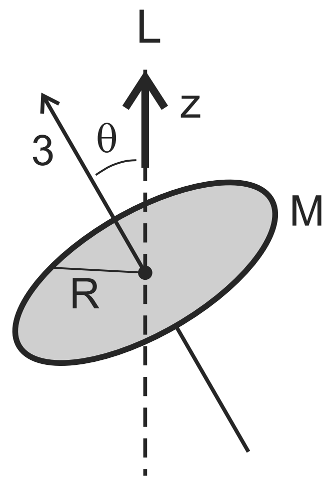
PHY820 Homework Set 11

1. [5 pts] Goldstein, Problem 5-29.

2. [10 pts] (a) Within the Lagrangian approach to rotation, use conservation laws to arrive at analytical solutions for the Euler angles as a function of time, of an axially symmetric body precessing uniformly in the absence of torques. Assume that the third body axis coincides with the axis of axial symmetry, $I_1 = I_2 \neq I_3$. Note that the requirement of a constant polar angle in the external frame imposes a condition on the behavior of the effective potential. (b) Use your results to express components of angular velocity $\vec{\omega}$ and angular momentum $\vec{L}$, along the principal axes, in terms of instantaneous Euler angles and generalized velocities. (c) By applying backward rotations, taking you from the body to the external frame, find the components of angular momentum in the external frame. Does the result make sense and why? (d) Assume now that the axially symmetric body ia a flat uniform disk of radius R and mass M. How are the principal moments I_1 and I_3 related to each other? How are $\dot{\Psi}$ and $\dot{\Phi}$ related to each other? The latter play the role of angular precession velocities for $\vec{\omega}$ in the body frame and external frame, respectively.



3. [10 pts] Investigate the motion of a heavy symmetric top of Sec. 5.7 for the case when the top's rotation axis is vertical. In the latter case, the third external and body axes coincide. By considering the effective potential around $\Theta = 0$, show that the motion is stable or unstable depending on whether $I_3\,\omega_3$ is greater or lesser than $2\sqrt{I_1\,M\,g\,\ell}$. Sketch the effective potential in the two cases. If the top is set spinning in the stable configuration, what is the effect as friction gradually reduces ω_3? *Hint:* Start out from the definitions of p_Ψ and p_Φ and demonstrate that these two momenta become identical when the top is set at $\Theta = 0$. The latter ensures a good behavior of $V'(\Theta)$ around $\Theta = 0$.

4. [10 pts] Goldstein, Problem 5-9. Hint: Start out by considering the components of the torque along the principal axes of the inertia tensor.

5. [5 pts] Goldstein, Problem 5-11. Hint: Consider the magnitude of angular momentum in the body frame. What does the precession imply for the magnitude of angular momentum?