Geometrical Optics

The light source for this experiment is a low-power helium-neon laser with a wavelength of 632.8 nm. **Never look directly at a laser beam nor permit anyone else to do so!** Exposure to the direct or reflected beam for more than a few seconds will cause serious eye damage. Do not pick up the lasers and shine them around the room. If these simple precautions are taken then there will be no risk associated with the lasers as they are of relatively low power. Special note: if you have long hair, bringing something to keep your hair out of the laser beam is a good idea.

In this experiment you will be measuring the index of refraction. This is a number $n \geq 1$ which represents the ratio c/v, where v is the speed of light in the medium and c is the speed of light in vacuum. For the measurements you make, you should find values of n between 1.4 and 1.6. Your measurements should allow you to determine n within 1-3%.

Theory

The two basic principles of geometrical optics are the law of reflection and the law of refraction (Snell's Law). The law of reflection states that the angle of incidence is equal to the angle of reflection, both angles being measured from the normal to the surface, and that the incident ray, the normal and reflected ray are coplanar.
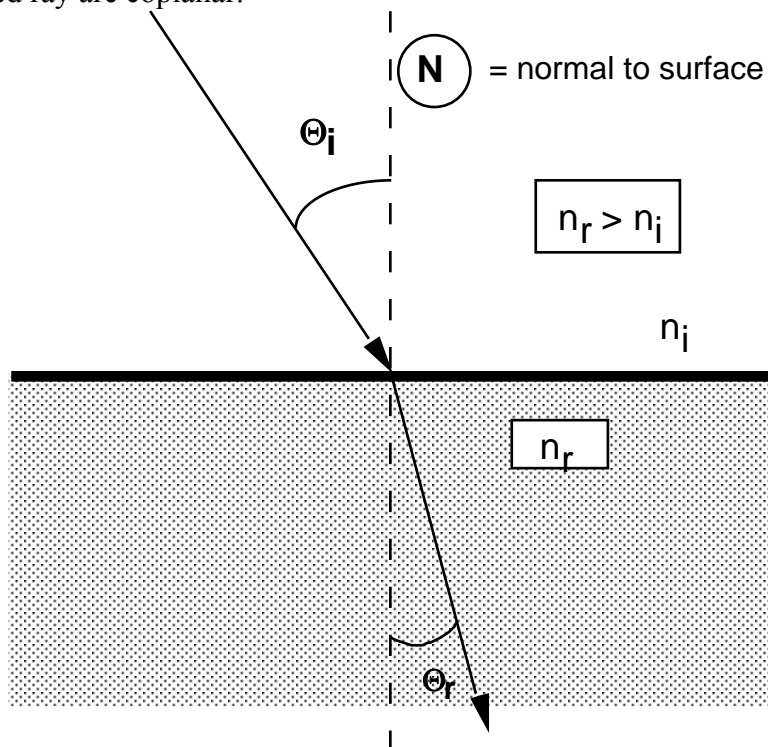

Fig. 1: Refraction at an interface between dissimilar media

Snell's law states that the incident ray, the refracted ray and the surface normal are coplanar and that:

$$n_i \, sin \, \Theta_i = n_r \, sin\Theta_r \qquad (1)$$

where n_i and $\underline{n_r}$ are the indices of refraction of the two media involved. The medium with the larger index of refraction is said to be optically denser than the one with the smaller index. If the ray goes from the optically less dense to the optically more dense medium, then it is refracted <u>toward</u> the normal. If the ray enters from the optically more dense medium, it is refracted <u>away from</u> the normal. In that case, the maximum possible angle of refraction is 90° (along the interface) and the corresponding angle of incidence is then (from Eq. 1):

$$\Theta_i = \Theta_c = \, \sin^{-1}\left(\frac{n_r}{n_i}\right) \qquad (2)$$

which is called the <u>critical angle</u>. If the angle of incidence is larger than the critical angle, there cannot be a refracted ray and the beam is **totally internally reflected**.
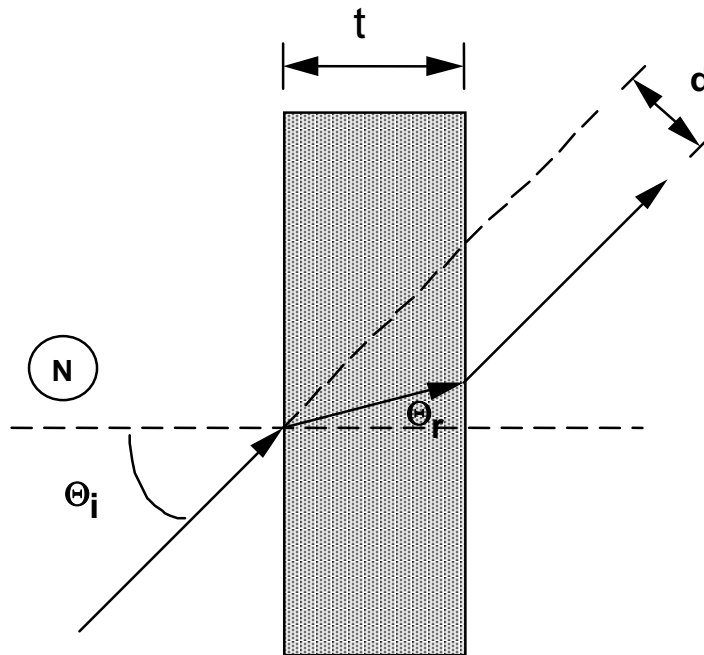

Fig. 2: Refraction through a slab.

The **optical micrometer**, a device for precisely displacing a light beam, is shown in figure 2 above. A light ray traversing a plate of material with parallel entrance and exit faces is displaced laterally without a change in direction. This displacement effect is very useful for laser surveying and laser control of instruments. With high-quality apparatus, displacements as small as one-tenth of one millimeter can be measured and controlled over sizable distances. It can be shown from Snell's law that the lateral displacement d of the beam is given by:

$$d = \frac{t}{\cos \Theta_r} \, \sin(\Theta_i - \Theta_r) \qquad (3)$$

where t is the thickness of the slab and Θ_i , Θ_r are the angles of incidence and refraction of the *entering* beam. [The roles of these two angles are reversed for the exit beam].

Extra Credit: Derive Eq 3 and explain why *n* doesn't appear in it.

Procedure

Be sure to ***make sketches*** of all your setups and indicate trajectories of rays.

1.   Set up the optical table so that the laser beam is parallel to axis of the optical beam and also parallel to the surface of the table, 2 to 3 mm above it, and traverses the table directly above the zero degree median line of the protractor circle. Lock the table in place. Beam positions can always be easily located by interposing a file card into the path of the beam.

2.   Look ahead through the procedure and decide on a good position for the protractor optical table and the moveable screen. Mount the semicircular lens on the protractor circle with the flat surface facing the laser with the pointer at 0°. Make sure that the beam is positioned over the center of the semicircular lens. Autocollimate the beam by rotating the optical table until the reflected beam bounces back into the opening on the laser face. You might not actually see this, but you can see when the reflected beam hits to the right or to the left of the opening and adjust accordingly. From now on you should only rotate the protractor. The protractor sits on the optical table, and allows you to change the orientation of the lenses.

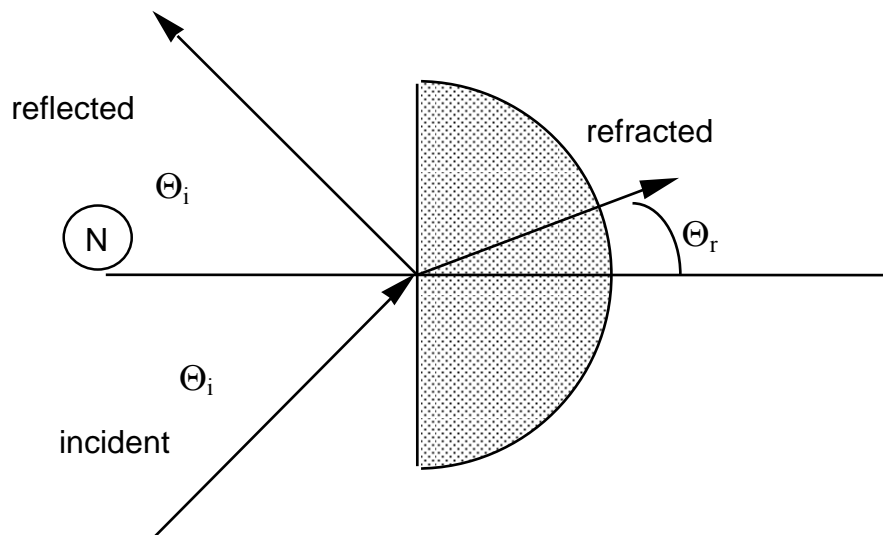**Q1**) Why is it important that the beam pass over the center of the lens?

Fig. 3: Reflection and refraction from a semicircular lens.

3.   Record the position of the refracted laser beam for a series of incident angles in 10° steps. Don't go past 50° or the optical table supporting the protractor may rotate; be sure this optical table isn't rotated between measurements. Note that the beam leaves the lens at right angles to the curved surface and is therefore not refracted on leaving. Qualitatively observe the intensities of the reflected and refracted beams as the angle of incidence is varied and note this in your lab book. You should measure for rotations to the left and to the right. Consistent asymmetries are

suggestive of systematic errors. Such systematic errors can be cancelled to first order by *averaging the left and right angles*.

5.   For each of your measurements separately, calculate the index of refraction of the plastic material from Snell's law, then average your calculated values to get the mean value and determine the standard deviation and the standard deviation of the mean. ***You will need to have a good sketch*** of your setup to relate the measured angles to those needed to apply Snell's Law. Take 1.0 as the value of n_{air} .

Q2) **Extra Credit**: Compare the fractional uncertainty of a typical n measurement with what you might have expected from the input data. *Hint*: find *SD(n)/n*. Compare it with your inputs for θi = 20°: $\delta\theta i/\theta i \oplus \delta x/\theta x$ if you measured a position on the screen, or $\delta\theta i/\theta i \oplus \delta\theta o/\theta o$ if you measured an outgoing angle. Which input of the two uncertainties do you expect to be more important? Why?

6.   Rotate the protractor by 180 degrees and so the curved surface is toward the laser. Observe the behavior of the beams reflected and refracted at the flat surface. Note that the entering beam is not refracted. Again observe the intensities qualitatively for angles of incidence smaller and larger than the critical angle. ***Make a sketch*** *in your lab book explaining your criterion for defining when you are at the critical angle.* Determine the critical angle, and from it calculate the index of refraction. Repeat this measurement of the index of refraction several times, then compute the mean value and the standard deviation for the mean of the index of refraction and compare it to your previous measurement (in step 4).
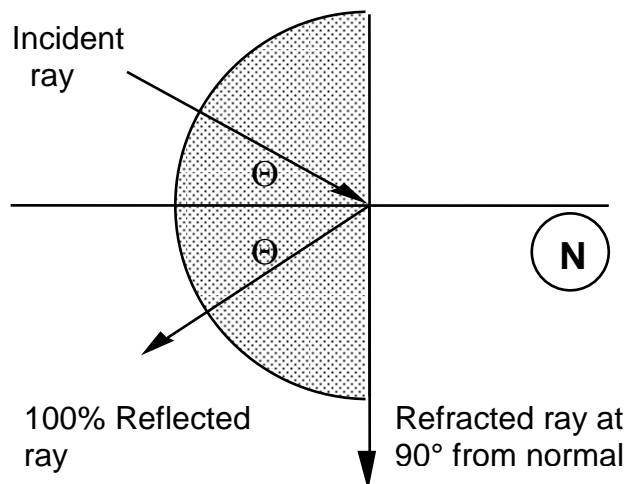

Fig. 4: Total internal reflection; the angles θ depend on the material.

Q3) Calculate δn from $\delta\theta c$. Compare it with SD(n) from the critical angle measurements. Do you think your $\delta\theta c$ was appropriate? Why compare with SD instead of SD of mean?

Extra Credit: take and analyze refraction measurements like steps 3-4 in the geometry of Fig. 4.

7.   **Optical Micrometer Configuration.** Mount the flat plate. For these measurements, which way should it be oriented? *Make a sketch* in your lab book. Autocollimate the beam. You should do

this set of measurements with a smaller distance between the screen and the optical table, and again with a larger distance, a distance L farther away, having measured the distance L between your near and far points of observation. For each set, record the lateral displacement *d* of the laser beam for a series of angles of incidence in 10° steps.   Measure *t*, the thickness of your plate, with a caliper. If you don't know how to use the caliper scale, look in the reference guide. Estimate your measurement errors for *d* and *t*.

8.   For each of your angles of incidence separately, "predict" the displacement *d* from Eq. 3, using the value for the index of refraction measured in part 4 above. Compare your calculations with the measured values of *d* by calculating the ratio *r* = *d*/*d*predicted, and calculating mean, standard deviation, and standard deviation of the mean of *r*.

   Q4) Which uncertainty of your measurements would you expect to be most important for this comparison   *Hint*: Compare the fraction uncertainties of $\delta\theta i/\theta i$ $\delta t/t$, $\delta n/n$, $\delta d/d$ for 20° incidence.

9.   In practice, the laser beam may emerge not only parallel-displaced laterally as the theory of the optical micrometer suggests, but also making some angle Θ_{dev} with respect to the incident path. This might happen, for example, if the slab is not a perfect rectangle. Referring to figure 5 below, you will measure the beam displacement as a function of the incidence angle both for a screen near the optical table, and for one a distance L farther away. If the displacement is strictly parallel, the displacements for the near and far positions should be identical for each incidence angle. Use your section 7 data and the equation below for the deviation angle in *radians*:

$$\Theta_{dev} = \frac{d_{far} - d_{near}}{L} \qquad (4)$$

to calculate the angle of deviation and its uncertainty, for each of your measurement pairs (near and far) at each incidence angle. This equation assumes the angle is small, and in this case $\theta \approx \tan\theta \approx \sin\theta$. Note that the angle can be either positive or negative. You may also find it useful to calculate the mean and standard deviation. Discuss your findings.
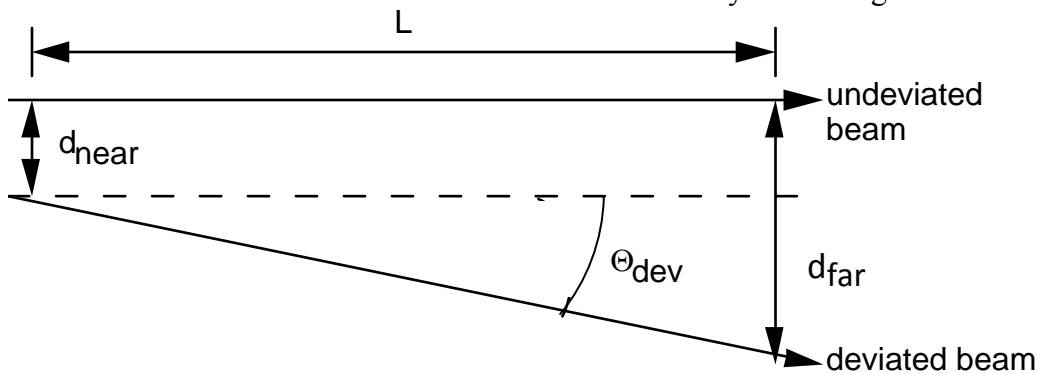

Fig. 5: Deviation of beam going through slab.

   Q5) In your measurements, do the 0° values agree with the position of the beam with no optical element in place? If not, what might this be due to?