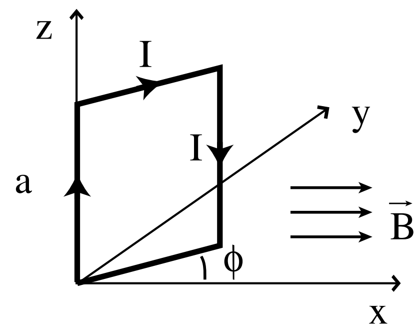
PHY820 Homework Set 14

1. [5 pts] Goldstein, Problem 8-14.

2. [10 pts] An exam problem: An ideally conductive square loop can rotate around its side placed on the z-axis, as shown, within a constant uniform magnetic field $\vec{B}$ along the x-axis. The loop's side length is a, moment of inertia is J and self-inductance is $\mathcal{L}$. As generalized coordinates describing the loop, one can use the angle ϕ of the loop relative to the x-axis and the net charge q that passed around the loop in the clockwise direction. (The current is $I \equiv \dot{q}$.) In terms of these coordinates, the Lagrangian for the loop can be written as



$$L(\phi, \dot{\phi}, q, \dot{q}) = \frac{1}{2}\, J\, \dot{\phi}^2 + \frac{1}{2}\, \mathcal{L}\, \dot{q}^2 - \dot{q}\, a^2\, B\, \sin\phi\,.$$

Here, one can recognize the rotational and inductive energies of the loop and an interaction term of the loop's magnetic moment with the field. (a) From the Lagrangian, find the conserved quantities for the motion of the loop. Can you interpret those quantities? (b) Obtain a Hamiltonian for the loop in terms of the specified coordinates and generalized momenta. (c) Exploit the conservation laws from (a) to obtain an effective potential $U_{eff}(\phi)$ for the motion of the loop in ϕ. Sketch the potential and discuss qualitatively the possible motions in ϕ depending on initial conditions.

3. [5 pts] Goldstein, Problem 8-27.

4. [5 pts] Goldstein, Problem 9-39(a) *only.*

5. [5 pts] A canonical transformation, representing the rotation by an angle α in the phase-space, is given by the equations

$$Q = q\cos\alpha + \lambda\, p \sin\alpha\,, \qquad P = p\cos\alpha - \frac{1}{\lambda}\, q \sin\alpha\,,$$

where λ is some scale parameter. (a) Find the equations for an inverse transformation. (b) Obtain $p = p(q, P)$ and $Q = Q(q, P)$. (c) Determine the generating function $F(q, P)$ for the above canonical transformation.

6. [10 pts] The Hamiltonian for a particle, described in terms of a cartesian position vector $\vec{r}(t)$ and conjugate momentum $\vec{p}(t)$, is $H = p^2/2m + V(\vec{r})$. Construct generators of infinitesimal canonical transformations representing changes that $\vec{r}$ and $\vec{p}$ undergo when the observer moves to a frame (a) moving at a constant velocity along direction $\vec{n}$ and (b) rotating at a constant angular velocity around an axis passing through the coordinate origin, directed along $\vec{n}$. What are the small parameters of transformation in these two cases? Are the transformations restricted? Are values of the Hamiltonian changed?

7. [10 pts] Goldstein, Problems 9-4 *and* 9-22.