular speed, 47 rev/s. Tip has tangential speed of 54 m/s . What is length of nylon string?

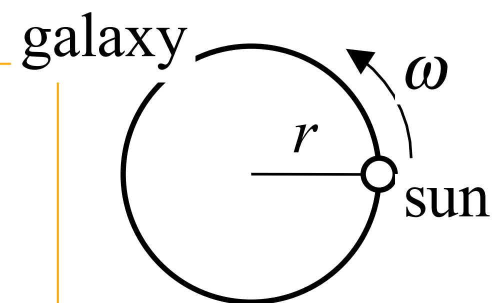
$$\omega = 47 \text{ rev/s} = 2\pi(47) \text{ rad/s} = 295 \text{ rad/s}$$

$$v_T = 54 \text{ m/s}, \; v_T = \omega r = \omega L$$

$$L = v_T / \omega = (54 \text{ m/s})/(295 \text{ rad/s}) = 0.18\text{m}$$



8.51 Sun moves in circular orbit, radius r, around the center of the galaxy at an angular speed, omega. a) tangential speed of sun? b) centripetal force?



$$r = (2.3\times10^4 \text{ l-yr})9.5\times10^{15}\text{m/l-yr} = 2.2\times10^{20}\text{m}$$

$$\omega = 1.1\times10^{-15} \text{ rad/s}, \quad m = 2\times10^{30}\text{kg}$$

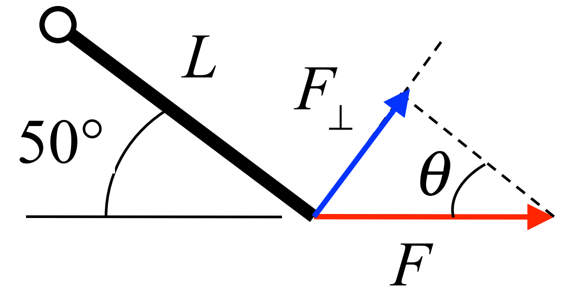
a) $v_T = \omega r = 2.4\times10^5 \text{ m/s}$

b) $F = ma_c = m(v_T^2/r) = m\omega^2 r$

$$= (2\times10^{30})(1.1\times10^{-15})^2(2.2\times10^{20}\text{m}) = 5.6\times10^{20}\text{N}$$

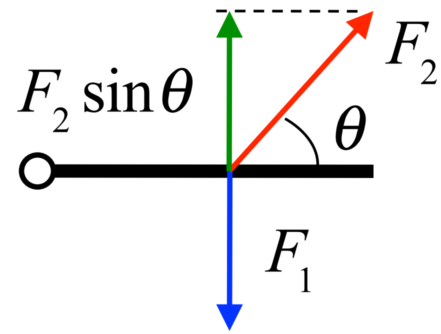
9.3 Wrench length L. Hand force F, to get torque 45 Nm?

$$\tau = F_{\perp} L = \left(F \sin\theta\right) L$$
$$F = \tau / L \sin\theta = 45\text{Nm}/(0.28\text{ m})\sin 50°$$
$$= 210\text{ N}$$

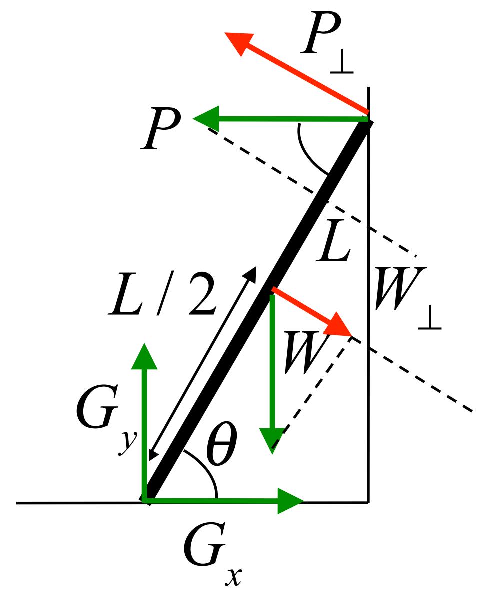


9.9 F_1=38N perpendicular, F_2= 55N at angle theta.
What is angle of F_2 for net torque = zero?

$$\tau_{Net} = \left(F_2 \sin\theta\right) L - F_1 L = 0$$
$$\sin\theta = F_1 / F_2 = 38/55 = 0.69$$
$$\theta = 43.7°$$



9.23 Board against wall, coefficient of friction of ground 0.650. What is smallest angle without slipping? (*See Example 4*)



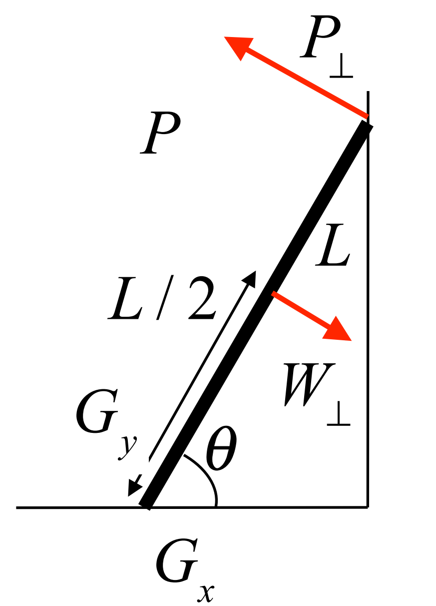
Forces:

$$G_y = W$$
$$P = G_x = \mu G_y = \mu W$$

Torque:

$$\tau_P = +P_\perp L = (P\sin\theta)L$$
$$\tau_W = -W_\perp (L/2) = -(W\cos\theta)(L/2)$$
$$\tau_W + \tau_P = 0 \Rightarrow (P\sin\theta) = (W\cos\theta)/2$$
$$W = 2P\sin\theta/\cos\theta$$

$$W = 2P\sin\theta/\cos\theta = 2\mu W\tan\theta$$
$$\tan\theta = 1/(2\mu) = 1/(1.3) = 0.77$$
$$\theta = 37.6^\circ$$



9.51 Energy 1.2 x 10^9 J . Flywheel M = 13 kg, R = 0.3m.

$$KE = \tfrac{1}{2} I\omega^2; \quad I_{disk} = \tfrac{1}{2} MR^2$$

$$= \tfrac{1}{2}\left(\tfrac{1}{2} MR^2\right)\omega^2$$

$$\omega^2 = \frac{4KE}{MR^2} = \frac{4.8\times 10^9\,\mathrm{J}}{(13\mathrm{kg})(0.3\mathrm{m})^2}$$

$$\omega = \sqrt{4.1\times 10^9} = 6.4\times 10^4\,\mathrm{rad/s}$$

$$= 6.4\times 10^4\,\mathrm{rad/s}\left(\mathrm{rev}/(2\pi)\mathrm{rad}\right)$$

$$= 1.02\times 10^4\ \mathrm{rev/s}\left(60\,\mathrm{s/min}\right) = 6.1\times 10^5\ \mathrm{rpm}$$