PHY820 Homework Set 6

1. [10 pts] An exam problem: Discuss the 2-dimensional motion of a particle moving in an attractive central-force described by the force law $f(r) = -k/r^{\alpha}$, where k is positive and $3 > \alpha > 2$.
   (a) Write down the equations of motion in polar coordinates;
   (b) Show how conservation laws can be used to derive the formal equation for the orbit of motion;
   (c) Describe the nature of the orbits for various possible initial energies and angular momenta. (Graphical methods can be very useful.)

2. [10 pts] Goldstein, Problem 3-12. Refrain from following the book's advice of $m = +1$; for convergence, the potential must vanish faster with the distance. The integral in relative distance, that eventually needs to be evaluated, can be expressed in terms of the Gamma function.

3. [10 pts] Consider a particle of mass m, moving at angular momentum ℓ, under the influence of the central force of the form
$$F(r) = -\frac{\alpha}{r^2} - \frac{\beta}{r^4},$$
where $\alpha, \beta > 0$. (a) Find minimum ℓ for which circular orbits, $r = const$, are possible. How many of those orbits emerge? (b) Examine mathematically the stability of those orbits. (c) Sketch V_{eff} for the three cases: when no circular orbits are possible, only one and more are possible. (d) If only one circular orbit is possible, is it stable or unstable and why?

4. [10 pts] Goldstein, Problem 3-13 (a)-(c). Consider a particle of mass m, moving over an orbit of radius R, at angular momentum ℓ. For (a), establish $r = r(\theta)$ and turn to the differential orbit equation for $u(\theta)$, where $u = 1/r$. For (c), use the second Kepler's law.

5. [5 pts] Goldstein, Problem 3.14.

6. [5 pts] The addition to the potential energy $V = -k/r$ of a small correction $\delta V(r)$ makes the bounded orbits deviate from closed; after each turn, the perihelion shifts by a small angle $\delta\theta$. Find $\delta\theta$ for (a) $\delta V = \beta/r^2$ and (b) $\delta V = \gamma/r^3$. *Hint:* Use the integral representation for the angle covered when moving between r_{min} and r_{max}. Expand the subintegral function in δV. *Note:* General relativity introduces corrections of type (b) to Newtonian gravity.