PHY 491 - 2013

Atomic, Molecular, and Condensed Matter Physics

Problem Set 9

1. In elastic neutron scattering, a flux of neutrons is sent onto a crystal and the scattered beam is observed for the neutrons with the same energy as in the incident beam. What is the typical range of neutron energies appropriate for such experiments? (3 pt) In fact, Bragg neutron scattering is used to obtain almost monochromatic neutron beams, i.e., beams where the energies of the neutrons lie in a very narrow range and the neutrons are propagating almost in the same direction. How does it work? (3 pt)

2. Find the density of lattice points per unit area in a lattice plane given that the spacing between neighboring planes of the family is d and the volume of the primitive cell is V_c. (4 pt)

3. Starting with the Hamiltonian $H = -\frac{\hbar^2}{2m_e}\boldsymbol{\nabla}^2 + U(\mathbf{r})$ with a periodic potential $U(\mathbf{r})$, derive the Schrödinger equation for the periodic part of the Bloch wave function $u_{\mathbf{k}}(\mathbf{r})$, $H_{\mathbf{k}}u_{\mathbf{k}}(\mathbf{r}) = E_{\mathbf{k}}u_{\mathbf{k}}(\mathbf{r})$. Show that the resulting eigenvalue problem, with account taken of the boundary conditions, has real eigenvalues. (4 pt) Use this equation to show that $\partial_{\mathbf{k}}E_{\mathbf{k}} = \int u^*_{\mathbf{k}}(\mathbf{r})\partial_{\mathbf{k}}H_{\mathbf{k}}u_{\mathbf{k}}(\mathbf{r})d\mathbf{r}$. (4 pt) Use the result to show that $\int d\mathbf{r}\psi^*_{\mathbf{k}}(\mathbf{r})(-im_e^{-1}\hbar\boldsymbol{\nabla})\psi_{\mathbf{k}}(\mathbf{r}) = \hbar^{-1}\partial_{\mathbf{k}}E_{\mathbf{k}}$ (5 pt) Please, follow the sequence of calculations as formulated in the problem, **do not** use the derivation in the textbook

You are supposed to get 23 points. The solution is due on November 13.