

PHY 491 - 2013

Atomic, Molecular, and Condensed Matter Physics

Problem Set 6

1. Calculate the ground state energy of the hydrogen atom using the variational wave function of the form $\psi(\mathbf{r}) = A \exp(-r^2/a^2)$ with one variational parameter a ; the normalization constant A depends on a . Compare with the exact value of the ground-state energy. Explain the sign of the difference. (5 pt)
2. Consider a system of two identical particles, each with spin S . Show that the spin states can be separated into $(S+1)(2S+1)$ symmetric and $S(2S+1)$ antisymmetric states with respect to spin permutation (6 pt)
3. For a hydrogen molecule H_2 , find the total nuclear spin in the symmetric (ortho) and anti-symmetric (para) state. Compare the state degeneracies with the result of Problem 2. Calculate the ratio of the densities of para- and ortho-hydrogen for high temperatures (6 pt)
4. Use the argument of the symmetry of the wave function to determine whether ortho- or para-state of a hydrogen molecule has lower ground-state energy; think of the rotational part of the energy (8 pt).

The required total number of points is 20. The solution is due on October 16.