The Grating Spectrometer and Atomic Spectra

Introduction

In the previous experiment diffraction and interference were discussed and at the end a diffraction grating was introduced. In this lab most of the things learned in the last experiment will be applied to measure the wavelengths of the lines in the visible spectrum of some elements. You will use a diffraction grating mounted on a spectrometer to measure the interference maxima for various wavelengths of light. Knowing the grating spacing, a measurement of the angle gives the wavelength. This should be a high precision experiment: you should be able to measure wavelengths within 2% before calibration, and .5% after calibration.

Theory

Grating Spectrometer

In the diffraction experiment we showed that the conditions for constructive interference or maxima in the intensity for a double slit are:

$$d \sin \theta_m = m\lambda \quad m = 0, \pm 1, \pm 2, \pm 3 \cdots \qquad (1)$$

Here d is the distance between the two slits, θ is the angle between the incoming beam and the location where we observe the intensity (see Fig. 4 of experiment 4) and m is the order number of the interference. The m=0 peak is what you observe at 0 degrees. As you go left or right, you will see colored stripes corresponding to m= ±1 for different wavelengths, and perhaps at m= ±2 at larger angles.

In the previous experiment no focusing lenses were used because the fringes were observed relatively far away. The spectrometer in this experiment uses collimator lenses as the large distance approximations no longer apply and it has the added advantage that the angle θ is measured directly.

The extension from an N = 2 double slit to a grating where N is large is relatively straightforward and Eq. 1 remains valid except that d is much smaller than it was for N = 2. If the wavelength is fixed and $m = 1$, then the angle "θ" at which the first maximum appears increases as d is decreased (N is increased). This is exactly what we studied in the last part of the diffraction experiment. The higher the grating constant, $G = 1/d$ (usually given as lines per mm), the larger the angle for the maxima and the better the wavelength can be measured.

Question 1:   When the fringe pattern behind the slits is viewed there is always diffraction as well as interference, as we learned in the previous experiment. The individual slits (lines) of the grating, however, are usually so narrow that the central maximum of diffraction for each slit sends light into a wide range of angles. The gratings we use have G = 600 lines/mm. Assuming that the transparent and opaque segments are of equal widths ($a = d/2$), calculate the angular locations of the first diffraction minimum for the approximate limits of visible light (roughly 400 nm to 700 nm).

Atomic Spectra

When the wavelengths of light emitted by excited atomic gases were first studied with precision spectrometers, scientists observed that most of the light was emitted at a few discrete wavelengths. In this experiment, you will observe the discrete emission lines of hydrogen and a few other atomic gases. These discrete lines originate from the de-excitation of atoms. That the lines are discrete was initially a great surprise because it implied that the valid energies of electrons in atoms were also discrete. This

discovery led to the development of quantum mechanics, which is one of the foundations of modern physics.
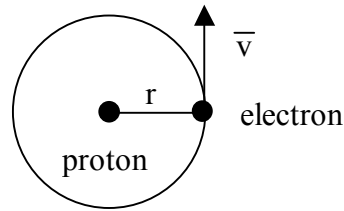
To put this discovery in perspective, consider what one might have expected if Newton's laws governed the energies of atomic electrons. Atomic hydrogen consists of a proton and an electron, which is bound to the proton by the attractive Coulomb force $F_C = k\dfrac{e^2}{r^2}$, where k=8.99x10^9 N·m^2/C^2, e=1.6x10^{-19}C is the magnitude of the electron and proton charges, and r is the distance between them. If the electron is in a circular orbit about the proton, as shown above, Newton's second law, (mass)x(centripetal acceleration)=Coulomb force, implies:

$$m\frac{v^2}{r} = k\frac{e^2}{r^2} \qquad (2)$$

(The proton is so much heavier than the electron that one can assume that it remains stationary while the electron orbits about it.) The potential energy U(r) of the electron can be obtained by integrating the Coulomb force from ∞; doing this provides:

$$U(r) = -\int_{\infty}^{r} d\vec{r} \cdot \overrightarrow{F_c} = -k\frac{e^2}{r} \ . \qquad (3)$$

Thus, the total energy E=1/2mv^2+U can be obtained from Equations (2) and (3) and is given by:

$$E = \frac{1}{2}mv^2 + U = -\frac{1}{2}\frac{e^2}{r} \ . \qquad (4)$$

It is negative because the electron is bound in the attractive Coulomb potential of the proton electric field.

This derivation, which follows from Newton's laws, implies that energy of the electron is negative and has a arbitrary magnitude that depends only on the radius r of the electron orbit. This turns out to be incorrect for small systems like an atom. The electron in a hydrogen atom cannot be in any arbitrary orbit. The energy is *quantized* to only those orbits with energies given by the Bohr formula:

$$E_n = -\frac{13.6eV}{n^2} \qquad (5)$$

where *n* = 1, 2, 3, 4..... and eV is an atomic-scale energy unit (KE of an electron accelerated through a voltage difference of one Volt). One eV= 1.6x10^{-19} J

When an electron makes a transition between an orbit with a higher energy (larger n_i) to one with a lower energy (smaller n_f), it emits a photon which carries off the energy lost by the electron. This implies the energies of photons emitted by a hydrogen gas are given by

$$E_{ph} = E_i - E_f = 13.6 \cdot \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) eV \cdot \qquad (6)$$

The wavelength of the photon can be obtained from its energy by the Planck formula:

$$\lambda = \frac{1240 eV \cdot nm}{E_{ph}} \qquad (7)$$

Question 2: **[Before class]** Only transitions that are in the "Balmer" series with $n_f = 2$, $n_i = 3,4,5,6...$ are visible to the human eye. Calculate the photon energies and wavelengths of the Balmer series for $n_i = 3,4,5,6$. These are the wavelengths you will be trying to measure.

Understanding the Spectrometer

A spectrometer is a device that allows precise measurements of the angle θ, between two rays of light. Because the light from the source to be studied is often not a narrow beam like a laser, it must be gathered and focused by lenses.

The light from the source is first passed through a slit, which should be as narrow as possible, consistent with letting enough light through to be visible. The light spreads out from the slit to the collimating lens, which makes it into a parallel beam (i.e. the slit is at the focal point of the lens). The parallel beam of light then strikes the "diffraction" grating (G=600 lines/mm to within 2%). Beyond the grating, the light travels in various directions due to diffraction and interference. If light of a particular wavelength and order of interference heads in the direction of the telescope, it will be focused down to an image of the slit.

Procedure

If the spectrometer is properly adjusted, the image falls on cross hairs at the focusing ring. This image is then examined with an eyepiece (a short focal length converging lens) to see how closely the image coincides with the cross hairs. If the telescope is first used to locate the 0th order of interference (straight ahead) and subsequently a first order line, then the difference in the Vernier readings is the desired angle Θ_i. **Diagrams in another online document show the location of the controls.**

Part 1

In this part we use the red line of a hydrogen lamp with a known wavelength (656.5 nm) as the source. We wish to adjust the spectrometer and to verify that we can actually measure the wavelength by measuring the position of the first and second order maxima.

Before attempting to move the telescope, be sure that the locking screw is loosened. When you are fine-setting on a line, the locking screw should be tightened and the final adjustment made with the fine-adjust screw (clockwise direction only).

1.   Adjusting the eyepiece. By turning the knurled knob, adjust the eyepiece until, with your *eye* relaxed for distant vision, the cross hairs are in focus.

2.   Adjust the telescope focus by loosening the set screw on top, then sliding the tube in or out to focus on a distant object, so that there is no parallax between cross hairs and slit image. This can be checked by moving your eye back and forth (from left to right). The slit edges should be crisp, and wider than the cross hairs but not enormously so. You may have to open the slits up a bit for dim lines, or tighten them to see closely separated bright lines.

3.   Now set the angle scale to 0 degrees at the straight-through position.

4.   Swing the telescope to the first order for the red line. Use the fine adjustment to make the line coincide with the cross hair and read the Vernier. Return to the central image, set the image on the cross hair and again read the Vernier. The difference in the readings is the angle θ_i. Now locate the first order image on the other side of the central image. How many higher orders can you locate? (Look by eye, with the telescope swung out of the way!) Estimate the accuracy of your angular reading.

5.   Calculate the wavelength from the position of the maxima and compare it to what you expect.

Question 3:   Assume that the error in the grating constant G to be negligible and the error in the angle measurement fixed (by the Vernier scale). (this is part of your uncertainty calculation). Based on this error analysis, is it better to measure λ with $m=1$ (first order interference maximum), or $m=2$ (second order interference)? Is it better to measure λ with large grating constant G or with a small G?

Part 2

Measure in first and higher orders as many hydrogen lines as you can see. If the light is favorable you may be able to see **four** (red, blue-green, purple and deep purple). You hope to see these 4 lines (for interference $m=\pm1$) on each side (left and right). Your analysis should compare the left and right angles for systematic effects, and average the left and right angles or wavelengths for your final analysis. You may be able to see some of the 4 lines in second order ($m=\pm2$) as well. The interpretation of these and other lines led Bohr to his model of the hydrogen atom which predicts a discrete spectrum of light emission rather than continuous. The spectrum consists of three distinct series called Balmer, Lyman and Paschen Series. Each Series represents certain energy transitions from higher levels to a specific lower level in the hydrogen atom. The specific lower levels are n = 1 for Lyman, n = 2 for Balmer and n = 3 for Paschen. As discussed previously, only the Balmer Series transitions are in the visible range of the spectrum. The Lyman Series is in the ultraviolet and Paschen Series is in the infrared. Compare the wavelengths you measure with the expected lines from the Balmer Series. Calculate the error in your measurement of the wavelength.

Question 4: How much does this left-right averaging improve your error? Do you have any evidence of L/R systematic error that this cancels? What might cause such a L/R asymmetry?

Calibration: *Calibration* is using known information to determine characteristics of your measurement instrument. You use a single diffraction grating during the whole lab. Thus its grating constant uncertainty is a systematic error for this experiment. Since the wavelength is directly proportional to d,

calculate the ratio λ/ λ_expected for the H lines you observe, and use this to find a best value for *d*. As a check, you should also report λ/ λ_expected after the calibration (see Question 5 below).

Question 5: How much does this calibration improve your uncertainty in the H lines? Compare δλ/ λ before and after calibration. Is the calibrated value of *d* compatible with the stated uncertainty, if you test by t value?

Part 3

Select a Hg light source from the ones that are available and measure the wavelength of a few of the strongest lines. You should use the calibrated value of d from part 2. (does this improve your agreement with the expected wavelengths for Hg?) You may in addition look at and describe another lamp with different atoms. Use the links below, or a Handbook of Chemistry and Physics to identify the prominent lines in Hg you picked, and compare them to your measurements.
http://www.uvp.com/pdf/wavelengths.pdf (a published paper, though for a pure isotope)
http://www.kayelaby.npl.co.uk/general_physics/2_5/2_5_5.html

In deciding whether your line coincides with the line of an element, you must have some estimate of the precision of your measurement. You must also consider only the most intense lines in the tables.

Question 6:   Which interference maximum gives the most accurate results? Does this agree with you answer to question 3? If not, why not?

Vernier Angle Scale: You can read the angle from a precise protractor scale on the outer rim of the table. Use the Vernier scale with the little magnifying glass to read the angle to the nearest arc minute. (1 arcmin = 1' = 1/ 60 degree.) **The following is an example**:
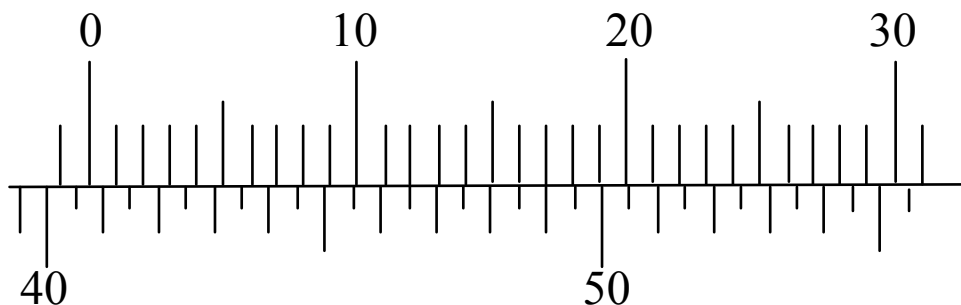

Figure 5

In this example, the zero reference line of the Vernier scale (the upper scale) is between 40.5° and 41°, so the angle is somewhere between 40° 30' and 41°. The Vernier scale tells exactly where in between. Look along the Vernier for the line that exactly lines up with the line below it. In this case, it's the 17' line. So the angle is 40° 47', which we get by adding 17' to 40° 30'. In order to use this measurement, we must convert it to decimal degrees: 40 + (47/60) degrees = 40.78°.