

# Kinematics

↳ the mathematical description of motion

The oldest topic in physics -

## MECHANICS

- KINEMATICS

- DYNAMICS

# Variables

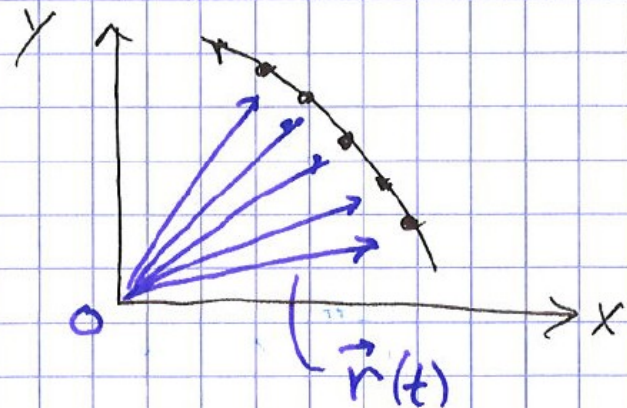
Time  $t$  [s]

Position  $\vec{r}(t)$  [m]

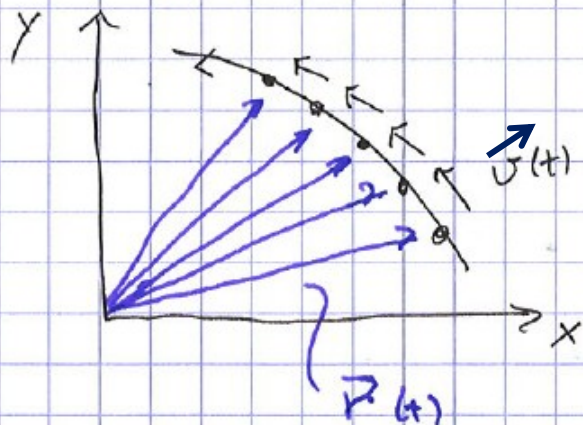
Velocity  $\vec{v}(t)$  [m/s]

Acceleration  $\vec{a}(t)$  [m/s/s]

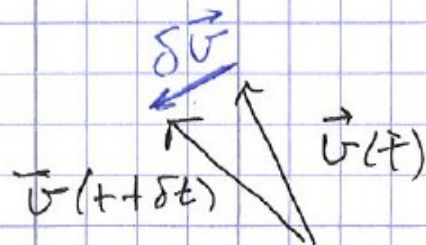
These are vectors



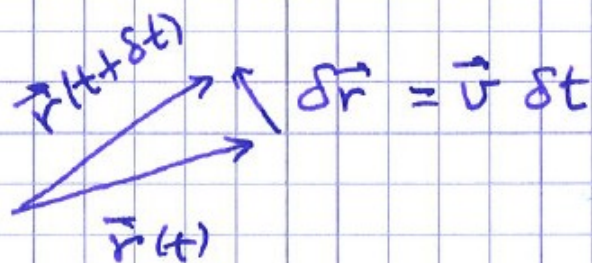
$$\vec{v} = \frac{\delta \vec{r}}{\delta t}$$



$$\vec{a} = \frac{\delta \vec{v}}{\delta t}$$



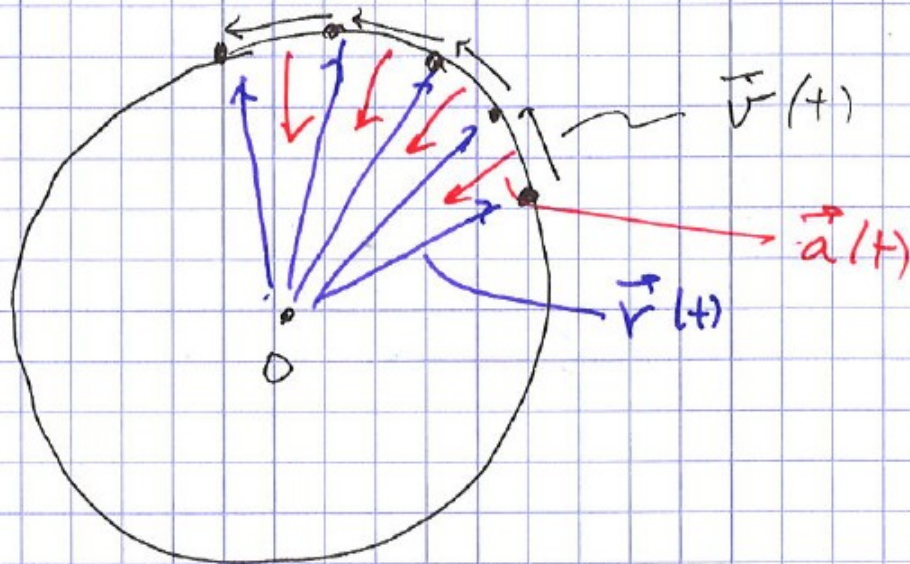
adding vectors:  
 $\vec{v}(t) + \delta \vec{v} = \vec{v}(t + \delta t)$



adding vectors:  $\vec{r}(t) + \delta \vec{r} = \vec{r}(t + \delta t)$



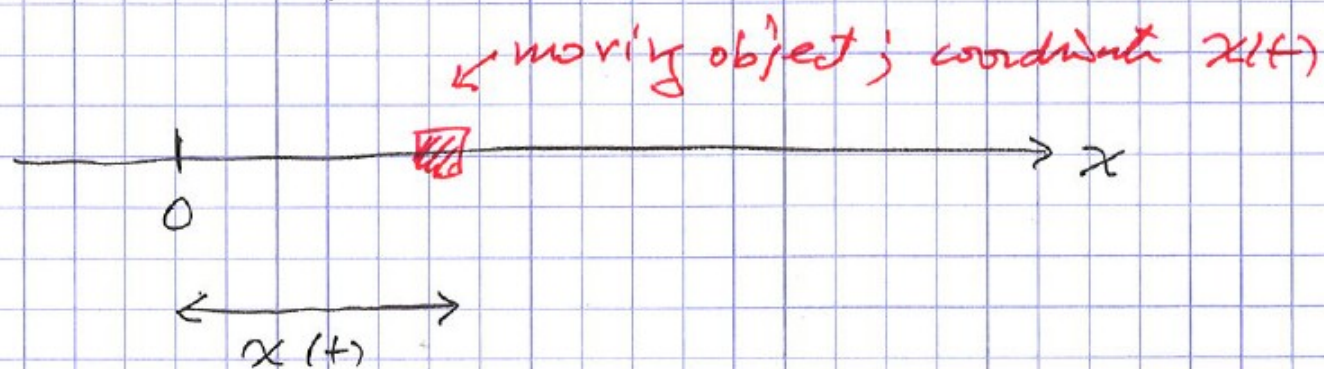
# Circular Motion



$$\vec{a}(t) = -\frac{v^2}{r} \hat{r}$$

"centripetal acceleration"

## Motion along a line



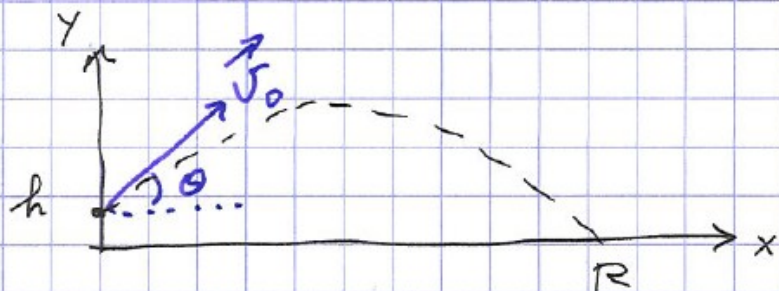
### Equations

Constant velocity :  $x(t) = x_0 + v_0 t$   
 $(\delta x / \delta t = v_0)$

Constant acceleration :  $x(t) = x_0 + v_0 t + \frac{1}{2} a_0 t^2$   
 $(\delta v / \delta t = a_0 ;$   
 $\delta x / \delta t = v_0 + a_0 t)$



## Example: Baseball home run



$$\vec{v}_0 = v_{0x} \hat{i} + v_{0y} \hat{j}$$

Neglect air resistance.

Then the horizontal (x) motion has constant velocity

$$\begin{aligned} x(t) &= x_0 + v_{0x} t \\ &= 0 + v_0 \cos \theta \cdot t \end{aligned}$$

The vertical (y) motion has constant acceleration

$$y(t) = y_0 + v_{0y} t + \frac{1}{2} a_{0y} t^2$$

$$y(t) = h + v_0 \sin \theta \cdot t - \frac{1}{2} g t^2$$

$$g = 9.8 \text{ m/s}^2 = 32 \text{ ft/s}^2$$

### Numerical example

Suppose  $v_0 = 90 \text{ mph} = 132 \text{ ft/s}$

and  $\theta = 45$  degrees; and

Calculate  $R$ .  $h = 3 \text{ ft}$

$$y(t) = 0 = h + \frac{v_0}{\sqrt{2}} t - \frac{1}{2} g t^2$$

$$t = \frac{v_0/\sqrt{2} + \sqrt{(v_0/\sqrt{2})^2 + 2gh}}{g}$$

(CALCULATOR)

$$t = 5.9 \text{ s}$$

$$R = \frac{v_0}{\sqrt{2}} t = \underline{\underline{547 \text{ ft}}}$$

makes sense ↑ ??? why not.