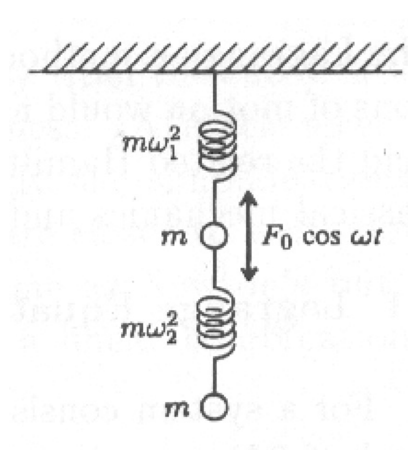
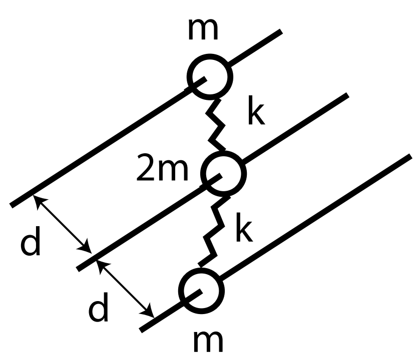
PHY820 Homework Set 13

1. [10 pts] For the system in problem 6-12 in Goldstein, determine the particle positions as a function of time, if, at $t = 0$, (a) the displacements and the velocity of the second particle are zero while the first particle moves at a velocity v, (b) the velocities and the displacement of the second particle are zero while the first particle is displaced by $+d$. (c) Find the general solution of the equations of motion if the particles get subjected to friction forces proportional to velocities, with a proportionality coefficient ν.

2. [5 pts] A mass m is suspended from a support by a spring with spring constant $m\,\omega_1^2$. A second mass m is suspended from the first by a spring with spring constant $m\,\omega_2^2$. A vertical harmonic force $F_0 \cos\omega t$ is applied to the upper mass. Find the steady-state motion for each mass. Examine what happens when $\omega = \omega_2$.



3. [10 pts] From a CM Final: Three beads of mass m, $2m$ and m, respectively, are threaded onto three parallel rods, a distance d apart from each other as shown. The beads are connected with springs characterized by a spring constant k. (Assume that the length of unstretched springs is zero.) The beads can move along the rods without friction. Find the normal modes of oscillation of the bead system (frequencies and amplitude vectors - no particular normalization required). Discuss those modes.



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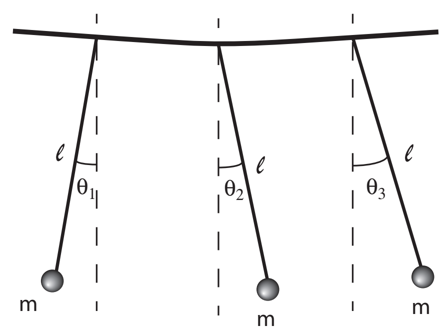
4. [5 pts] Consider three identical pendula suspended from a slightly yielding support. Because the support is not rigid, a coupling occurs between the pendula, making the potential energy approximately equal to:

$$U \approx \frac{1}{2}\, m\, g\, \ell\, (\theta_1^2 + \theta_2^2 + \theta_3^2) - \epsilon\, m\, g\, \ell\, (\theta_1\, \theta_2 + \theta_1\, \theta_3 + \theta_2\, \theta_3)\,,$$

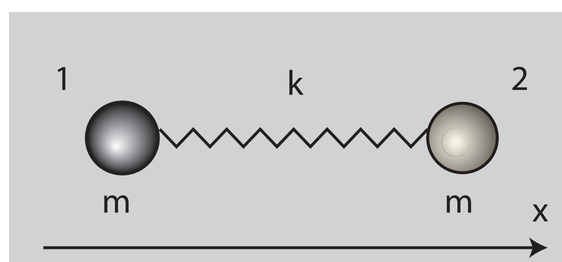
where $\epsilon \ll 1$, while the kinetic energy remains equal to

$$T = \frac{1}{2}\, m\, \ell^2\, (\dot{\theta}_1^2 + \dot{\theta}_2^2 + \dot{\theta}_3^2)\,.$$

Find the normal frequencies and normal modes for the coupled system. Note: Given the three degrees of freedom, three modes are expected. With the reflection and cyclic symmetries of the system, an individual mode can be expected to be either invariant under a symmetry or get interchanged with another mode. In the latter case, the frequency should not change. After you find the modes, classify their behavior under the symmetries.



5. [10 pts] Consider two identical particles, 1 and 2, of mass m, connected by a massless spring of spring constant k, moving in one dimension, parametrized in terms of x, in a fluid. The particles are subject to drag forces from the fluid, respectively $-b\,\dot{x}_1$ and $-b\,\dot{x}_2$. (a) Ignore any other forces, but the drag and spring forces. Start with the Newton's equations for the particles, turn to the motion of the center of mass and the motion in the relative separation and solve the equations to arrive at the general form of t-dependence for $x_1(t)$ and $x_2(t)$. You do <u>not</u> need to relate the arbitrary constants in the solution to the initial conditions at $t = 0$. (b) Next turn to the matrix approach for small oscillations. What eigenvalues would you expect, on the basis of (a), as solutions to the determinant equation? Show that the expected eigenvalues indeed satisfy the determinant equation. Find the amplitude eigenvectors corresponding to the eigenvalues.



6. [10 pts] Goldstein, Problem 6-18.