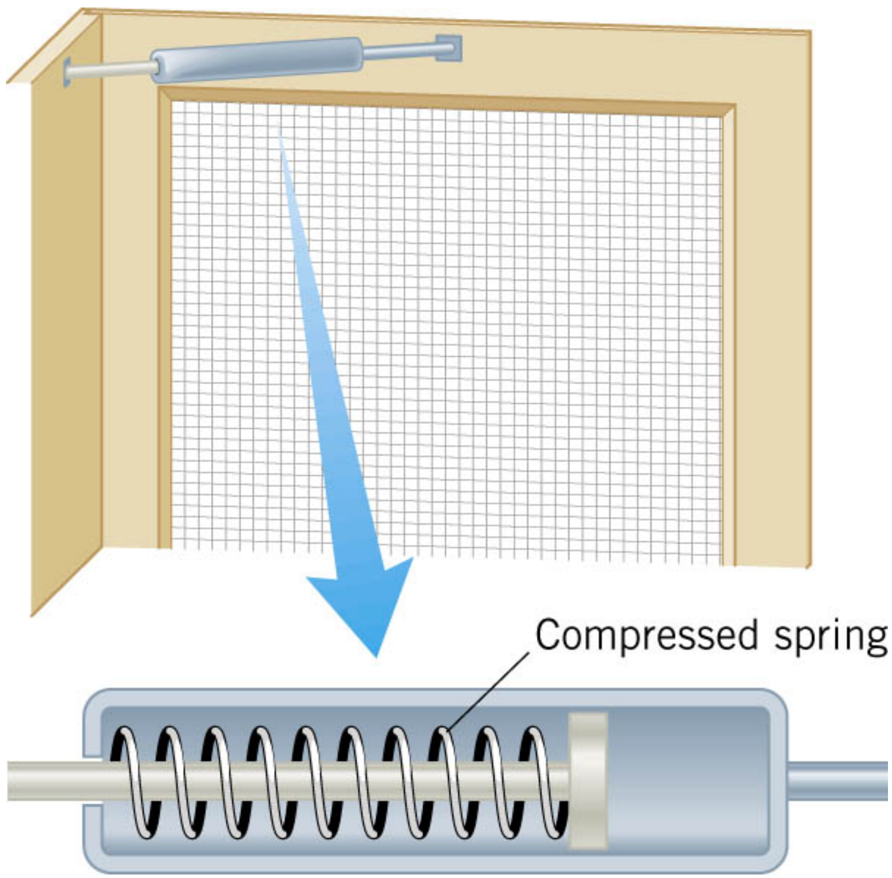
Chapter 10

Simple Harmonic Motion and Elasticity

continued

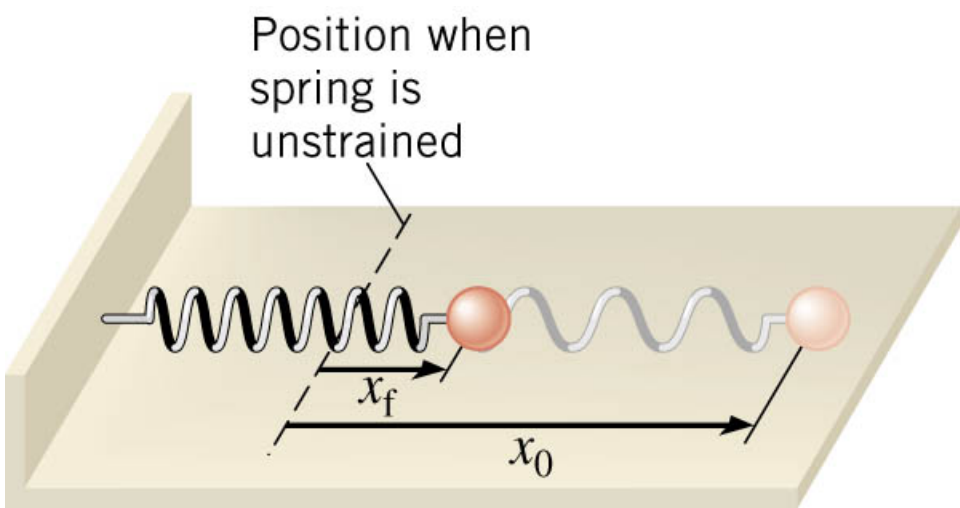
A compressed spring can do work.



A compressed spring can do work.

$$W_{\text{elastic}} = \left(\bar{F}\cos\theta\right)s = \tfrac{1}{2}\left(kx_o + kx_f\right)\cos 0^\circ\left(x_o - x_f\right)$$

$$W_{\text{elastic}} = \tfrac{1}{2}kx_o^2 - \tfrac{1}{2}kx_f^2$$

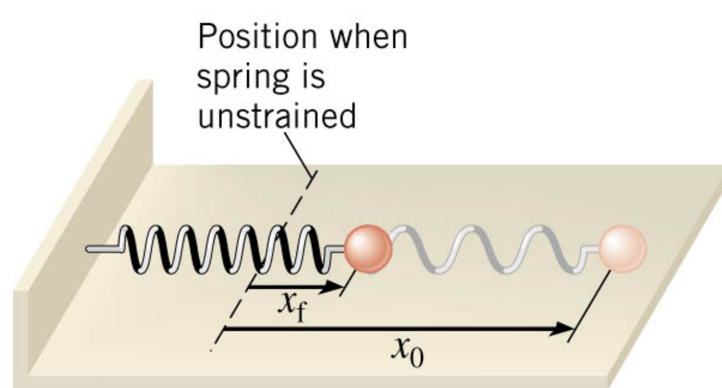


DEFINITION OF ELASTIC POTENTIAL ENERGY

The elastic potential energy is the energy that a spring has by virtue of being stretched or compressed. For an ideal spring, the elastic potential energy is

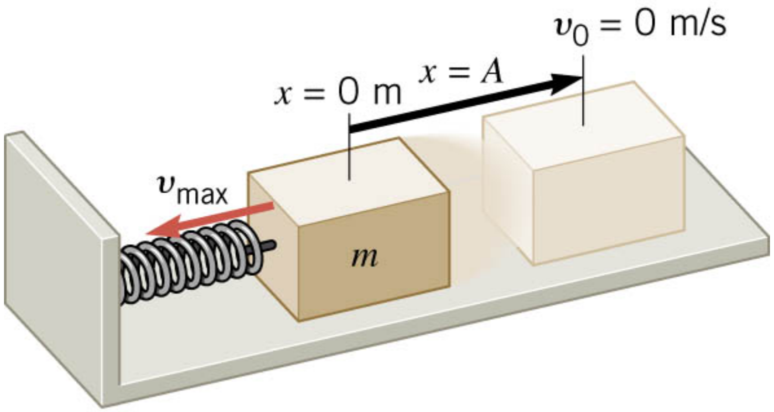
$$\mathrm{PE}_{\text{elastic}} = \tfrac{1}{2} k x^2$$

SI Unit of Elastic Potential Energy: joule (J)



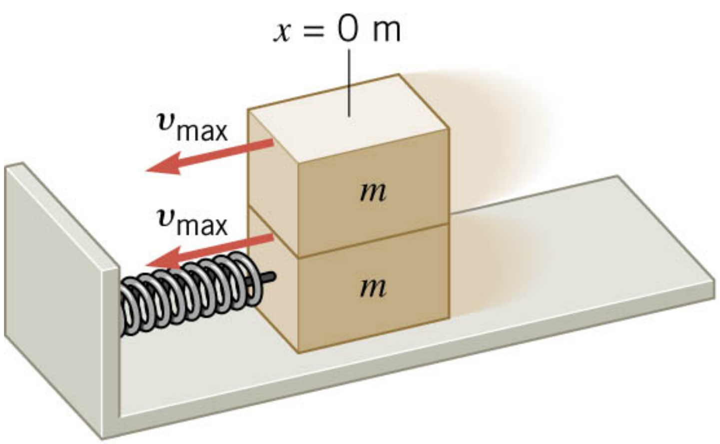
Conceptual Example 8 Changing the Mass of a Simple Harmonic Oscilator

The box rests on a horizontal, frictionless surface. The spring is stretched to x=A and released. When the box is passing through x=0, a second box of the same mass is attached to it. Discuss what happens to the (a) maximum speed (b) amplitude (c) angular frequency.



(a)

a) When 1st box reaches maximum velocity, second box added at the same velocity



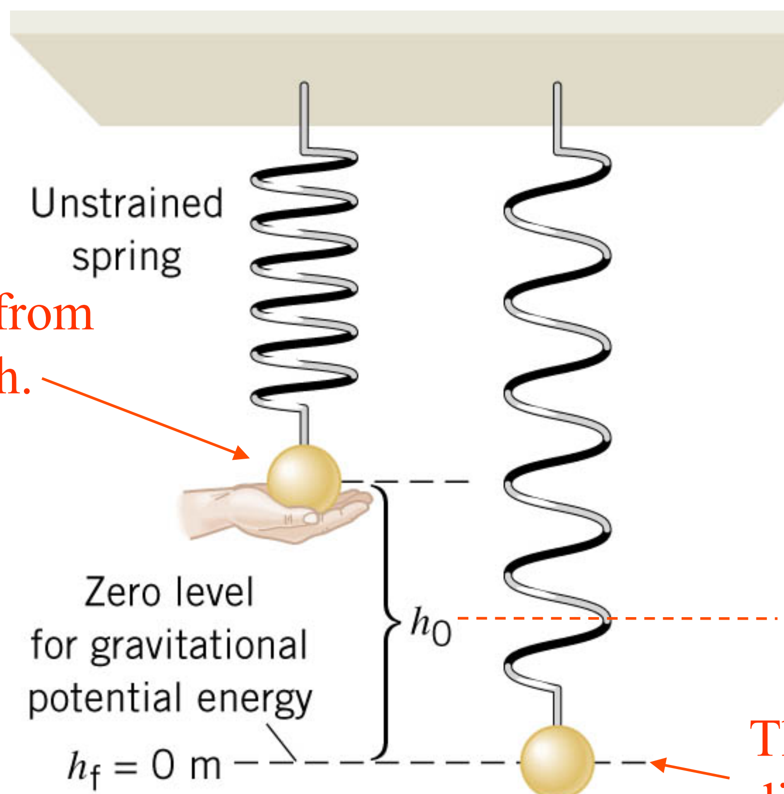
(b)

In homework, the mass is added when mass reaches maximum displacement, and velocity is zero.

Example 8 **Changing the Mass of a Simple Harmonic Oscilator**

A 0.20-kg ball is attached to a vertical spring. The spring constant is 28 N/m. When released from rest, how far does the ball fall before being brought to a momentary stop by the spring?

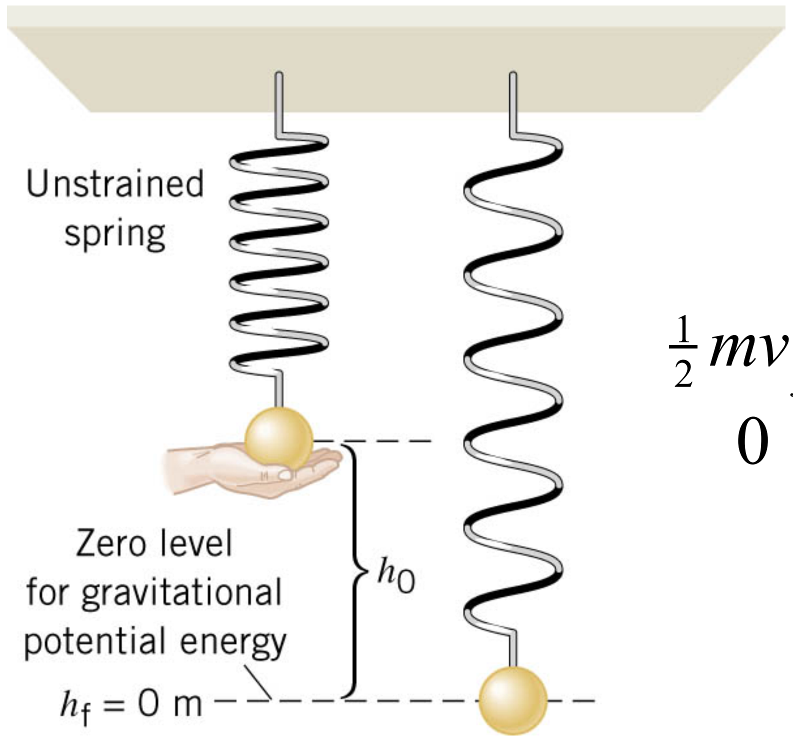
The ball is releasted from the unstretched length.



The ball reaches maximum speed at the equilibrium point.

The ball reaches maximum displacement when the velocity reaches zero.

After release, only conservative forces act.



Energy Conservation

$$E_f = E_o$$

$$\tfrac{1}{2}mv_f^2 + mgh_f + \tfrac{1}{2}ky_f^2 = \tfrac{1}{2}mv_o^2 + mgh_o + \tfrac{1}{2}ky_o^2$$

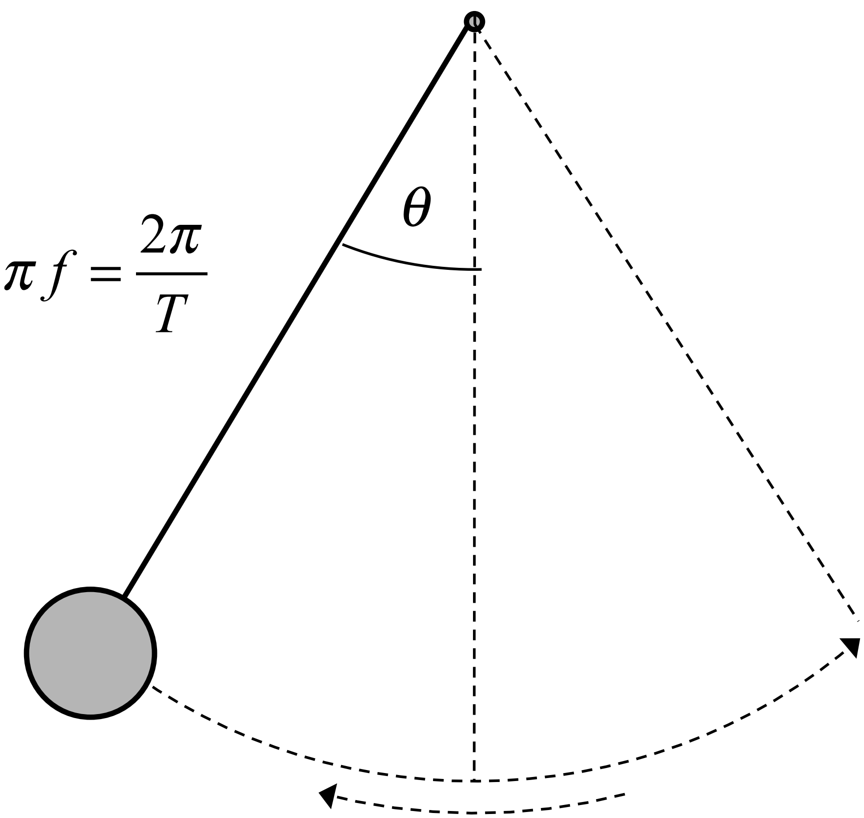
0 0 0 0

$$\tfrac{1}{2}kh_o^2 = mgh_o$$

CYU: Gravitational potential energy converted to elastic potential energy

$$h_o = \frac{2mg}{k} = \frac{2(0.20\text{ kg})(9.8\,\text{m/s}^2)}{28\,\text{N/m}} = 0.14\text{ m}$$

A ***simple pendulum*** consists of a particle attached to a frictionless pivot by a cable of negligible mass.



Angular frequency

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\omega = \sqrt{\frac{g}{L}} \qquad \text{(small angles only)}$$

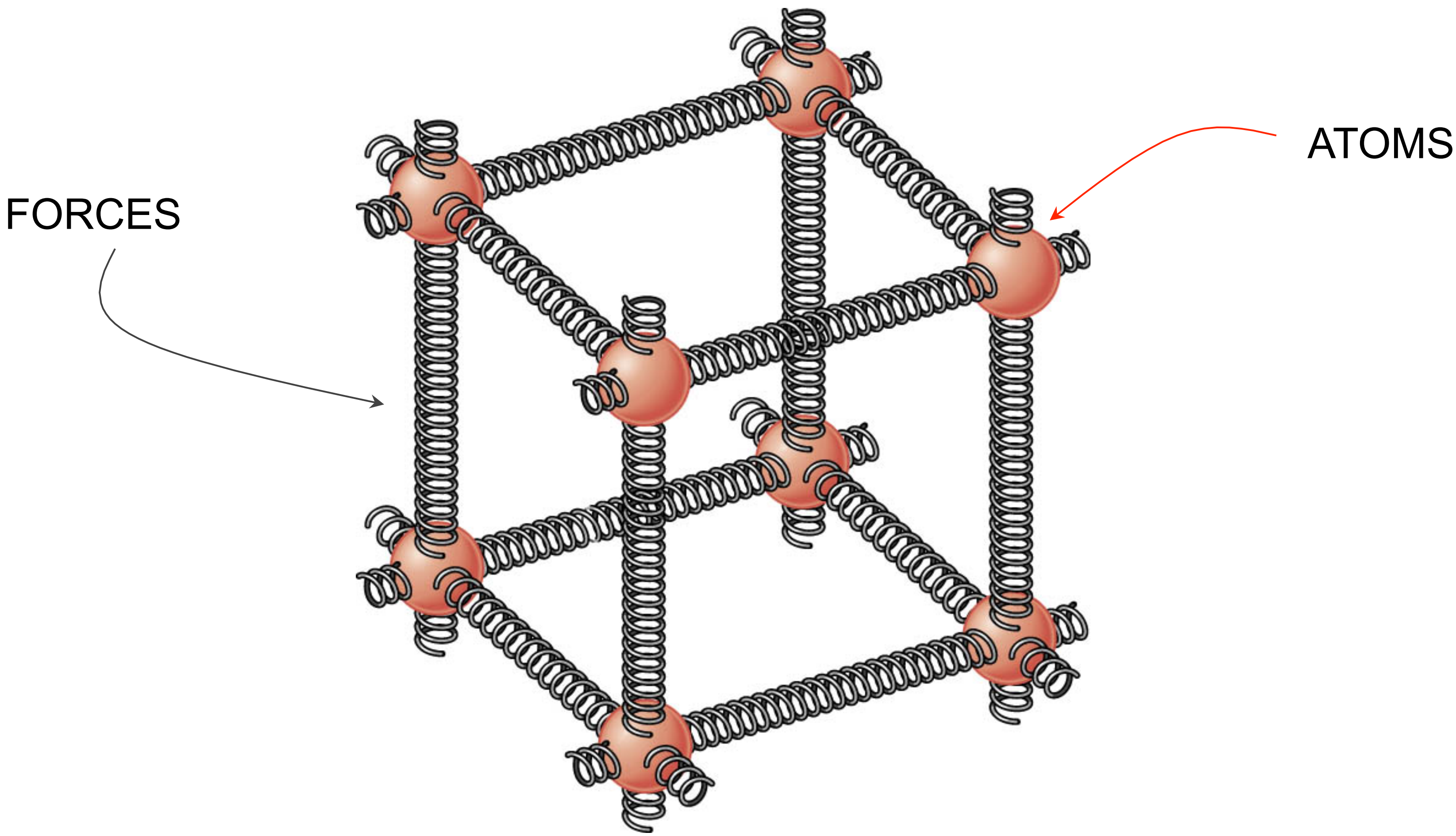
$$I = mL^2$$

$$\frac{1}{L} = \frac{mL}{I}$$

$$\omega = \sqrt{\frac{mgL}{I}} \qquad \text{(small angles only)}$$

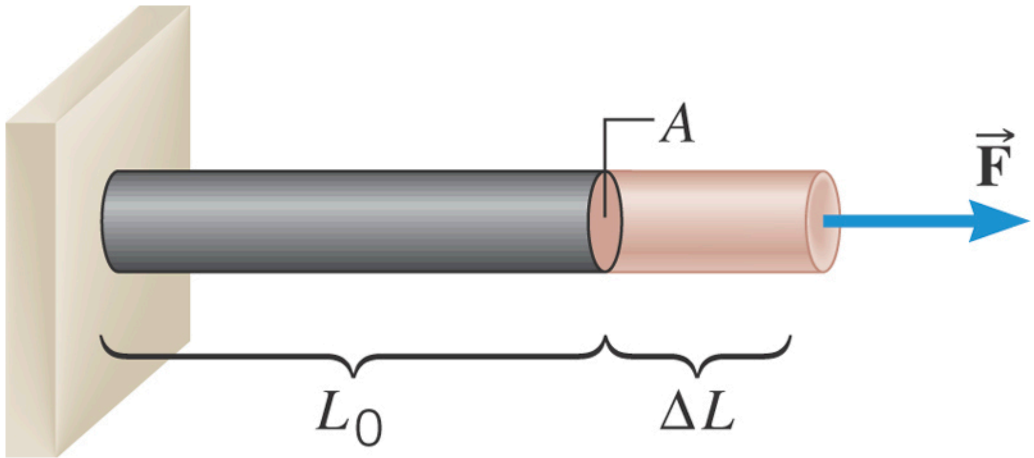
10.7 Elastic Deformation

Because of these atomic-level “springs”, a material tends to return to its initial shape once forces have been removed.



10.7 *Elastic Deformation*

STRETCHING, COMPRESSION, AND YOUNG'S MODULUS



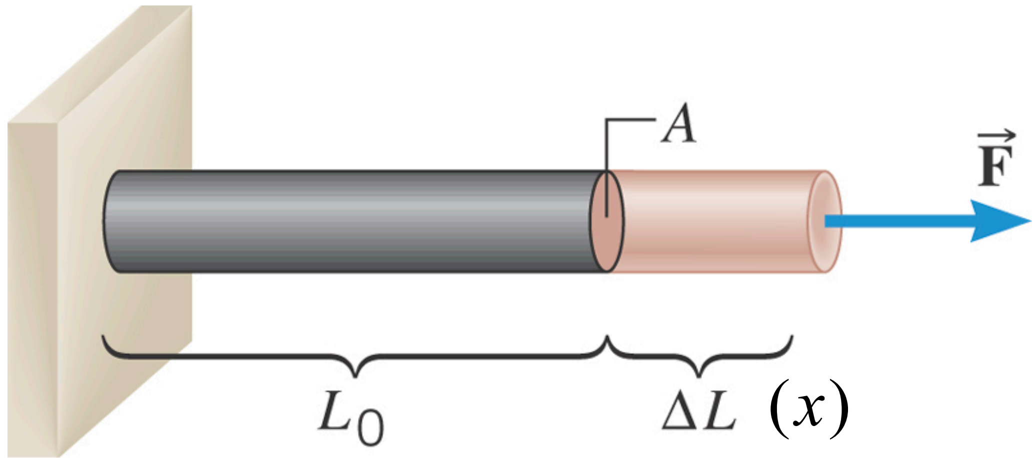
$$F = Y\left(\frac{\Delta L}{L_o}\right)A$$

Young's modulus has the units of pressure: N/m^2

Young's modulus is a characteristic of the material (see table 10.1)

$$Y_{\text{Steel}} = 2.0\times 10^{11}\ \text{N/m}^2$$

Spring Constants and Young's Modulus



Y : Young's Modulus

A, L_0 : Area and length of rod

ΔL : Change in rod length (x)

$$F = Y\left(\frac{\Delta L}{L_o}\right)A = \left(\frac{YA}{L_0}\right)\Delta L; \quad \text{let } \Delta L = x$$

$$= kx, \quad k = \left(\frac{YA}{L_0}\right)$$

10.7 *Elastic Deformation*

Table 10.1 Values for the Young's Modulus of Solid Materials

Material	Young's Modulus Y (N/m^2)
Aluminum	6.9×10^{10}
Bone	
Compression	9.4×10^{9}
Tension	1.6×10^{10}
Brass	9.0×10^{10}
Brick	1.4×10^{10}
Copper	1.1×10^{11}
Mohair	2.9×10^{9}
Nylon	3.7×10^{9}
Pyrex glass	6.2×10^{10}
Steel	2.0×10^{11}
Teflon	3.7×10^{8}
Titanium	1.2×10^{11}
Tungsten	3.6×10^{11}

Note: 1 Pascal (Pa) = 1 N/m^2

$$1 \text{ GPa} = 1\times10^{9}\ N/m^2$$

10.8 Stress, Strain, and Hooke's Law

In general the quantity *F/A* is called the **Stress**.

The change in the quantity divided by that quantity is called the **Strain**:

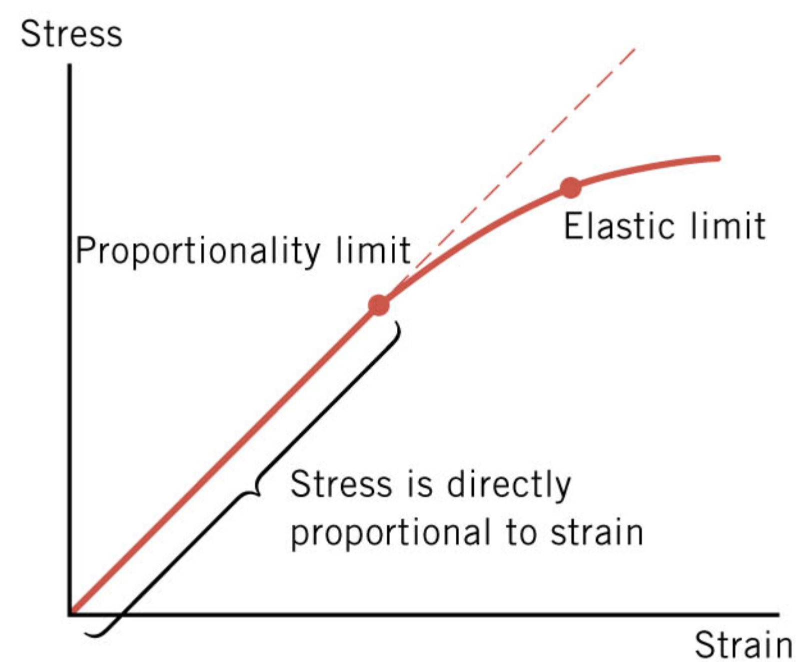
$$\Delta V/V_o \quad \Delta L/L_o \quad \Delta x/L_o$$

HOOKE'S LAW FOR STRESS AND STRAIN

Stress is directly proportional to strain.

Strain is a unitless quantitiy.

SI Unit of Stress: N/m^2



Example 12 Bone Compression

In a circus act, a performer supports the combined weight (1080 N) of a number of colleagues. Each thighbone of this performer has a length of 0.55 m and an effective cross sectional area of 7.7×10^{-4} m^2. Determine the amount that each thighbone compresses under the extra weight.

$$F = Y\left(\frac{\Delta L}{L_o}\right)A$$

$$\text{each leg} = \frac{1080\ \text{n}}{2}$$

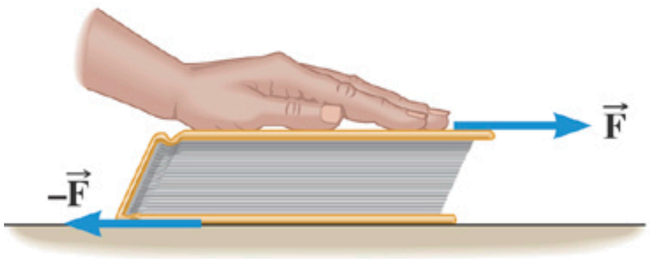
$$\Delta L = \frac{FL_o}{YA}$$

$$= \frac{(540\ \text{N})(0.55\ \text{m})}{(9.4\times10^{9}\ \text{N/m}^2)(7.7\times10^{-4}\ \text{m}^2)}$$

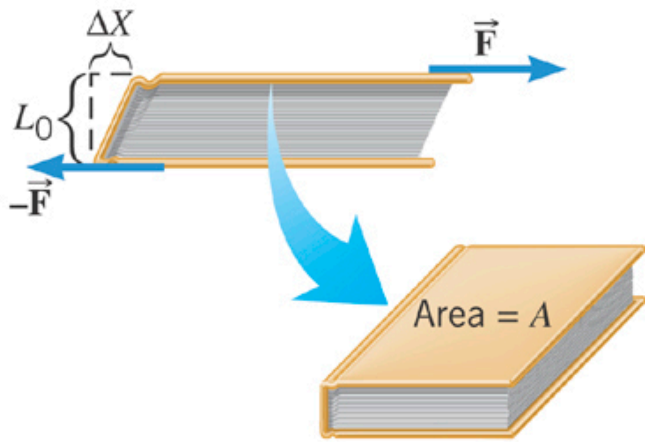
$$= 4.1\times10^{-5}\,\text{m} = 0.041\text{mm}$$

10.7 *Elastic Deformation*

SHEAR DEFORMATION AND THE SHEAR MODULUS



(a)

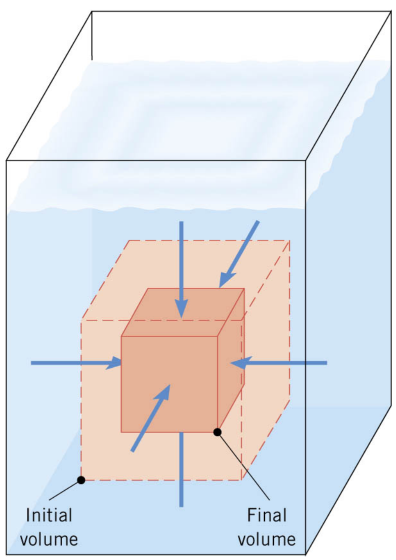


(b)

$$F = S\left(\frac{\Delta x}{L_o}\right)A$$

S: Shear modulus
Table 10.2

VOLUME DEFORMATION AND THE BULK MODULUS



Pressure Change

$$\Delta P = -B\left(\frac{\Delta V}{V_o}\right)$$

B: Bulk modulus
Table 10.3

Chapter 11

Fluids

11.1 Mass Density

DEFINITION OF MASS DENSITY

The mass density of a substance is the mass of a substance divided by its volume:

$$\rho = \frac{m}{V}$$

SI Unit of Mass Density: kg/m^3

Table 11.1 Mass Densities[a] of Common Substances

Substance	Mass Density ρ (kg/m^3)
Solids	
Aluminum	2700
Brass	8470
Concrete	2200
Copper	8890
Diamond	3520
Gold	19 300
Ice	917
Iron (steel)	7860
Lead	11 300
Quartz	2660
Silver	10 500
Wood (yellow pine)	550
Liquids	
Blood (whole, 37 °C)	1060
Ethyl alcohol	806
Mercury	13 600
Oil (hydraulic)	800
Water (4 °C)	1.000×10^3
Gases	
Air	1.29
Carbon dioxide	1.98
Helium	0.179
Hydrogen	0.0899
Nitrogen	1.25
Oxygen	1.43

[a] Unless otherwise noted, densities are given at 0 °C and 1 atm pressure.

Example 1 Blood as a Fraction of Body Weight

The body of a man whose weight is 690 N contains about $5.2\text{x}10^{-3}$ m^3 of blood.

(a) Find the blood's weight and (b) express it as a percentage of the body weight.

$$m = \rho V$$

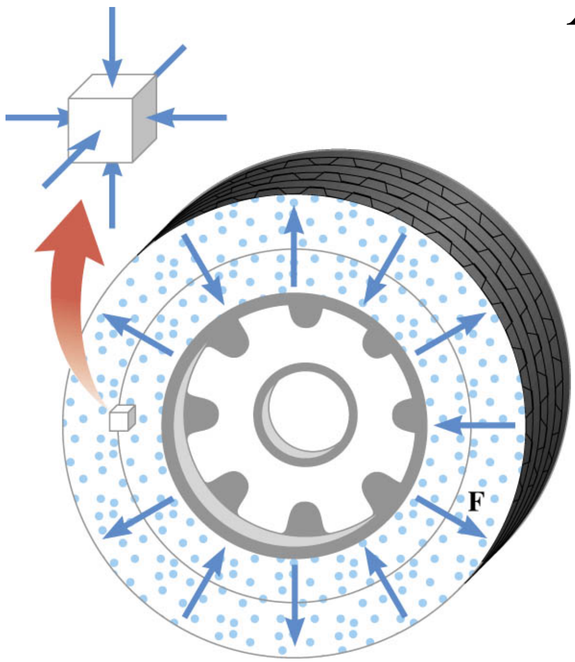
$$\text{(a) } W = mg = \rho V g = \left(1060 \text{ kg/m}^3\right)\left(5.2\times10^{-3} \text{ m}^3\right)\left(9.80\,\text{m/s}^2\right) = 54 \text{ N}$$

$$\text{(b) Percentage} = \frac{54 \text{ N}}{690 \text{ N}} \times 100\% = 7.8\%$$

11.2 Pressure

$$P = \frac{F}{A}$$

Pressure = Force per unit Area



The same pressure acts inward in every direction on a small volume.

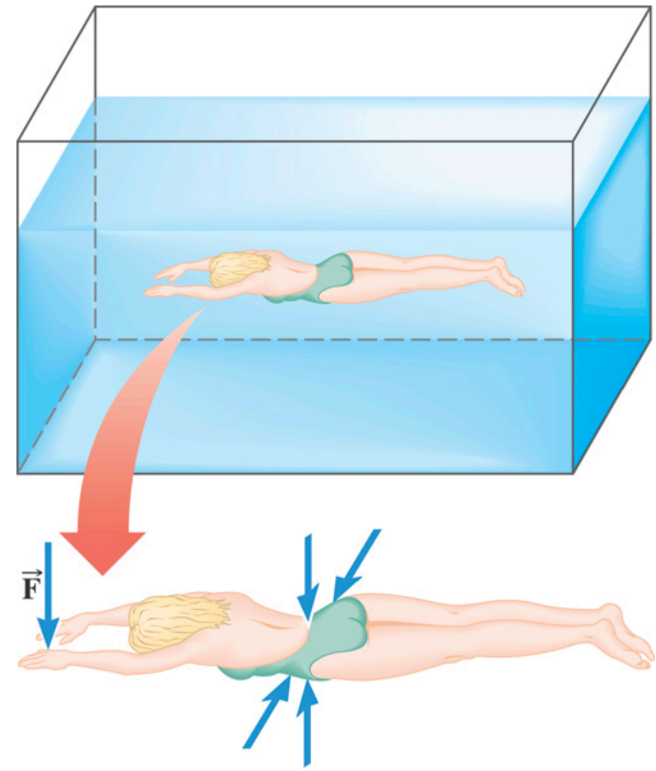
SI Unit of Pressure: $1\ N/m^2 = 1 Pa$

Pascal

Example 2 The Force on a Swimmer

Suppose the pressure acting on the back of a swimmer's hand is 1.2×10^{5} Pa. The surface area of the back of the hand is $8.4 \times 10^{-3} m^{2}$.

(a) Determine the magnitude of the force that acts on it.
(b) Discuss the direction of the force.



Force per unit area: $P = \dfrac{F}{A}$

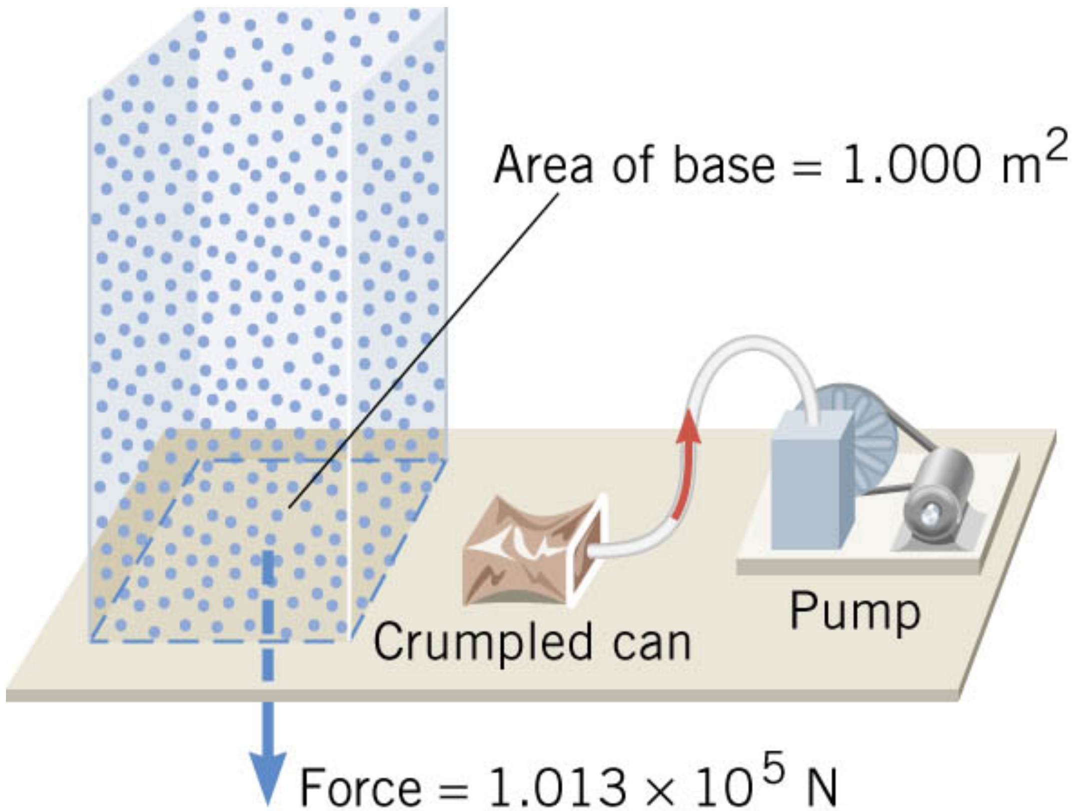
$$F = PA = \left(1.2 \times 10^{5}\,\mathrm{N/m^{2}}\right)\left(8.4 \times 10^{-3}\,\mathrm{m^{2}}\right)$$
$$= 1.0 \times 10^{3}\ \mathrm{N}$$

Since the water pushes perpendicularly against the back of the hand, the force is directed downward in the drawing.

Pressure on the underside of the hand is slightly greater (greater depth). So force upward is slight greater - bouyancy

11.2 Pressure

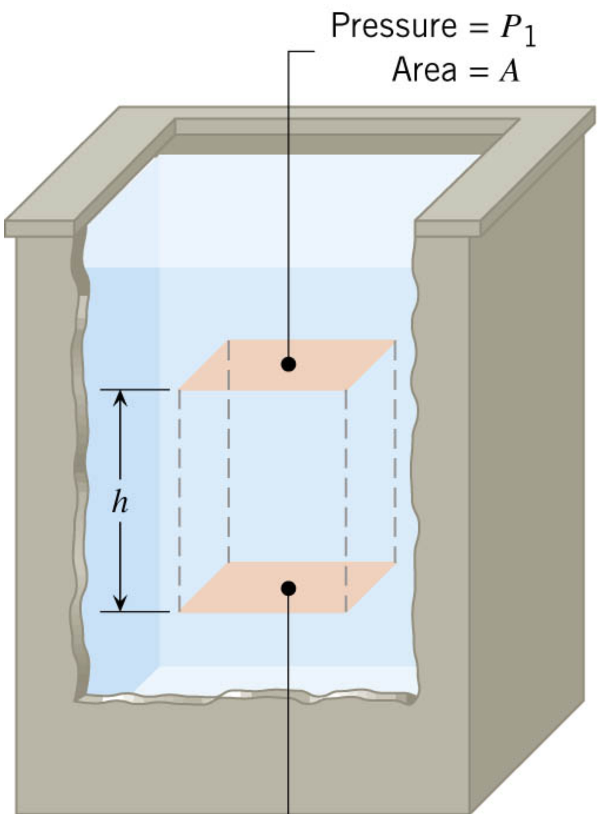
Atmospheric Pressure at Sea Level: 1.013×10^5 Pa = 1 atmosphere



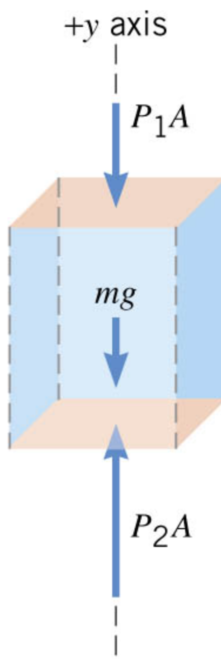
11.3 Pressure and Depth in a Static Fluid

Fluid density is ρ

Equilibrium of a volume of fluid



(*a*)



(*b*) Free-body diagram of the column

$$F_2 = F_1 + mg$$

with $F = PA,\ m = \rho V$

$$P_2 A = P_1 A + \rho V g$$

with $V = Ah$

$$\boxed{P_2 = P_1 + \rho g h}$$

Pressure grows linearly with depth (h)

Conceptual Example 3 The Hoover Dam

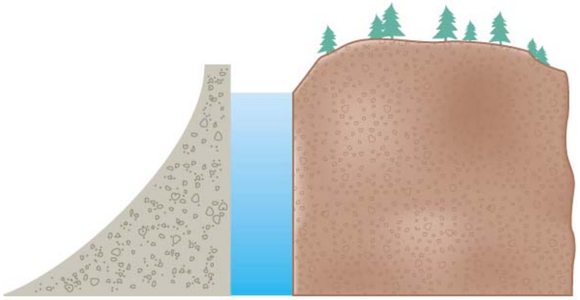
Lake Mead is the largest wholly artificial reservoir in the United States. The water in the reservoir backs up behind the dam for a considerable distance (120 miles).

Suppose that all the water in Lake Mead were removed except a relatively narrow vertical column.

Would the Hoover Dam still be needed to contain the water, or could a much less massive structure do the job?

Pressure depends only on depth (h)

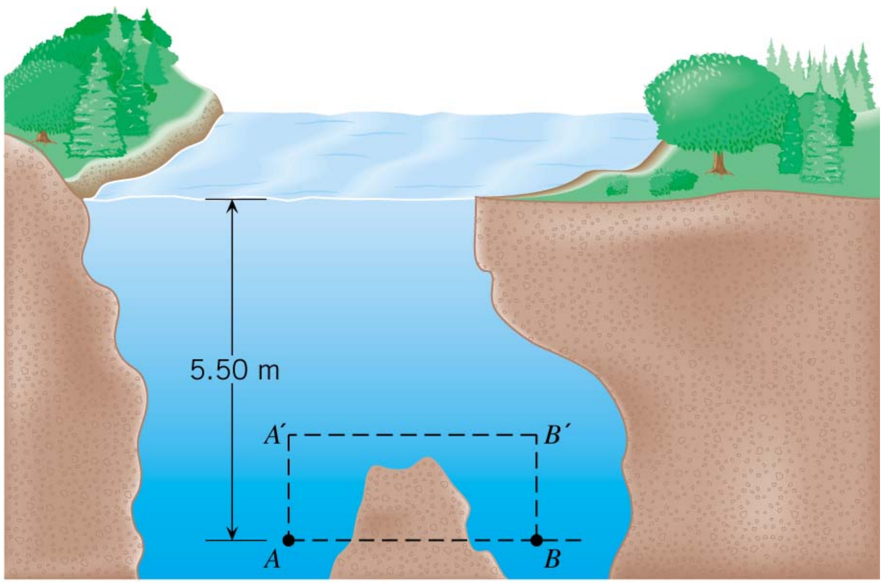
(a)



(b)

Example 4 The Swimming Hole

Points A and B are located a distance of 5.50 m beneath the surface of the water. Find the pressure at each of these two locations.



Atmospheric pressure

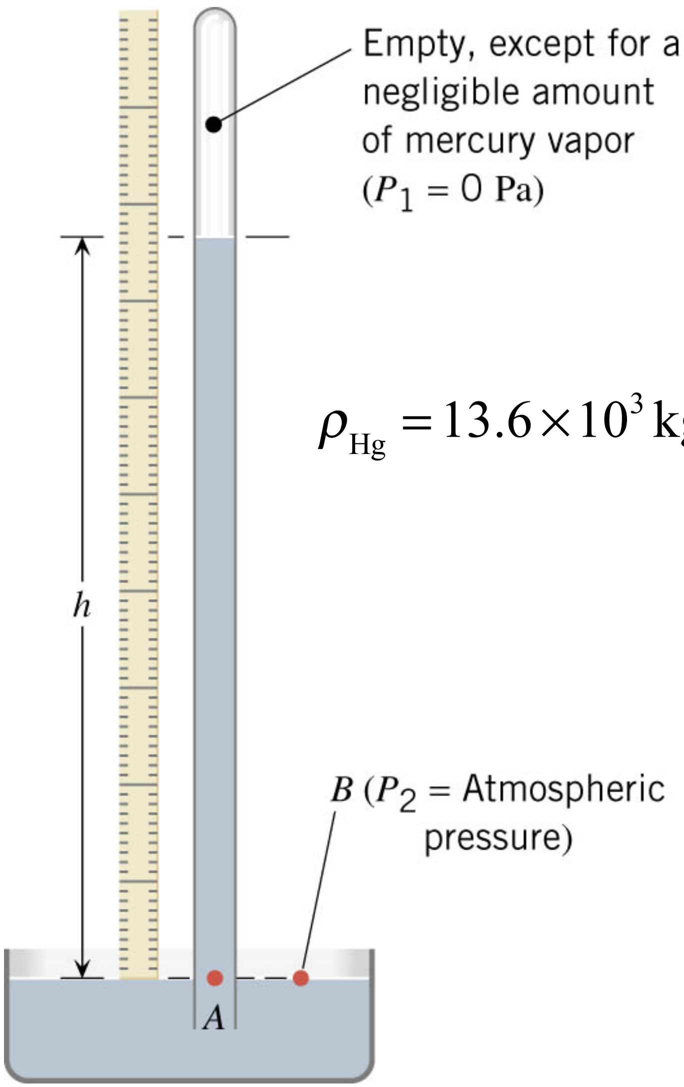
$$P_1 = 1.01\times10^5\ \mathrm{N/m^2}$$

$$P_2 = P_1 + \rho g h$$

$$P_2 = \overbrace{\left(1.01\times10^5\ \mathrm{Pa}\right)}^{\text{atmospheric pressure}} + \left(1.00\times10^3\,\mathrm{kg/m^3}\right)\left(9.80\,\mathrm{m/s^2}\right)\left(5.50\ \mathrm{m}\right)$$

$$= 1.55\times10^5\ \mathrm{Pa}$$

11.4 Pressure Gauges



$$P_2 = P_1 + \rho g h$$

$$0$$

$$P_{atm} = \rho g h$$

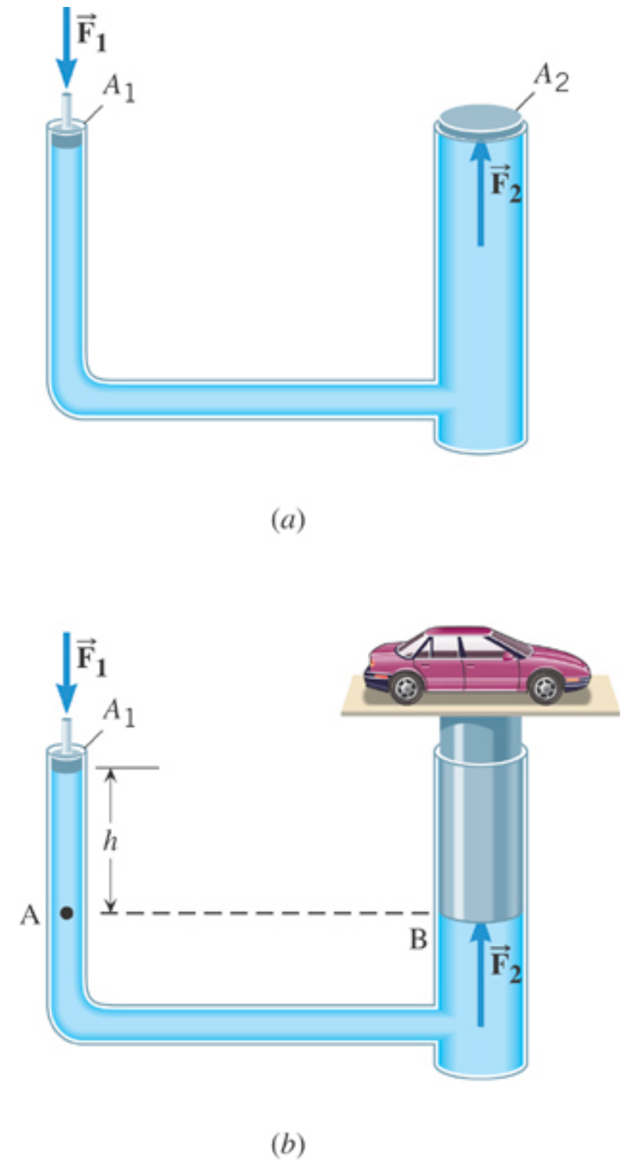
$$\rho_{\mathrm{Hg}} = 13.6\times 10^3\,\mathrm{kg/m^3}$$

$$h = \frac{P_{atm}}{\rho g} = \frac{\left(1.01\times 10^5\ \mathrm{Pa}\right)}{\left(13.6\times 10^3\,\mathrm{kg/m^3}\right)\left(9.80\,\mathrm{m/s^2}\right)}$$

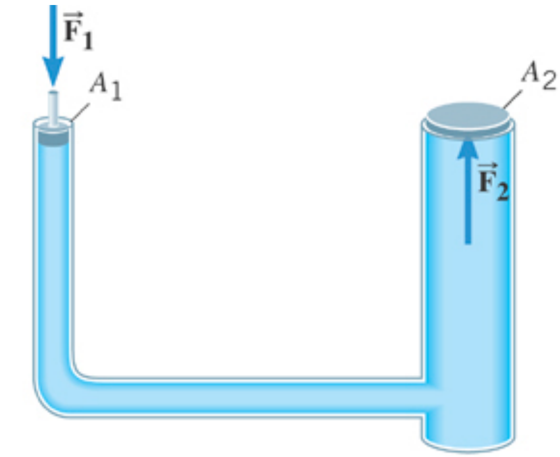
$$= 0.760\ \mathrm{m} = 760\ \mathrm{mm\ of\ Mercury}$$

PASCAL'S PRINCIPLE

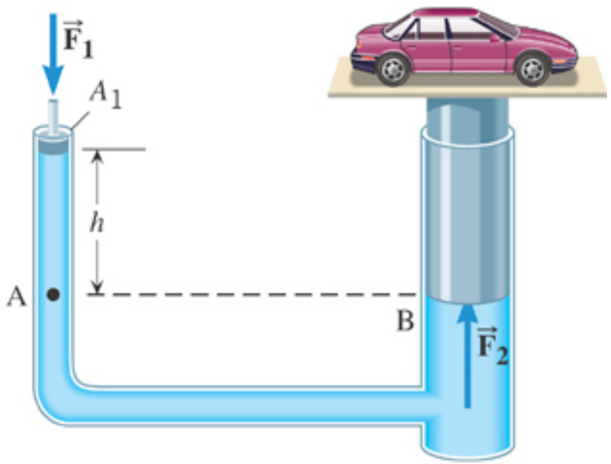
Any change in the pressure applied to a completely enclosed fluid is transmitted undiminished to all parts of the fluid and enclosing walls.



11.5 Pascal's Principle



(a)



(b)

$$P_2 = P_1 + \rho\, g(0\,\mathrm{m})$$

$$\Downarrow$$

$$\frac{F_2}{A_2} = \frac{F_1}{A_1}$$

$$\Downarrow$$

$$F_2 = F_1\left(\frac{A_2}{A_1}\right)$$

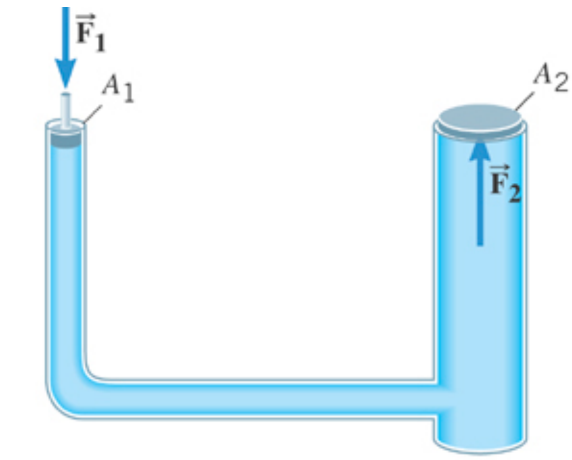
Example 7 A Car Lift

The input piston has a radius of 0.0120 m and the output plunger has a radius of 0.150 m.

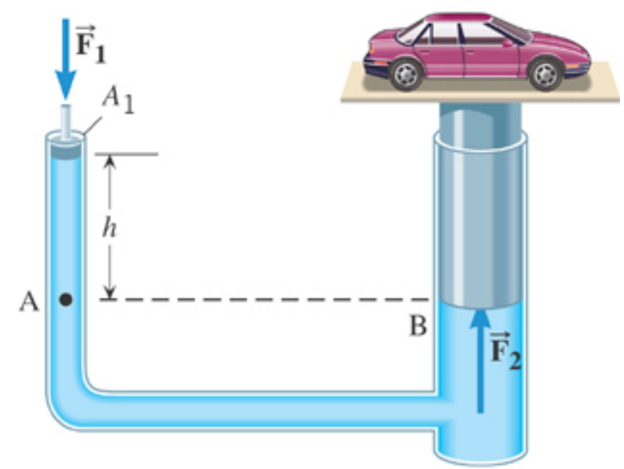
The combined weight of the car and the plunger is 20500 N. Suppose that the input piston has a negligible weight and the bottom surfaces of the piston and plunger are at the same level. What is the required input force?

$$F_2 = F_1\left(\frac{A_2}{A_1}\right)$$

$$= (20500\ \text{N})\frac{\pi(0.0120\ \text{m})^2}{\pi(0.150\ \text{m})^2} = 131\ \text{N}$$

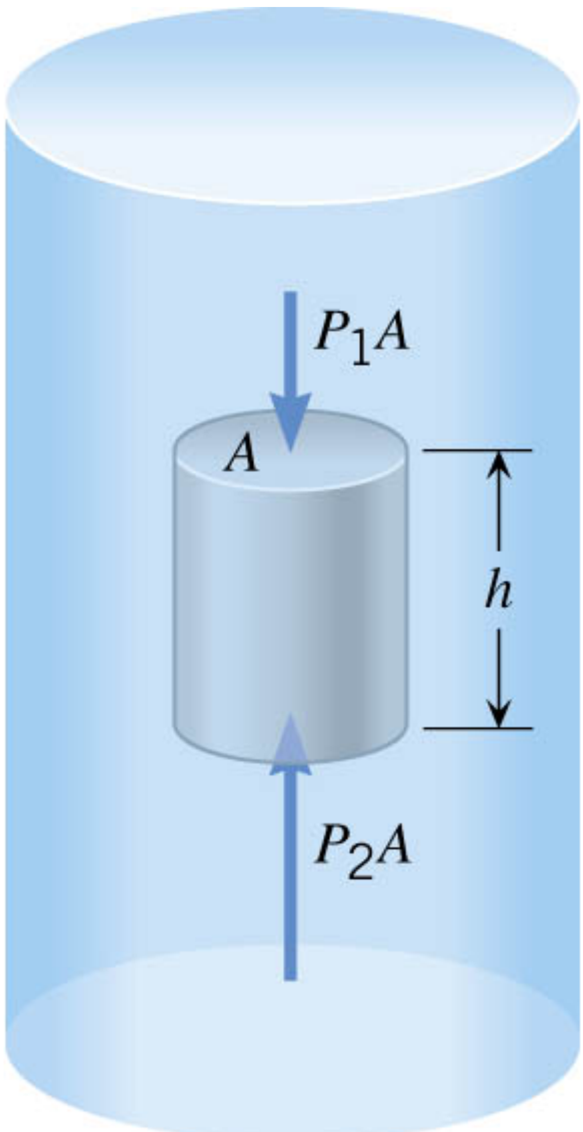


(a)



(b)

11.6 Archimedes' Principle



Buoyant Force

$$F_B = P_2A - P_1A = \left(P_2 - P_1\right)A$$

$$= \rho g h A \qquad\qquad P_2 = P_1 + \rho g h$$

$$\underbrace{= \rho V}_{\text{mass of displaced fluid}} g \qquad\qquad V = hA$$

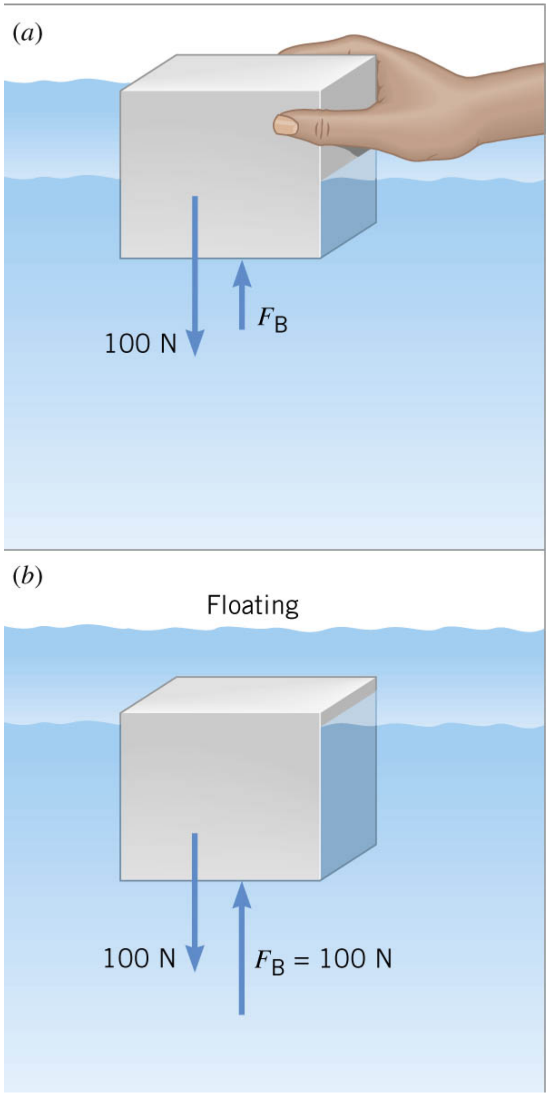
Buoyant force = Weight of displaced fluid

ARCHIMEDES' PRINCIPLE

Any fluid applies a buoyant force to an object that is partially or completely immersed in it; the magnitude of the buoyant force equals the weight of the fluid that the object displaces:

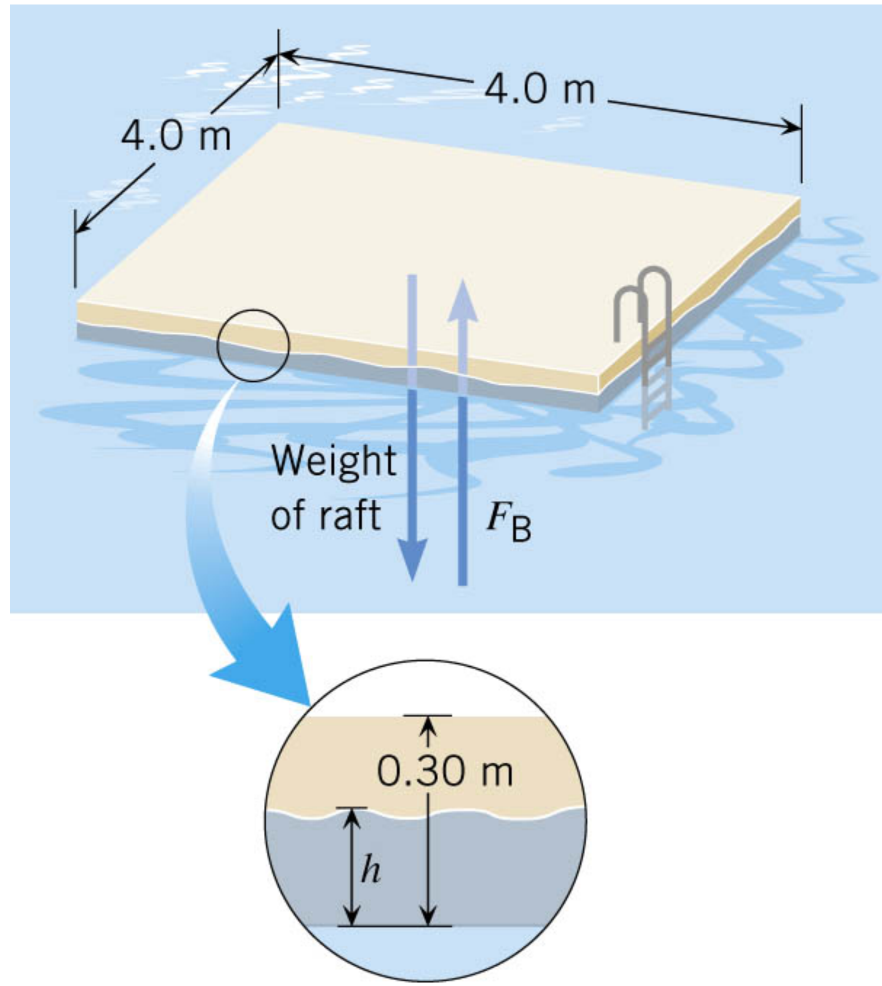
$$\underbrace{F_B}_{\substack{\text{Magnitude of} \\ \text{buoyant force}}} = \underbrace{W_{\text{fluid}}}_{\substack{\text{Weight of} \\ \text{displaced fluid}}}$$

If the object is floating then the magnitude of the buoyant force is equal to the magnitude of its weight.



Example 9 A Swimming Raft

The raft is made of solid square pinewood. Determine whether the raft floats in water and if so, how much of the raft is beneath the surface.



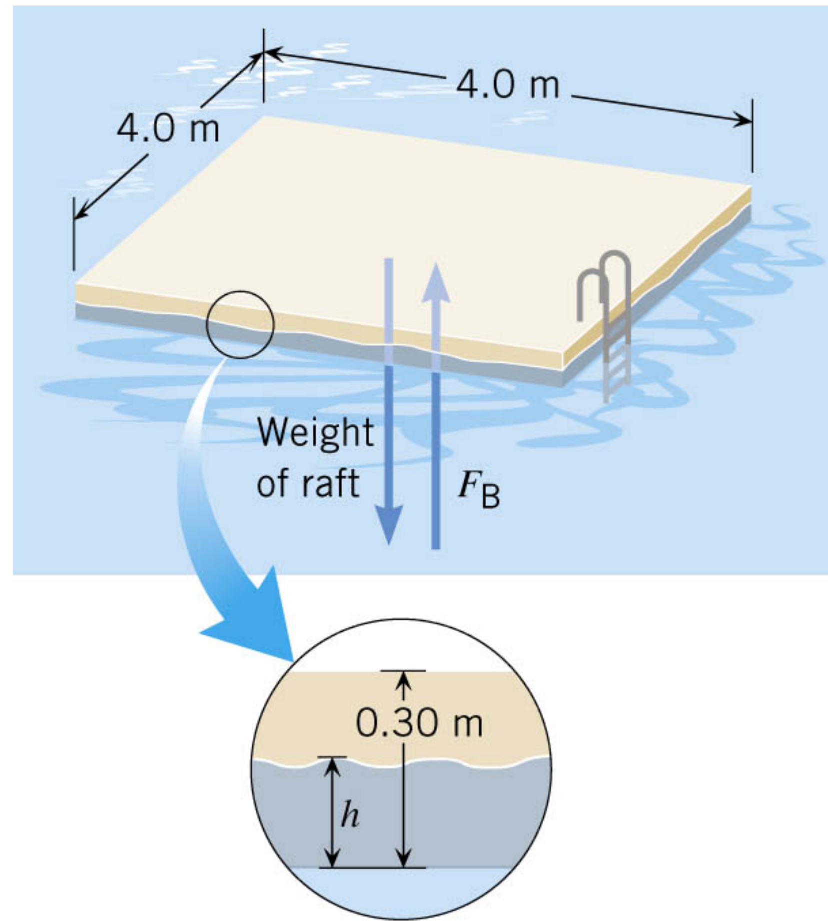
$$V_{raft} = (4.0\text{ m})(4.0\text{ m})(0.30\text{ m}) = 4.8\text{ m}$$

Max Buoyant Force

$$F_B^{\max} = \rho V g = \rho_{water} V_{water} g$$

$$= (1000\,\text{kg/m}^3)(4.8\,\text{m}^3)(9.80\,\text{m/s}^2)$$

$$= 47000\text{ N}$$



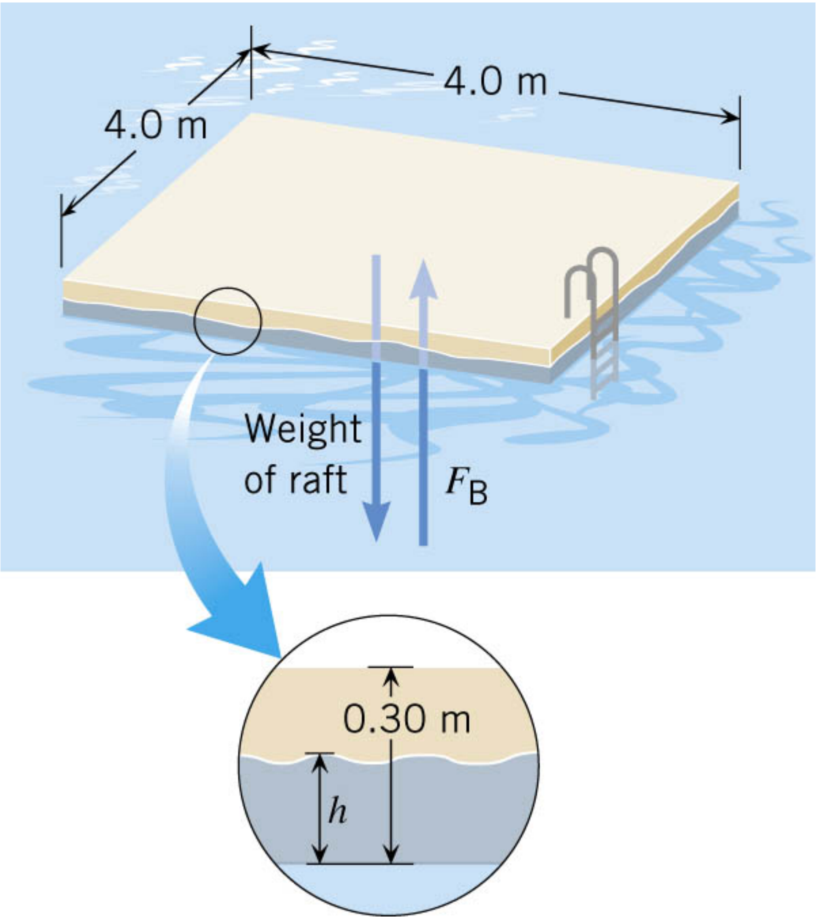
Raft weight

$$W_{raft} = m_{raft} g = \rho_{pine} V_{raft} g$$

$$= (550\,\text{kg/m}^3)(4.8\,\text{m}^3)(9.80\,\text{m/s}^2)$$

$$= 26000\text{ N} < 47000\text{ N}$$

The raft floats!



If the raft is floating:

$$W_{\text{raft}} = F_{\text{B}}$$

$$W_{\text{raft}} = \rho_{\text{water}} g V_{\text{water}}$$
$$= \rho_{\text{water}} g (A_{\text{water}} h)$$

$$h = \frac{W_{\text{raft}}}{\rho_{\text{water}} g A_{\text{water}}} = \frac{26000\text{N}}{\left(1000\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left(16.0\ \text{m}^2\right)}$$

$$= 0.17\ \text{m}$$