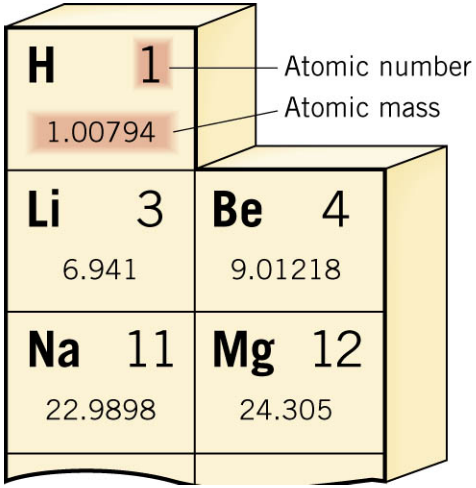
Chapter 14

The Ideal Gas Law and Kinetic Theory

14.1 Molecular Mass, the Mole, and Avogadro's Number

The ***atomic number*** of an element is the # of protons in its nucleus.
Isotopes of an element have different # of neutrons in its nucleus.

The ***atomic mass unit*** (symbol u) is used to compare the mass of elements. The reference is the most abundant isotope of carbon, which is called carbon-12.



$$\boxed{1\ \mathrm{u} = 1.6605\times10^{-24}\,\mathrm{g} = 1.6605\times10^{-27}\,\mathrm{kg}}$$

The atomic mass is given in atomic mass units. For example, a Li atom has a mass of 6.941u.

One ***mole*** (mol) of a substance (element or molecule) contains as many particles as there are atoms in 12 grams of the isotope carbon-12. The number of atoms in 12 grams of carbon-12 is known as Avogadro's number, N_A.

Avogadro's number

$$\boxed{N_A = 6.022\times10^{23}\,\mathrm{mol}^{-1}}$$

14.1 Molecular Mass, the Mole, and Avogadro's Number

The mass per mole (in g/mol) of a substance has the same numerical value as the atomic or molecular mass of the substance (in atomic mass units).

For example Hydrogen has an atomic mass of 1.00794 g/mol, while the mass of a single hydrogen atom is 1.00794 u.

$$\begin{aligned} N &: \text{ \# of atoms or molecules,} \\ n &: \text{ \# of moles of element or molecule} \\ m_p &: \text{ atomic mass (amu)} \Rightarrow \text{also grams/mole} \end{aligned}$$

$$\boxed{\begin{aligned} N &= nN_{\mathrm{A}} \\ m &= nm_p \end{aligned}}$$

Example 1 Hope Diamond & Rosser Reeves Ruby

The Hope diamond (44.5 carats) is almost pure carbon. The Rosser Reeves ruby (138 carats) is primarily aluminum oxide (Al_2O_3). One carat is equivalent to a mass of 0.200 g. Determine (a) the number of carbon atoms in the Hope diamond and (b) the number of Al_2O_3 molecules in the ruby.

$$\boxed{\left[2(26.98)+3(15.99)\right]\text{g/mole}}$$

(a)

$$n=\frac{m}{\text{Mass per mole}}=\frac{(44.5\text{ carats})\left[(0.200\text{ g})/(1\text{ carat})\right]}{12.011\text{g}/\text{mol}}=0.741\text{ mol}$$

$$\boxed{N=nN_A=(0.741\text{ mol})\left(6.022\times10^{23}\text{mol}^{-1}\right)=4.46\times10^{23}\text{atoms}}$$

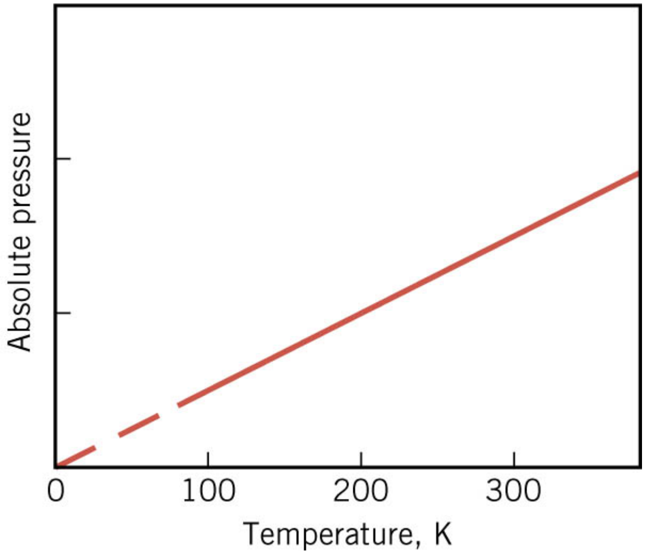
(b)

$$n=\frac{m}{\text{Mass per mole}}=\frac{(138\text{ carats})\left[(0.200\text{ g})/(1\text{ carat})\right]}{101.96\text{g}/\text{mol}}=0.271\text{ mol}$$

$$\boxed{N=nN_A=(0.271\text{ mol})\left(6.022\times10^{23}\text{mol}^{-1}\right)=1.63\times10^{23}\text{atoms}}$$

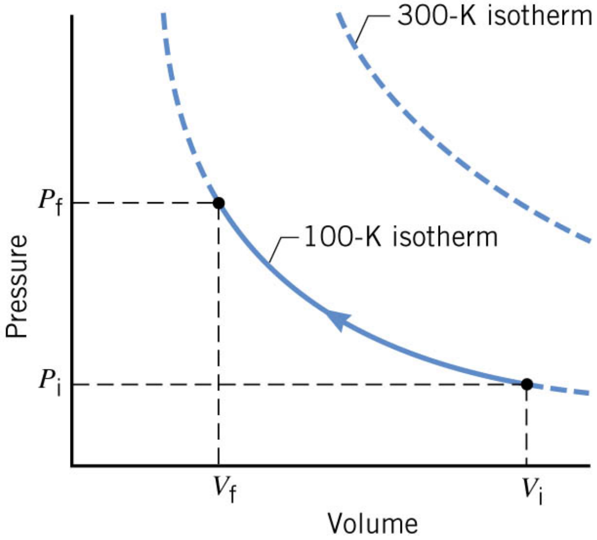
14.2 The Ideal Gas Law

An ***ideal gas*** is an idealized model for real gases that have sufficiently low densities, interacting only by elastic collisions.



At constant volume the pressure is proportional to the temperature.

$$P \propto T$$



At constant temperature, the pressure is inversely proportional to the volume.

$$P \propto 1/V$$

The pressure is also proportional to the amount of gas.

$$P \propto n$$

THE IDEAL GAS LAW

The absolute pressure of an ideal gas is directly proportional to the Kelvin temperature and the number of moles (*n*) of the gas and is inversely proportional to the volume of the gas.

$$P = \frac{nRT}{V}$$

$$\boxed{PV = nRT}$$

$$\boxed{R = 8.31\,\mathrm{J}/(\mathrm{mol}\cdot\mathrm{K})}$$

Another form for the Ideal Gas Law using the number of atoms (*N*)

$$PV = nRT$$

$$= N\left(\frac{R}{N_A}\right)T$$

$$\boxed{N = nN_A}$$

$$\boxed{PV = NkT}$$

$$\boxed{k = \frac{R}{N_A} = \frac{8.31\mathrm{J}/(\mathrm{mol}\cdot\mathrm{K})}{6.022\times10^{23}\mathrm{mol}^{-1}} = 1.38\times10^{-23}\,\mathrm{J}/\mathrm{K}}$$

Example 2 Oxygen in the Lungs

In the lungs, the respiratory membrane separates tiny sacs of air (pressure 1.00x10^5 Pa) from the blood in the capillaries. These sacs are called alveoli. The average radius of the alveoli is 0.125 mm, and the air inside contains 14% oxygen. Assuming that the air behaves as an ideal gas at 310K, find the number of oxygen molecules in one of these sacs.

$$PV = NkT$$

$$N_{tot} = \frac{PV}{kT} = \frac{\left(1.00\times10^{5}\,\mathrm{Pa}\right)\left[\tfrac{4}{3}\pi\left(0.125\times10^{-3}\,\mathrm{m}\right)^{3}\right]}{\left(1.38\times10^{-23}\,\mathrm{J/K}\right)\left(310\ \mathrm{K}\right)}$$

$$= 1.9\times10^{14}$$

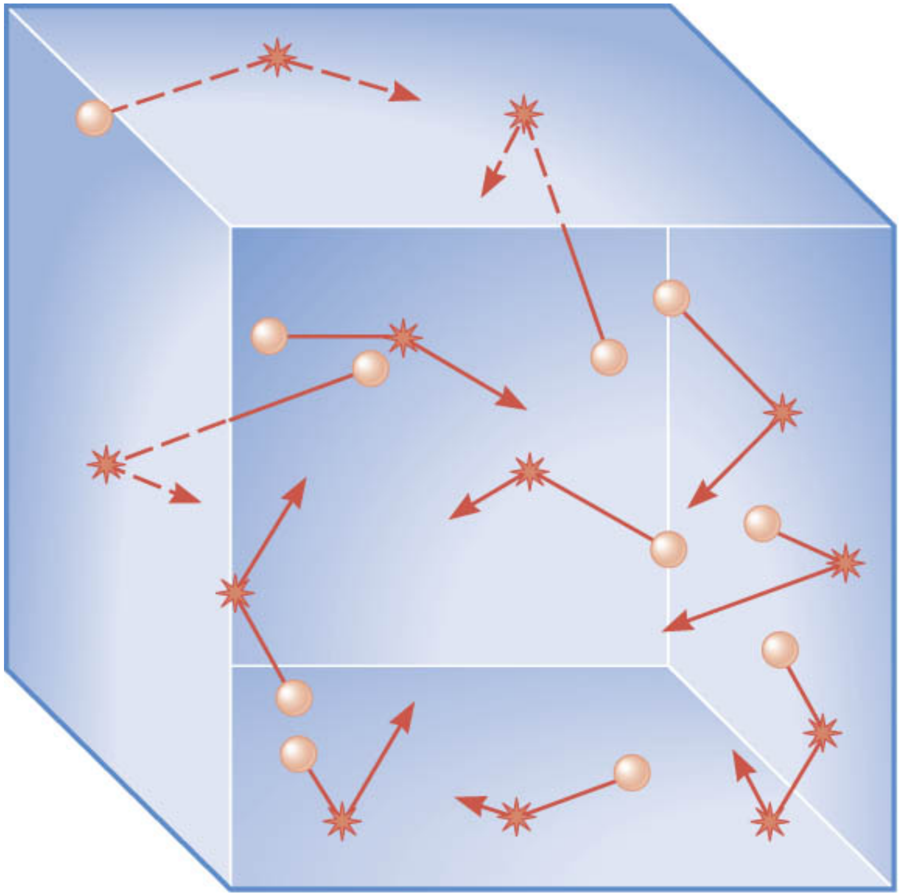
$$N_{\mathrm{Oxy}} = \left(1.9\times10^{14}\right)\times\left(0.14\right) = 2.7\times10^{13}$$

14.3 Kinetic Theory of Gases

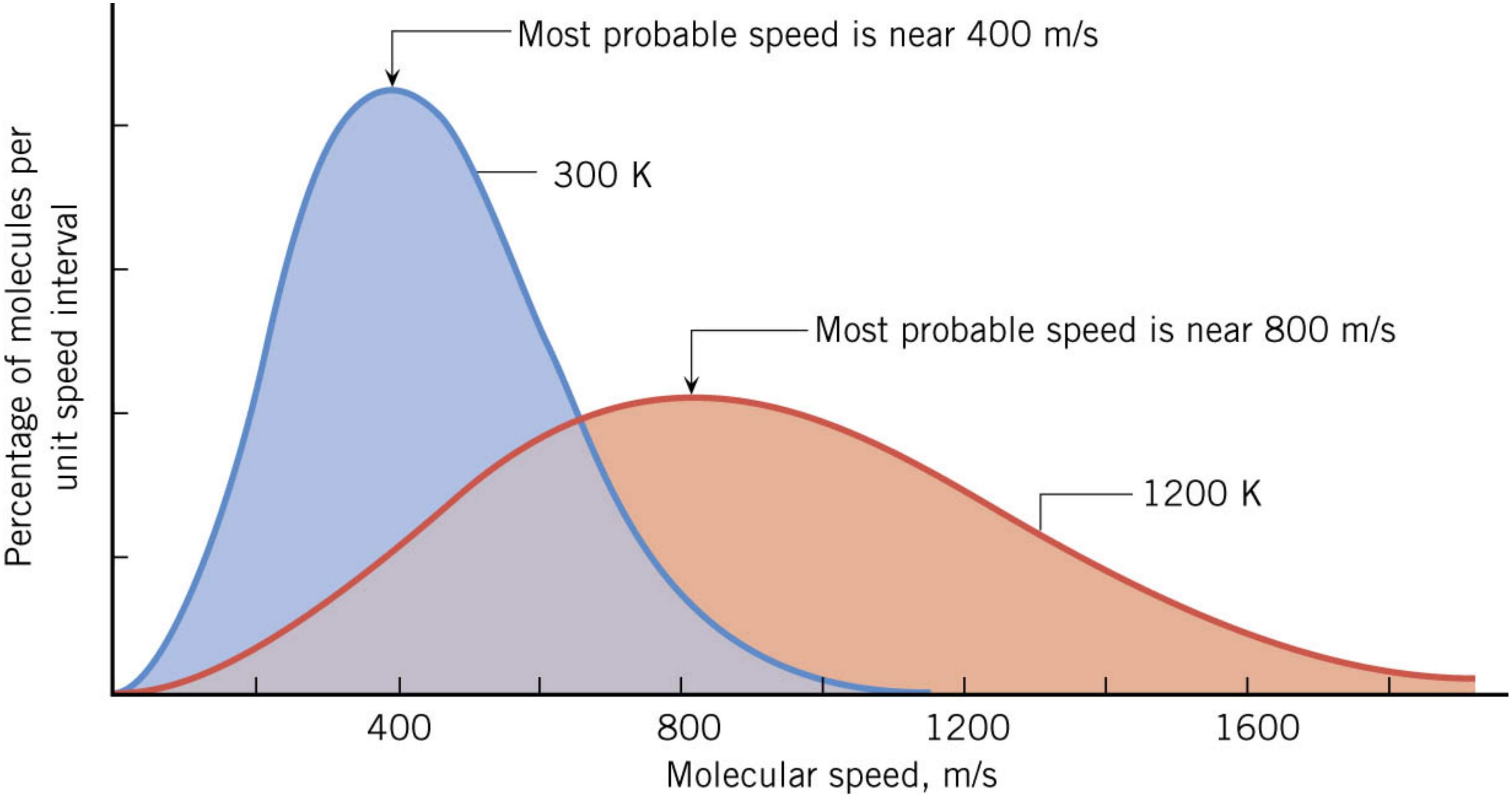
The particles are in constant, random motion, colliding with each other and with the walls of the container.

Each collision changes the particle’s speed.

As a result, the atoms and molecules have different speeds.

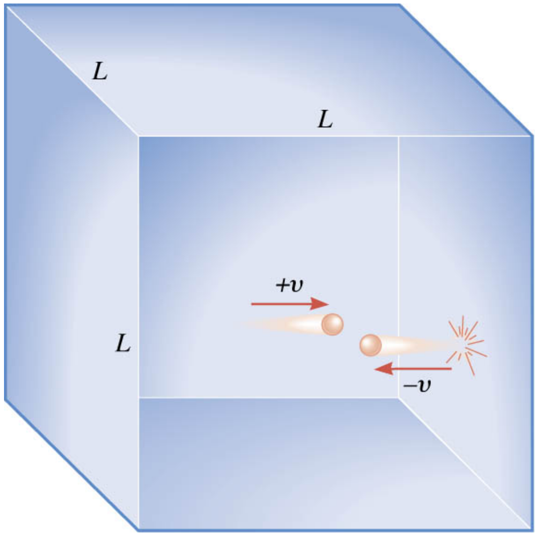


THE DISTRIBUTION OF MOLECULAR SPEEDS



14.3 Kinetic Theory of Gases

KINETIC THEORY



$$\sum F = ma = m\frac{\Delta v}{\Delta t} = \frac{\Delta(mv)}{\Delta t}$$

Average force on each gas molecule when hitting the wall $= \dfrac{\text{Final momentum-Initial momentum}}{\text{Time between successive collisions}}$

$$= \frac{(-mv)-(+mv)}{2L/v} = \frac{-mv^2}{L}$$

Average force on the wall

$$\overline{F} = \left(\frac{N}{3}\right)\left(\frac{m\overline{v^2}}{L}\right) \Rightarrow P = \frac{\overline{F}}{A} = \frac{\overline{F}}{L^2} = \left(\frac{N}{3}\right)\left(\frac{m\overline{v^2}}{L^3}\right)$$

$$PV = \left(\frac{N}{3}\right)m\overline{v^2} = \tfrac{2}{3}N\left(\tfrac{1}{2}m\overline{v^2}\right)$$

$$\boxed{PV = NkT}$$

$$\boxed{\overline{KE} = \tfrac{1}{2}m\overline{v^2}}$$

$$\boxed{v_{rms} = \sqrt{\overline{v^2}}}$$

root mean square speed

Temperature reflects the average Kinetic Energy of the molecules

$$\boxed{\tfrac{3}{2}kT = \tfrac{1}{2}mv_{rms}^2 = \overline{KE}}$$

$$\boxed{k = 1.38\times10^{-23}\ \mathrm{J/K}}$$

Example 6 The Speed of Molecules in Air

Air is primarily a mixture of nitrogen N_2 molecules (molecular mass 28.0u) and oxygen O_2 molecules (molecular mass 32.0u). Assume that each behaves as an ideal gas and determine the rms speeds of the nitrogen and oxygen molecules when the temperature of the air is 293K.

$$\tfrac{1}{2} m v_{rms}^2 = \tfrac{3}{2} kT$$

$$\boxed{v_{rms} = \sqrt{\frac{3kT}{m}}}$$

T must be in Kelvin

(K = C°+273)

$$\boxed{\begin{aligned} &\underline{\text{Nitrogen molecule}} \\ m &= \frac{28.0\,\text{g/mol}}{6.022\times10^{23}\,\text{mol}^{-1}} \\ &= \underline{4.65\times10^{-26}\,\text{kg}} \end{aligned}}$$

$$\begin{aligned} v_{rms} &= \sqrt{\frac{3kT}{m}} \\ &= \sqrt{\frac{3\left(1.38\times10^{-23}\,\text{J/K}\right)\left(293\,\text{K}\right)}{4.65\times10^{-26}\,\text{kg}}} = 511\,\text{m/s} \end{aligned}$$

Molecules are moving really fast but do not go very far before hitting another molecule.

14.3 *Kinetic Theory of Gases*

THE INTERNAL ENERGY OF A <u>MONATOMIC</u> IDEAL GAS

$$\overline{\mathrm{KE}} = \tfrac{1}{2} m v_{rms}^2 = \tfrac{3}{2} kT$$

Average KE per atom

⇓ multiply by the number of atoms

$$U = N \tfrac{3}{2} kT = \tfrac{3}{2} nRT$$

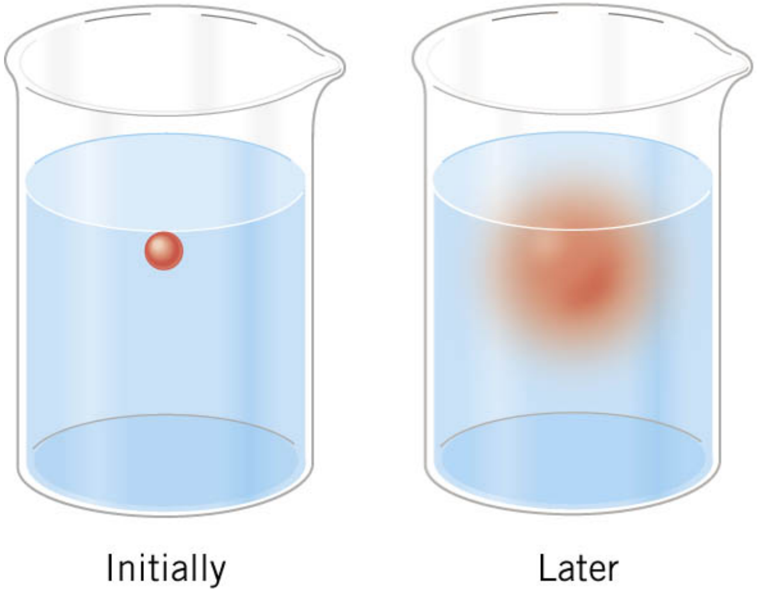
Total Internal Energy

THE INTERNAL ENERGY OF A MOLECULAR GAS MUST INCLUDE MOLECULAR VIBRATIONS!

H_2, N_2, H_2O, SO_2, CO_2, …(most gases except Nobel gases)

14.4 ***Diffusion***

The process in which molecules move from a region of higher concentration to one of lower concentration is called ***diffusion***.



FICK'S LAW OF DIFFUSION

The mass *m* of solute that diffuses in a time *t* through a solvent contained in a channel of length *L* and cross sectional area *A* is

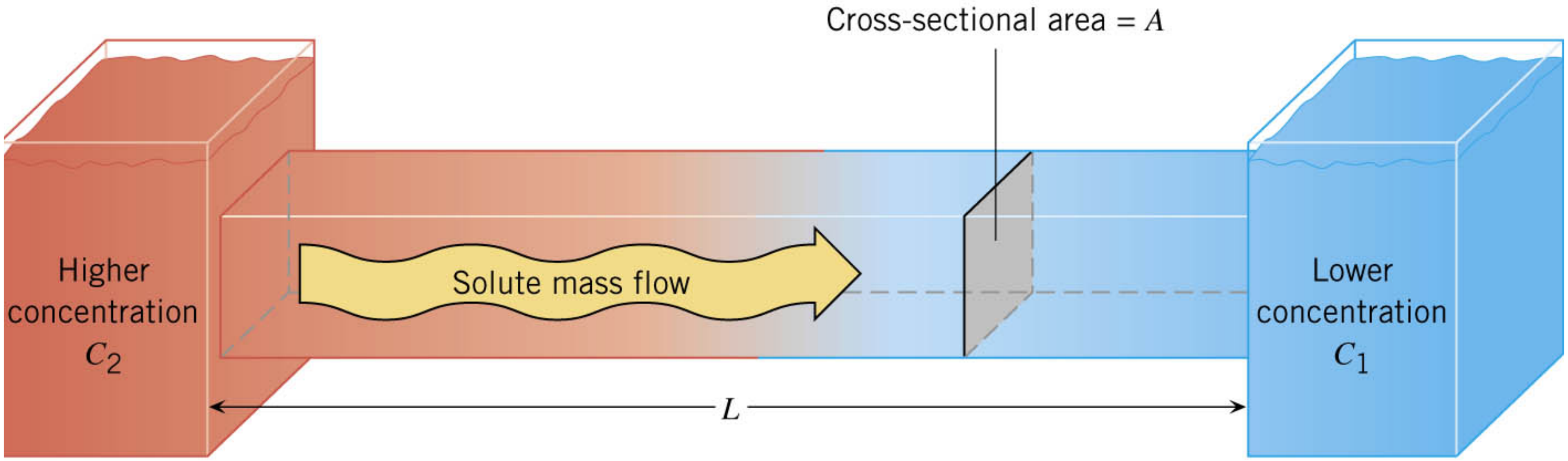
$$m = \frac{DA(\Delta C)t}{L}$$

D	Diffusion constant
ΔC	Concentration gradient between ends

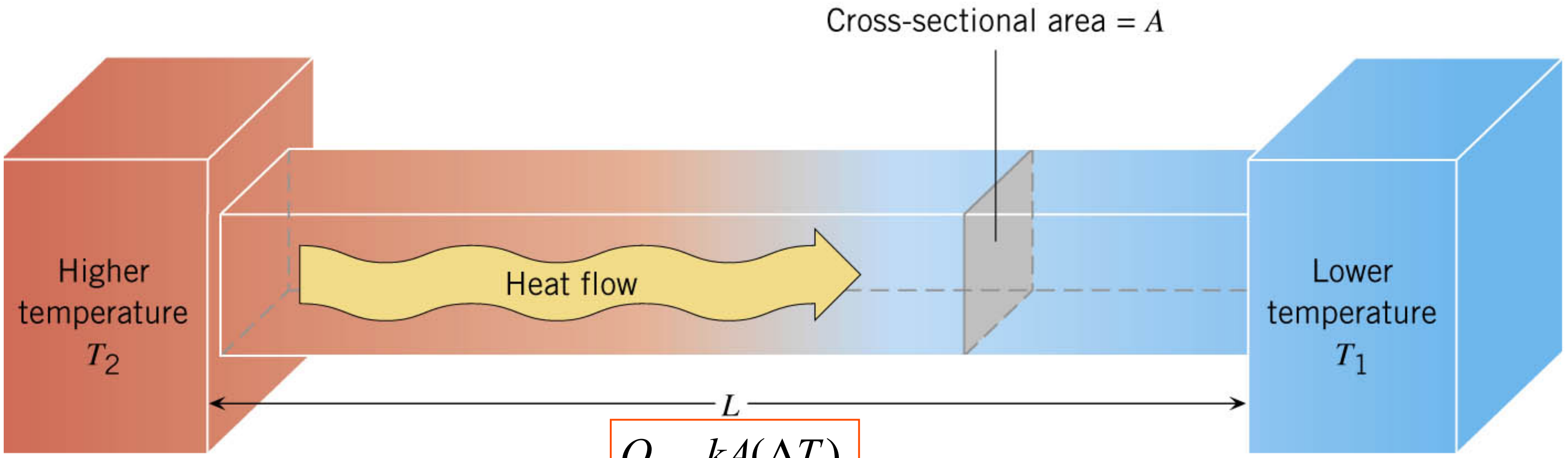
SI Units for the Diffusion Constant: m^2/s

14.4 **Diffusion**

Diffusion analogous to heat flow



$$\boxed{\frac{m}{t} = \frac{DA(\Delta C)}{L}}$$



$$\boxed{\frac{Q}{t} = \frac{kA(\Delta T)}{L}}$$

Chapter 15

Thermodynamics

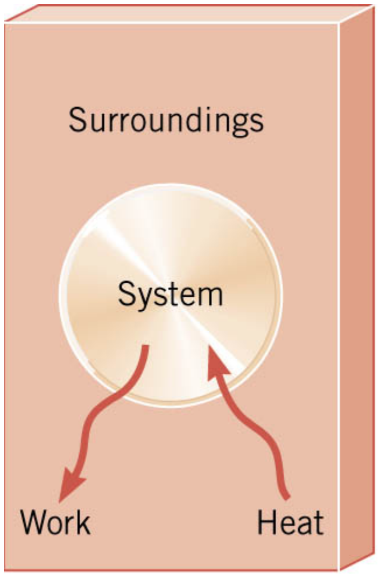
15.1 Thermodynamic Systems and Their Surroundings

Thermodynamics is the branch of physics that is built upon the fundamental laws that heat and work obey.

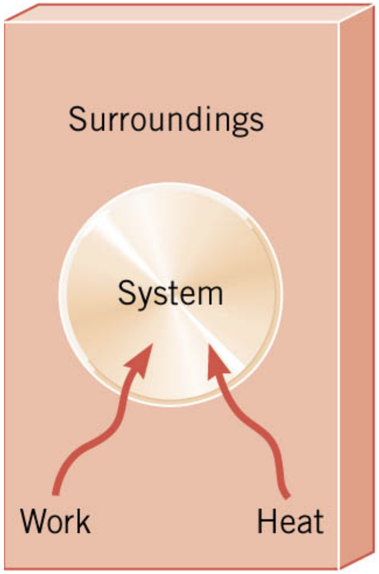
The collection of objects on which attention is being focused is called the ***system***, while everything else in the environment is called the ***surroundings***.

Walls that permit heat flow are called ***diathermal walls***, while walls that do not permit heat flow are called ***adiabatic*** (NO HEAT FLOW) ***walls***

To understand thermodynamics, it is necessary to describe the ***state of a system***.



(*a*)

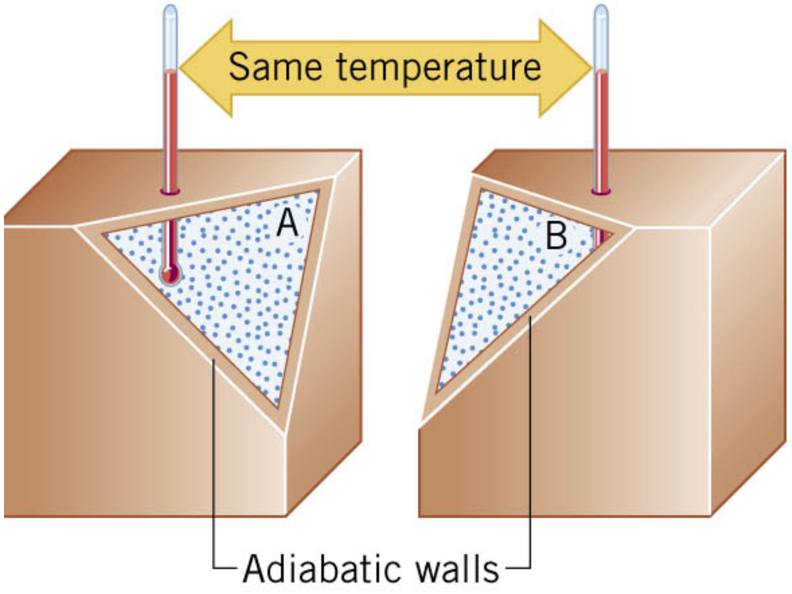


(*b*)

15.2 The Zeroth Law of Thermodynamics

Two systems are said to be in ***thermal equilibrium*** if there is no heat flow between them when in contact.

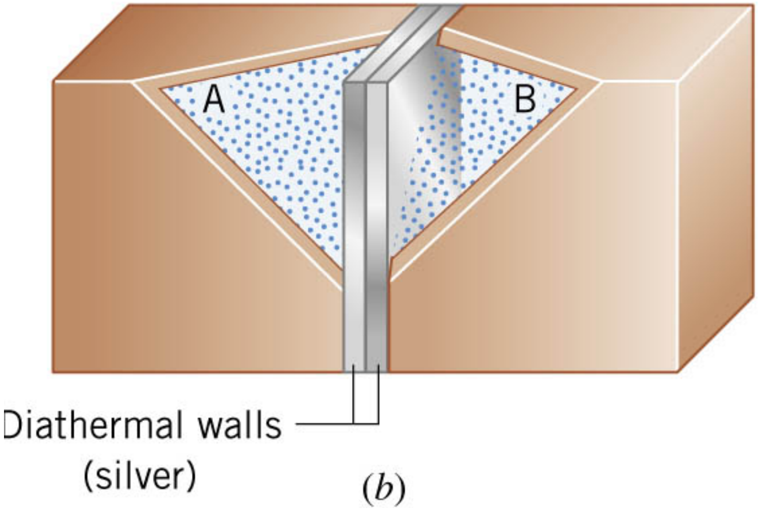
Temperature: there is no net flow of heat between two systems in thermal contact that have the same temperature.



(*a*)

THE ZEROTH LAW OF THERMODYNAMICS

Two systems individually in thermal equilibrium with a third system are in thermal equilibrium with each other.



(*b*)

15.3 *The First Law of Thermodynamics*

1) Suppose that a system only gains heat (Q) and nothing else changes. Consistent with the law of conservation of energy, the internal energy (U) of the system changes:

$$\Delta U = U_f - U_i = Q$$

$Q > 0$ system gains heat

2) If a system does work W on its surroundings but there is no heat flow, (adiabatic) conservation of energy indicates that the internal energy of the system will decrease:

$$\Delta U = U_f - U_i = -W$$

$W > 0$ if system does work

THE FIRST LAW OF THERMODYNAMICS

The internal energy of a system changes due to heat and work:

$$\boxed{\Delta U = U_f - U_i = Q - W}$$

$Q > 0$ system gains heat

$W > 0$ if system does work

Example 1 Positive and Negative Work

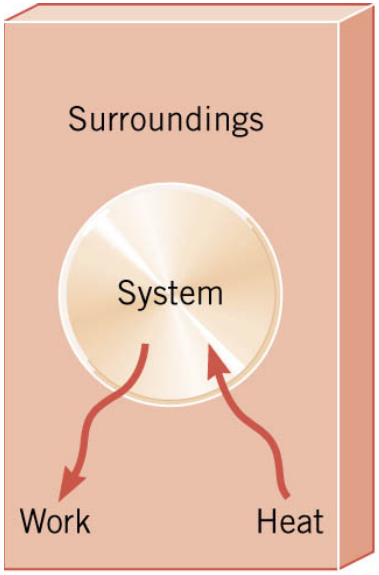
In part (*a*) of figure, the system gains 1500J of heat and 2200J of work is done by the system on its surroundings.

In part (*b*), the system also gains 1500J of heat, but 2200J of work is done on the system.

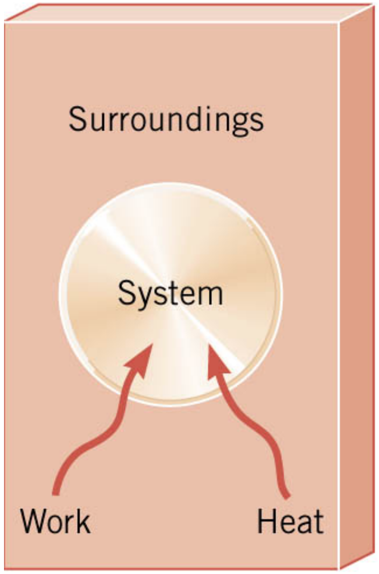
In each case, determine the change in internal energy of the system.

(*a*)
$$\Delta U = Q - W$$
$$= (+1500\ \text{J}) - (+2200\ \text{J}) = -700\ \text{J}$$

(*b*)
$$\Delta U = Q - W$$
$$= (+1500\ \text{J}) - (-2200\ \text{J}) = +3700\ \text{J}$$



(*a*)



(*b*)

Example 2 An Ideal Gas

The temperature of three moles of a monatomic ideal gas is reduced from 540K to 350K as 5500J of heat flows into the gas.

Find (a) the change in internal energy and (b) the work done by the gas.

$$\boxed{\Delta U = U_f - U_i = Q - W}$$

$$\boxed{U = \tfrac{3}{2} nRT}$$

(a)

$$\begin{aligned}\Delta U &= \tfrac{3}{2} nR\left(T_f - T_i\right)\\ &= \tfrac{3}{2}\left(3.0\ \text{mol}\right)\left(8.31\ \text{J}/\left(\text{mol}\cdot\text{K}\right)\right)\left(350\ \text{K} - 540\ \text{K}\right)\\ &= -7100\ \text{J}\end{aligned}$$

(b)

$$\begin{aligned}W &= Q - \Delta U\\ &= 5500\ \text{J} - \left(-7100\ \text{J}\right)\\ &= 12600\ \text{J}\end{aligned}$$

15.4 Thermal Processes

A ***quasi-static*** process is one that occurs slowly enough that a uniform temperature and pressure exist throughout all regions of the system at all times.

isobaric: constant pressure $$W = Fs = P(As) = P\Delta V$$

isochoric: constant volume $$W = P\Delta V = 0$$

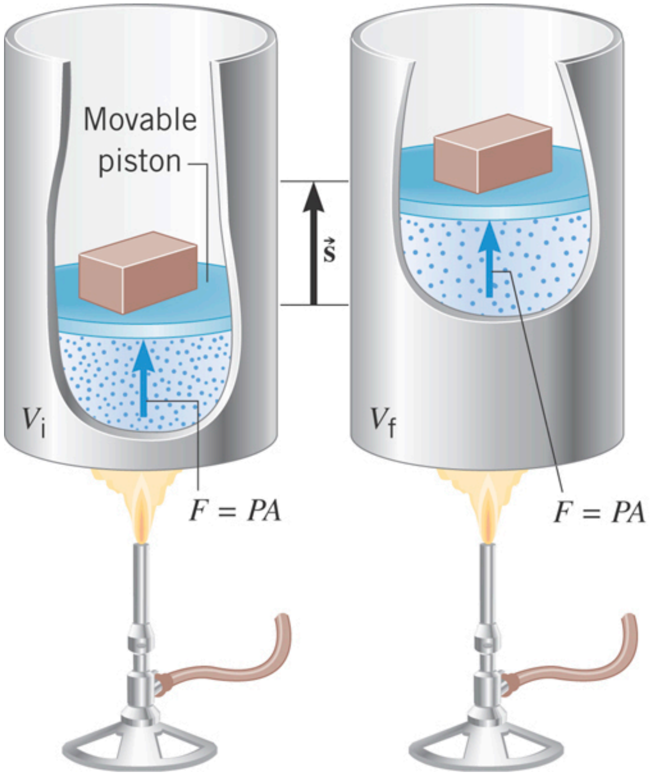
isothermal: constant temperature $$W = nRT\ln\left(V_f / V_i\right) \quad \text{ideal gas}$$

adiabatic: no transfer of heat $$W = \tfrac{3}{2}nR\left(T_f - T_i\right) \quad \text{ideal gas}$$

15.4 Thermal Processes

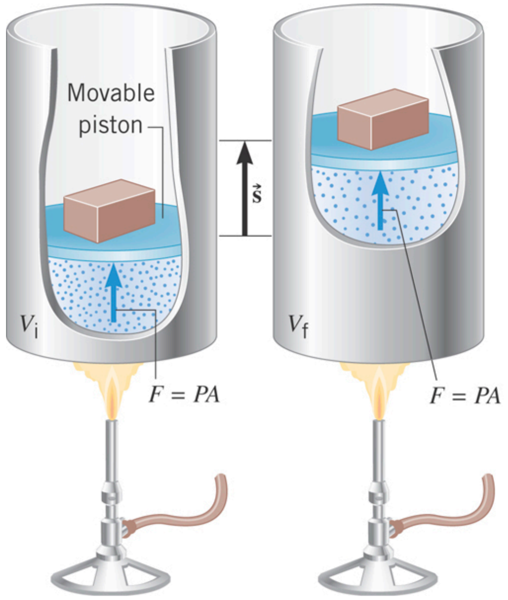
An ***isobaric*** process is one that occurs at constant pressure.

$$W = Fs = P\left(As\right) = P\Delta V$$



Isobaric process: $$W = P\Delta V = P\left(V_f - V_i\right)$$

Example 3 Isobaric Expansion of Water



One gram of water is placed in the cylinder and the pressure is maintained at $2.0\text{x}10^5$ Pa. The temperature of the water is raised by 31°C. The water is in the liquid phase and expands by the small amount of $1.0\text{x}10^{-8}$ m^3.

Find the work done and the change in internal energy.

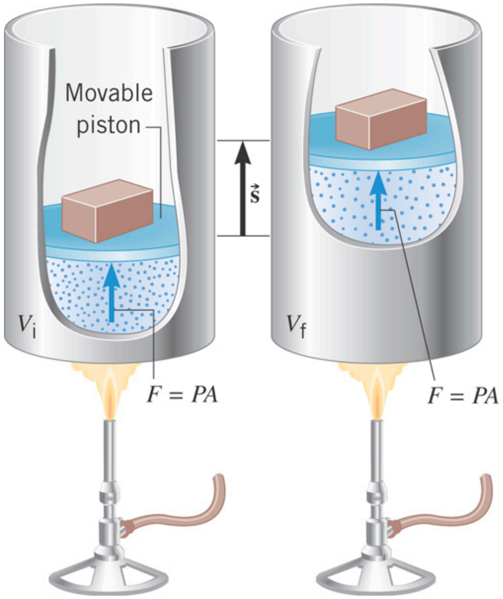
$$\boxed{\begin{aligned} W &= P\Delta V \\ &= \left(2.0\times10^{5}\,\text{Pa}\right)\left(1.0\times10^{-8}\,\text{m}^{3}\right) = 0.0020\text{J}\end{aligned}}$$

$$\boxed{\text{Liquid water } \Delta V \sim 0}$$

$$\begin{aligned} Q &= mc\Delta T \\ &= \left(0.0010\ \text{kg}\right)\left[4186\text{J}/\left(\text{kg}\cdot\text{C}^{\circ}\right)\right]\left(31\ \text{C}^{\circ}\right) = 130\ \text{J}\end{aligned}$$

$$\boxed{\Delta U = Q - W = 130\ \text{J} - 0.0020\ \text{J} = 130\ \text{J}}$$

Example 3 Isobaric Expansion of Water Vapor



One gram of water vapor is placed in the cylinder and the pressure is maintained at 2.0x10^5 Pa. The temperature of the vapor is raised by 31^{o}C, and the gas expands by 7.1x10^{-5} m^3. Heat capacity of the gas is 2020 J/(kg-C°).

Find the work done and the change in internal energy.

$$\boxed{\begin{aligned} W &= P\Delta V = (2.0\times10^5\ \text{Pa})(7.1\times10^{-5}\text{m}^3) \\ &= 14.2\ \text{J} \end{aligned}}$$

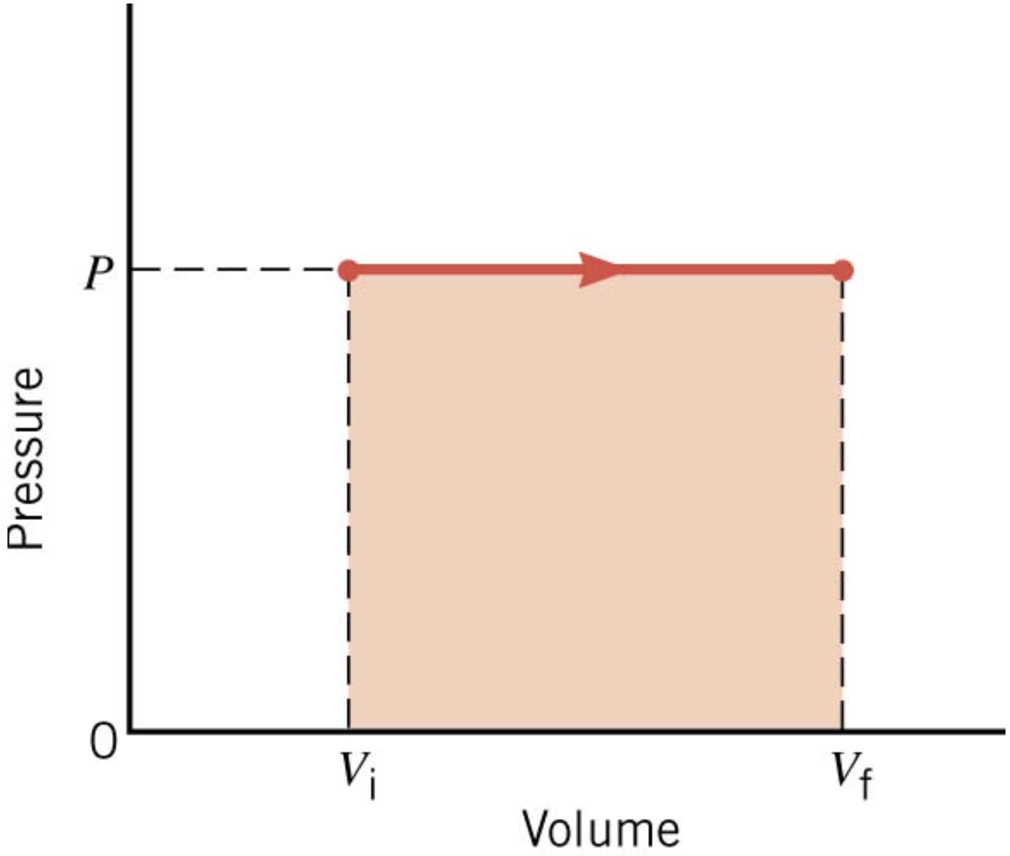
$$\begin{aligned} Q &= mc\Delta T \\ &= (0.0010\ \text{kg})\left[2020\,\text{J}/\left(\text{kg}\cdot\text{C}^\circ\right)\right]\left(31\ \text{C}^\circ\right) = 63\ \text{J} \end{aligned}$$

$$\boxed{\Delta U = Q - W = 63\ \text{J} - 14\ \text{J} = 49\ \text{J}}$$

15.4 Thermal Processes

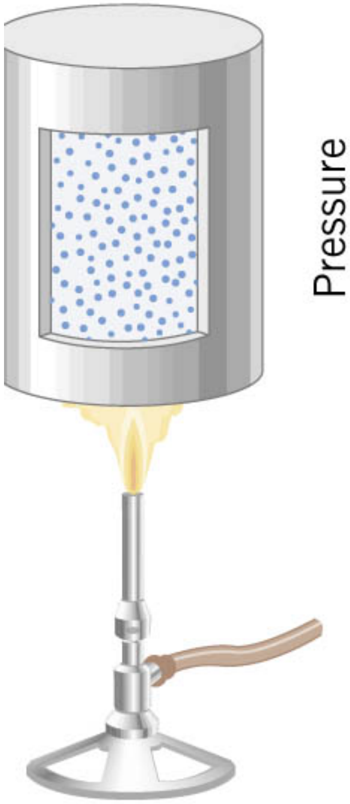
$$W = P\Delta V = P\left(V_f - V_i\right)$$

The work done at constant pressure the work done is the area under a P-V diagram.

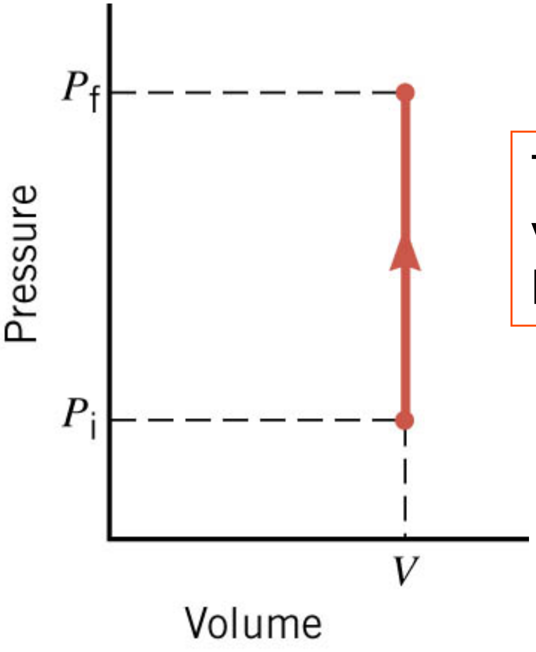


15.4 Thermal Processes

isochoric: constant volume



(a)



(b)

The work done at constant volume is the area under a P-V diagram. The area is zero!

$$W = 0$$

$$\Delta U = Q - W = Q$$

Change in internal energy is equal to the heat added.

Example 4 Work and the Area Under a Pressure-Volume Graph

Determine the work for the process in which the pressure, volume, and temperature of a gas are changed along the straight line in the figure.

The area under a pressure-volume graph is the work for any kind of process.

$$W = 9\left(2.0\times10^{5}\,\text{Pa}\right)\left(1.0\times10^{-4}\,\text{m}^{3}\right)$$
$$= +180\ \text{J}$$