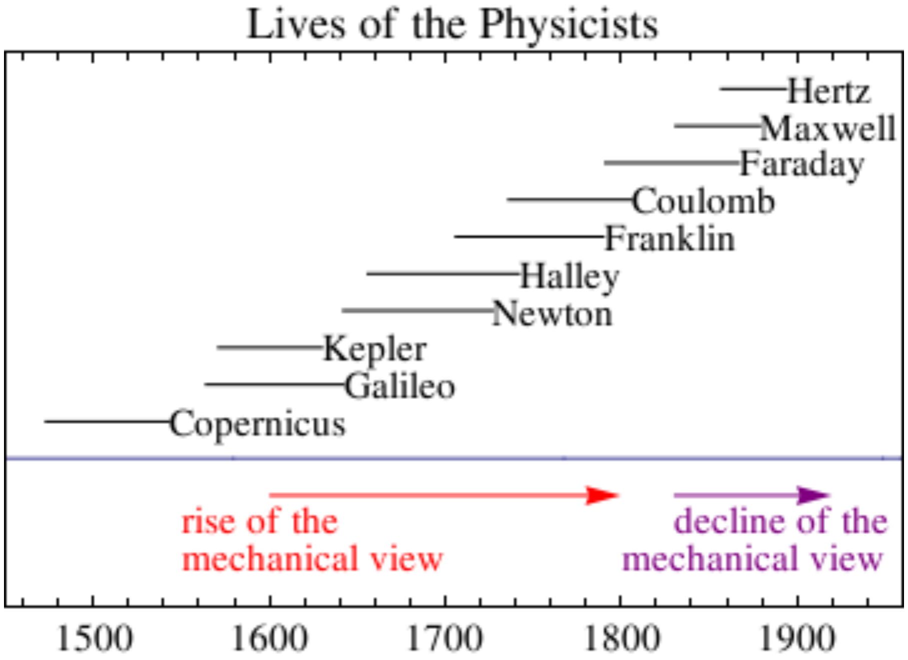
Charges and Fields - Part I

History of Physics



The mechanical view of physics: all physical phenomena are due to *materials* and *forces* acting in absolute space and time.

The modern view: some physical phenomena involve *non-material fields*. Space and time are relative coordinates; i.e., they depend on the observer.

The decline of the mechanical view occurred because of advances in electricity, magnetism, optics and relativity.

Charges and Fields - Part I

Who first discovered electricity?

Thales of Miletus (the first philosopher of Western Civilization; 624 - 546 BC) noted that when *amber* is rubbed with animal fur, it would then attract small bits of straw or feathers.

William Gilbert (the queen’s doctor; 1544 - 1603) coined the word “electricity” to describe this phenomenon, after the greek word ‘*electron*’ for amber.

The electric force

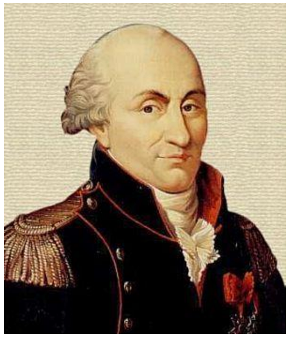
Many people following Gilbert studied electric phenomena.

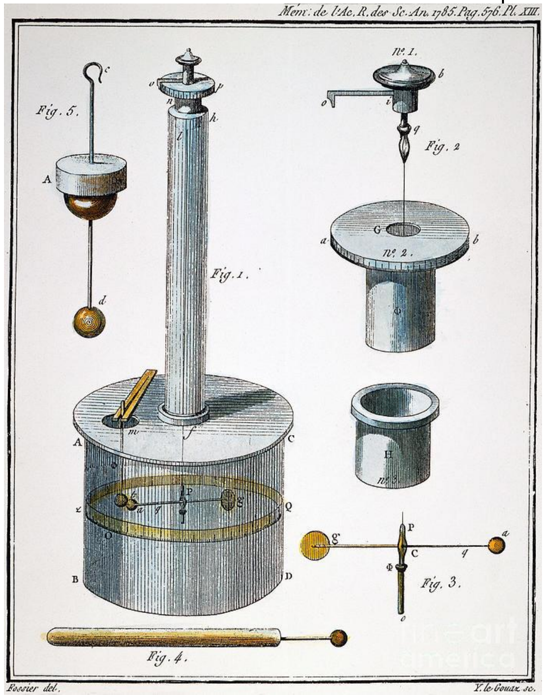
Benjamin Franklin developed a theory that “electricity” was a kind of fluid - a material substance - that would flow from one object to another; then the objects would be electrified, with either positive charge (excess fluid) or negative charge (deficit of fluid).

You learned in grade school: “like charges repel and unlike charges attract”.

__Who first measured the electric forces accurately?__

Charles Augustin de Coulomb





$$\mathbf{F} = \frac{K Q_1 Q_2}{r^2} \hat{\mathbf{r}}$$

$K = 8.99 \times 10^9$ N m^2 / C^2

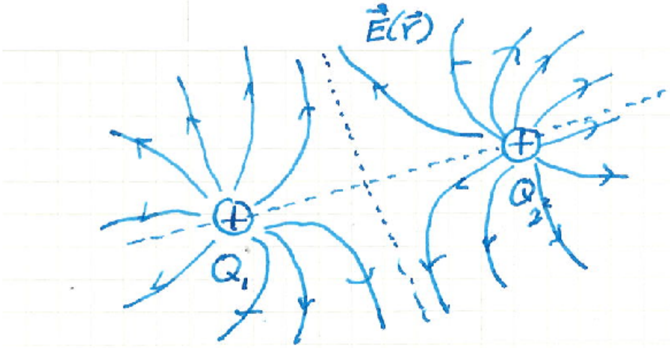
The unit of electric charge is the coulomb [C].

The charge of a single electron is -e (negative!) ; $e = 1.602 \times 10^{-19}$ C

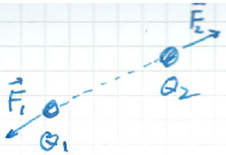
Faraday and Maxwell

Faraday, the most famous experimental physicist of his time, postulated that electric (and magnetic) effects are produced by "***lines of force***".

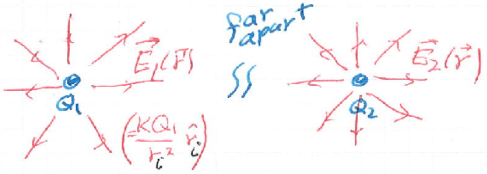
Maxwell, a little younger than Faraday and with greater mathematical knowledge, developed Faraday's idea into a full mathematical theory of electricity and magnetism; he chose the word "field" for the lines of force.

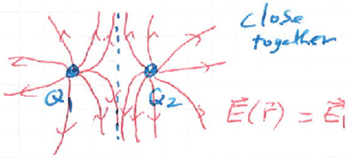


The field theory of electric forces



$\vec{F} = \frac{K Q_1 Q_2}{r^2} \hat{r}$





$\vec{E}(\vec{r}) = \vec{E}_1(\vec{r}) + \vec{E}_2(\vec{r})$

$\vec{E}(\vec{r}) = \sum_i \frac{KQ_i}{r_i^2} \hat{r}_i$

$\boxed{\epsilon_0 = \frac{1}{4\pi K}}$

Energy density $u_{el}(\vec{r}) = \frac{1}{2} \epsilon_0 \vec{E} \cdot \vec{E}(\vec{r})$

$u_{el} = \frac{\epsilon_0}{2} E_1^2 + \frac{\epsilon_0}{2} E_2^2 + \underbrace{\frac{\epsilon_0}{2} 2\vec{E}_1 \cdot \vec{E}_2}_{\text{interaction energy density}}$

Total interaction energy $= \sum \epsilon_0 \vec{E}_1 \cdot \vec{E}_2 \, \delta V$

(Calculus : $U = \int \epsilon_0 \vec{E}_1 \cdot \vec{E}_2 \, dV$)

$U = \epsilon_0 \, KQ_1 \, KQ_2 \underbrace{\int \frac{dV}{r_1^2 r_2^2}}_{= \frac{4\pi}{r_{12}}}$

$\downarrow$

$U(r_{12})$

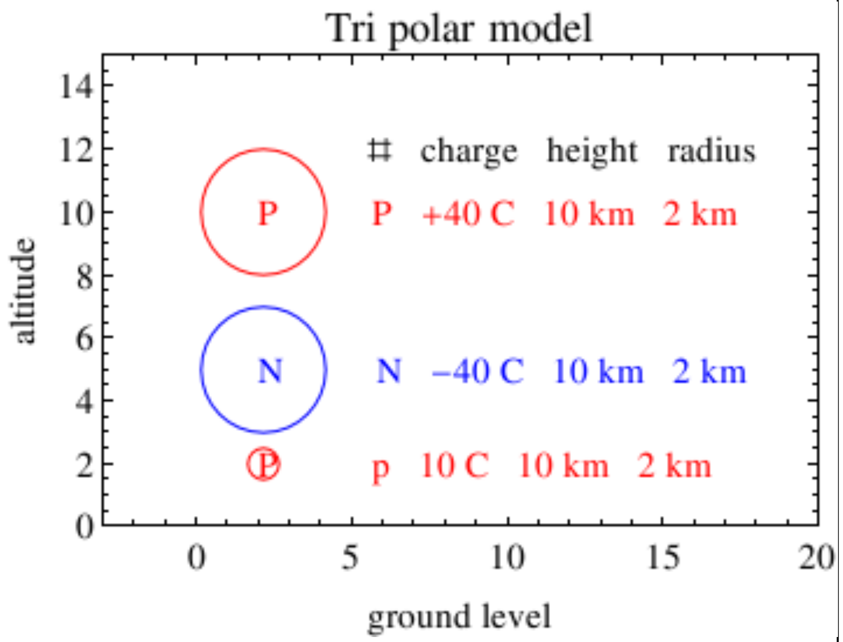
$U(r) = \frac{K Q_1 Q_2}{r}$; $\vec{F} = \frac{K Q_1 Q_2}{r^2} \hat{r}$

(Coulomb)

$\boxed{\delta U = -\vec{F} \cdot \delta \vec{r}}$

Atmospheric Electricity in a Thunderstorm

The past ~ 50 years of research on atmospheric electricity can be summarized in a simple ***model*** of a thunderstorm cloud.



Calculate the electric field at height 1.5 km (the base of the cloud).

$$\vec{E}(\vec{r}) = \sum_{i=1}^{3} \frac{kQ_i}{r_i^2}\hat{r}_i$$

$$E_z = K\left\{\frac{-10C}{(0.5km)^2} + \frac{40C}{(3.5km)^2} - \frac{40C}{(8.5km)^2}\right\}$$

$$K = 8.99 \times 10^9 \frac{Nm^2}{C^2}$$

Units: $\frac{Nm^2}{C^2} \cdot \frac{C}{m^2} = \frac{N}{C}$

<u>OR</u> $\frac{J\,m}{C\,m^2} = \frac{V}{m}$

(volt = joule/coulomb)

$$E_z = -3.35 \times 10^5 \ V/m$$

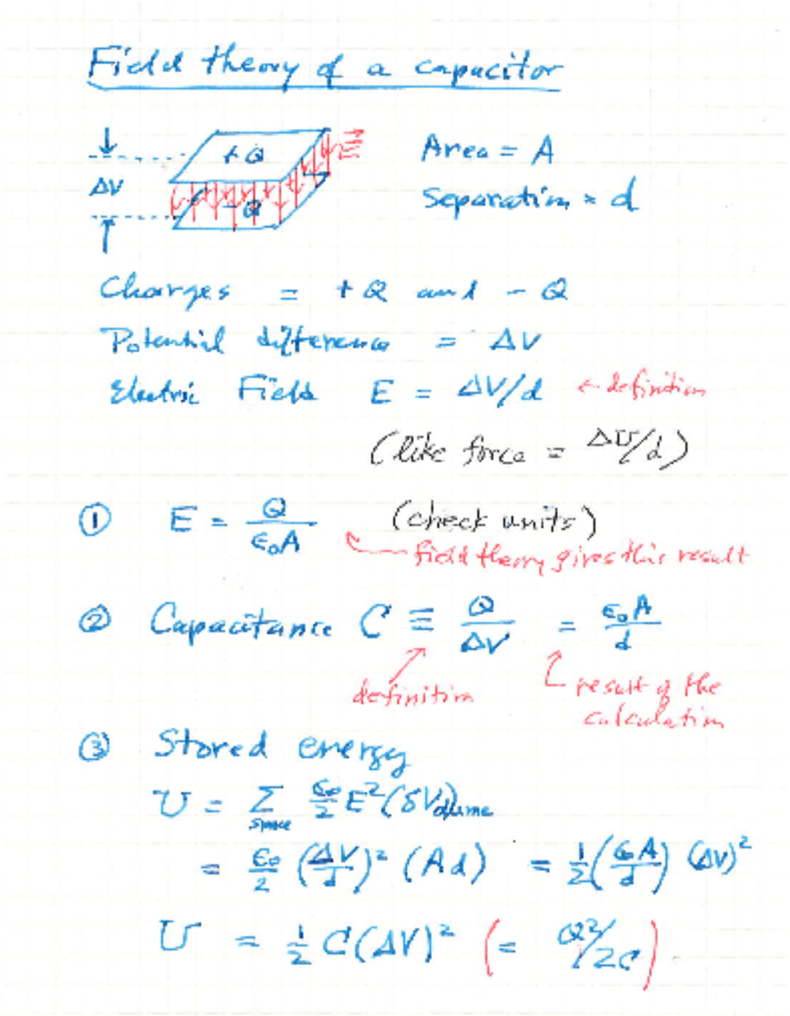
Dielectric breakdown of dry air occurs at approximately

$E_B = 3 \times 10^6$ V/m

Capacitors

A capacitor is a circuit device that stores separated charge. The simplest design is a parallel plate capacitor - two metal plates separated by an insulator (i.e., dielectric) material

Field theory of a capacitor



Field theory of a capacitor

+Q

ΔV

-Q

Area = A

Separation = d

Charges = +Q and -Q

Potential difference = ΔV

Electric Field $E = \Delta V/d$ ← definition

(like force = $\Delta U/d$)

① $E = \frac{Q}{\epsilon_0 A}$ (check units)

field theory gives this result

② Capacitance $C \equiv \frac{Q}{\Delta V} = \frac{\epsilon_0 A}{d}$

definition

result of the calculation

③ Stored energy

$$U = \sum_{\text{space}} \frac{\epsilon_0}{2} E^2 (\delta V_{\text{olume}})$$

$$= \frac{\epsilon_0}{2} \left(\frac{\Delta V}{d}\right)^2 (Ad) = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right) (\Delta V)^2$$

$$U = \frac{1}{2} C (\Delta V)^2 \left(= Q^2/2C\right)$$