

Chapter 9

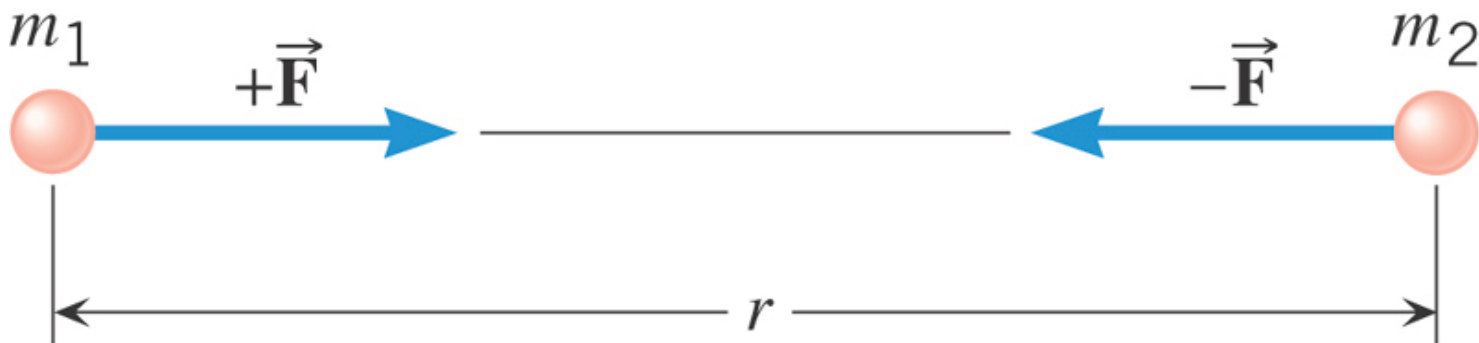
Gravitation ***continued***

9.1 The Gravitational Force

For two particles that have masses m_1 and m_2 and are separated by a distance r , the force has a magnitude given by

$$F = G \frac{m_1 m_2}{r^2}$$

$$G = 6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$$



the same magnitude of force acts on each mass, no matter what the values of the masses.

9.1 The Gravitational Force

Near the earth's surface

$$r = R_E = 6.38 \times 10^6 \text{ m}$$

Radius of the earth

$$mg = G \frac{mM_E}{R_E^2} \Rightarrow g = G \frac{M_E}{R_E^2}$$

$$g = \left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2 \right) \frac{\left(5.97 \times 10^{24} \text{ kg} \right)}{\left(6.37 \times 10^6 \text{ m} \right)^2}$$

$$= 9.81 \text{ m/s}^2$$

This is why acceleration due to gravity is this value on the earth.

Your WEIGHT on the earth

$$W = mg$$

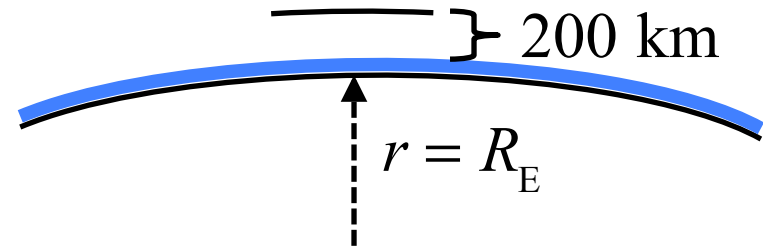
for example: $m = 80.0 \text{ kg}$,

$$W = mg = 784 \text{ N}$$

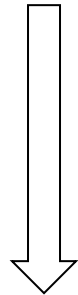
9.1 The Gravitational Force

Near the earth's surface

In orbit at altitude = 200 km



$$g = 9.81 \text{ m/s}^2$$



$$g' = 9.20 \text{ m/s}^2$$

At radius of the earth

At 200 km above the earth

$$r' = R_E + 200 \text{ km} = \underline{6.37 \times 10^6} + 0.20 \times 10^6 \text{ m}$$

$$= \underline{6.57 \times 10^6 \text{ m}}$$

$$g' = \frac{GM_E}{r'^2} = 9.21 \text{ m/s}^2$$

In low-earth orbit,
your weight is almost the same as on earth. NOT ZERO!

Clicker Question 9.1

A person weighs 500 N on the earth. Consider this person on planet P where the acceleration due to gravity is, $g_p = 4.9 \text{ m/s}^2$. Chose the answer that is *false* or answer e).

- a) On the earth, the mass of the person is 51 kg.
- b) Everywhere on the earth, the person has a mass of 51 kg.
- c) On the planet P, the mass of the person is 51 kg.
- d) On the planet P, the weight of the person is 250 N.
- e) All of the above are true.

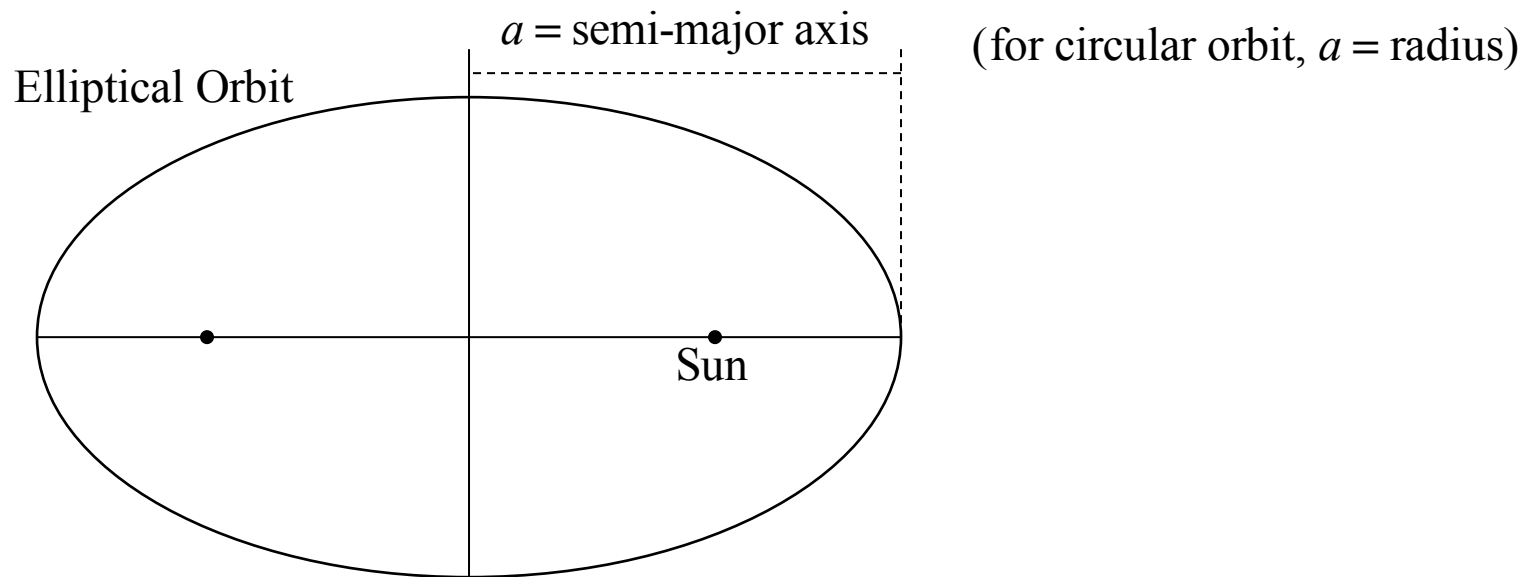
9.2 *Elliptical Orbits*

In general, the orbit of a satellite (around a planet) or planet (around a star) is an ellipse. Kepler was the first to describe this motion for planets around the sun that are a consequence of Newton's Universal Gravitational Force.

Kepler's Laws for planetary orbits (in homework)

1. Orbits are elliptical with the Sun at one focus.
2. In a given time a planet covers the same area anywhere in the orbit.
3. If T is the orbital period and a the semi-major axis then

$$\frac{a^3}{T^2} = C \text{ (constant)} \quad C_{\text{Sun}} = 3.36 \times 10^{18} \text{ m}^3/\text{s}^2$$



9.3 *Gravitational Potential Energy*

The Gravitational force gets smaller for large distances above the earth.

Gravitational Potential Energy for large distances above the earth requires that the Potential Energy be defined as zero at very large distances. The closer one gets to the earth the lower (the more negative) the potential energy becomes.

Near the surface of the earth we use:

$$U = mgy \quad (\text{puts } U = 0 \text{ at } y = 0)$$

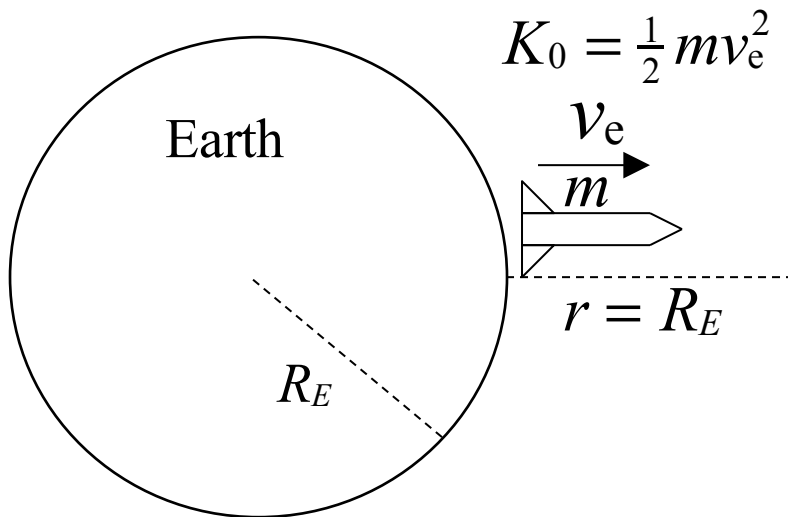
But for extreme heights above the earth:

$$U = -G \frac{mM}{r} \quad (\text{puts } U = 0 \text{ at } r = \infty)$$

9.3 Gravitational Potential Energy

Example: Escape velocity (Energy Conservation)

There is a velocity above which an object fired from the surface of the earth will never return to the earth. A rocket fires just long enough to get above the atmosphere and reach the escape speed.



$$K_0 = \frac{1}{2} m v_e^2$$

$$K = 0$$

$$U = 0$$

$$r \rightarrow \infty$$

Initial

Final

Kinetic Energy: $K_0 = \frac{1}{2} m v_e^2$

$$K = 0$$

$$U = -G \frac{m M_E}{r}$$

Potential Energy: $U_0 = -G \frac{m M_E}{R_E}$
 $r = R_E$

$$U = -G \frac{m M_E}{r} \rightarrow 0$$

$r \rightarrow \infty$

9.3 *Gravitational Potential Energy*

Example: Escape velocity

Energy Conservation:

$$K_0 + U_0 = K + U = 0$$

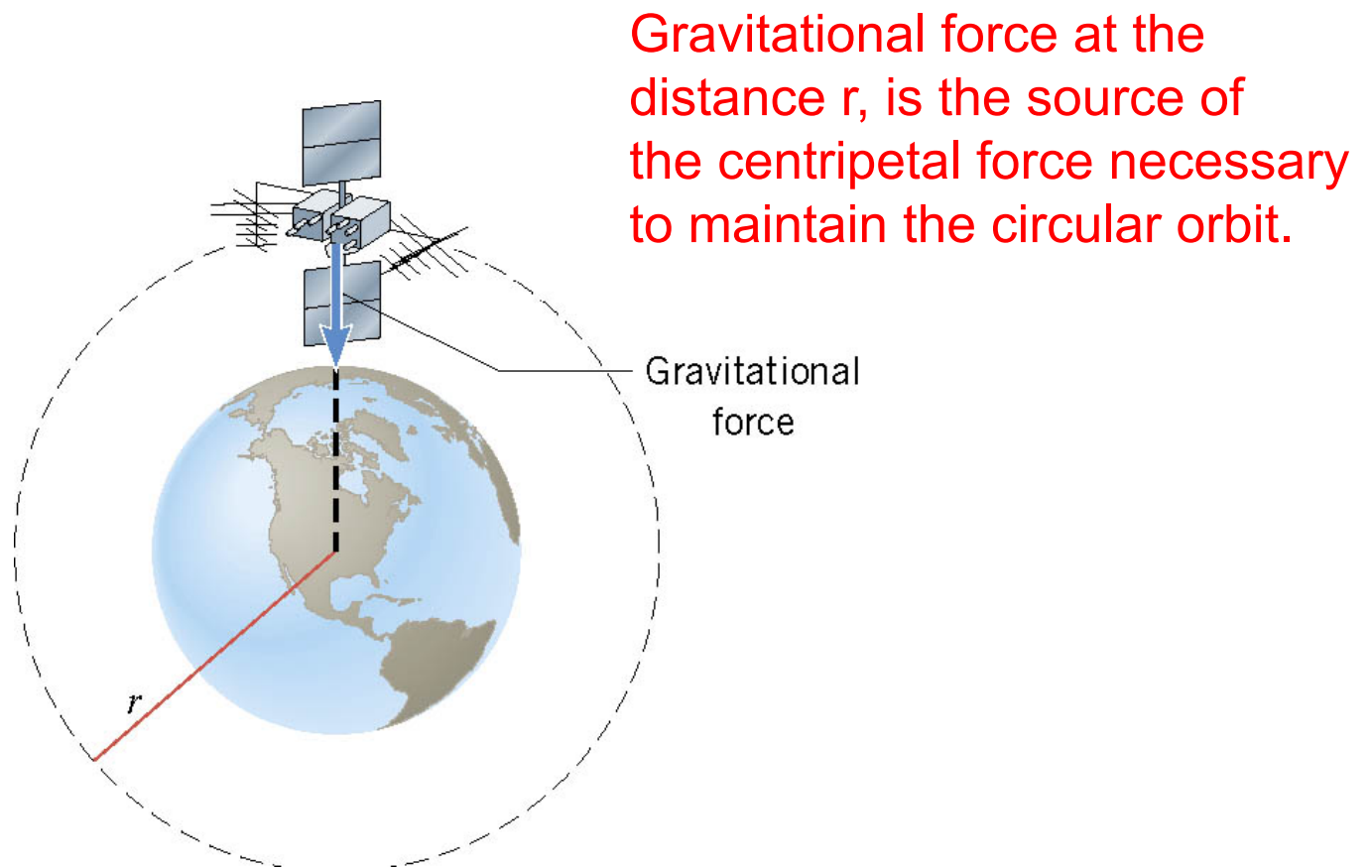
$$\frac{1}{2}mv_e^2 - G\frac{mM_E}{R_E} = 0$$

$$v_e^2 = 2G\frac{M_E}{R_E} \quad \text{earlier: } g = G\frac{M_E}{R_E^2}$$

$$\begin{aligned} v_e &= \sqrt{2gR_E} = \sqrt{2(9.81\text{m/s}^2)(6.37 \times 10^6 \text{ m})} = \\ &= 11.2\text{km/s} = 40.3 \times 10^3 \text{ km/hr } (\sim 25,000 \text{ miles/hr}) \end{aligned}$$

9.4 Satellites in Circular Orbits

There is only one speed that a satellite can have if the satellite is to remain in an orbit with a fixed radius.

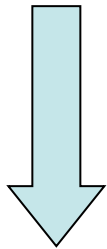


9.4 Satellites in Circular Orbits

Gravitational force
at the distance r

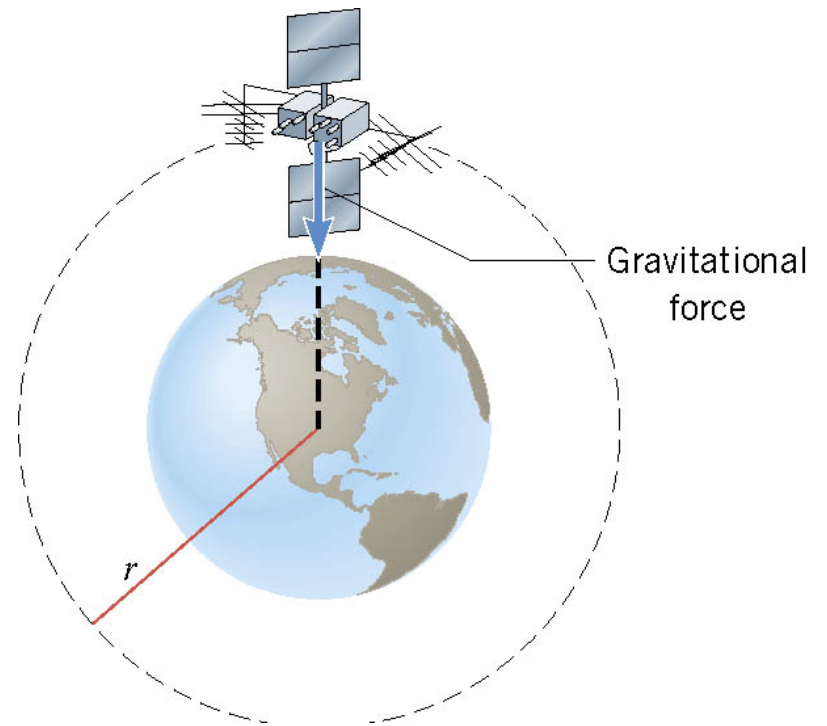
Centripetal force

$$F_C = G \frac{mM_E}{r^2} = m \frac{v^2}{r}$$



$$v = \sqrt{\frac{GM_E}{r}}$$

Speed to keep satellite in the orbit with radius r .



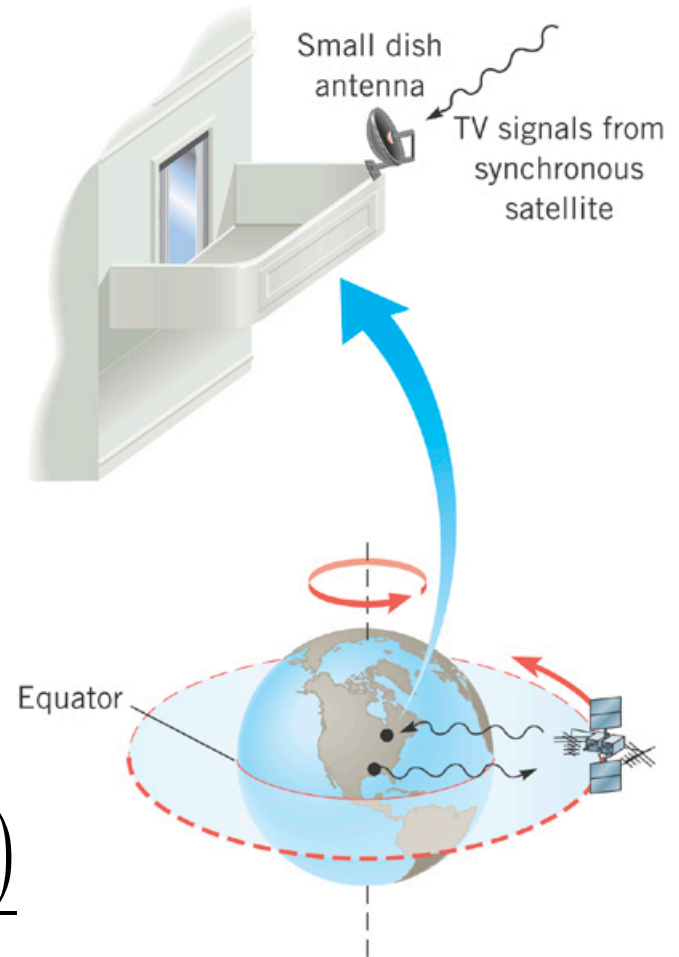
9.4 Satellites in Circular Orbits

There is a radius where the speed will make the satellite go around the earth in exactly 24 hours. This keeps the satellite at a fixed point in the sky.

$$\omega = \frac{2\pi}{(3600 \text{ s/hr})(24 \text{ hr})} \quad \text{same for Earth \& satellite}$$
$$= 72.7 \times 10^{-6} \text{ rad/s}$$

$$\omega r_s = v_T = \sqrt{\frac{GM_E}{r_s}}$$
$$r_s^3 = \frac{GM_E}{\omega^2}$$
$$= \frac{(6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})}{(72.7 \times 10^{-6} \text{ rad/s})^2}$$

$$r_s = 42,200 \text{ km (synchronous orbit)}$$



9.5 *Apparent Weightlessness*

Can you feel gravity? We previously determined that you can't.

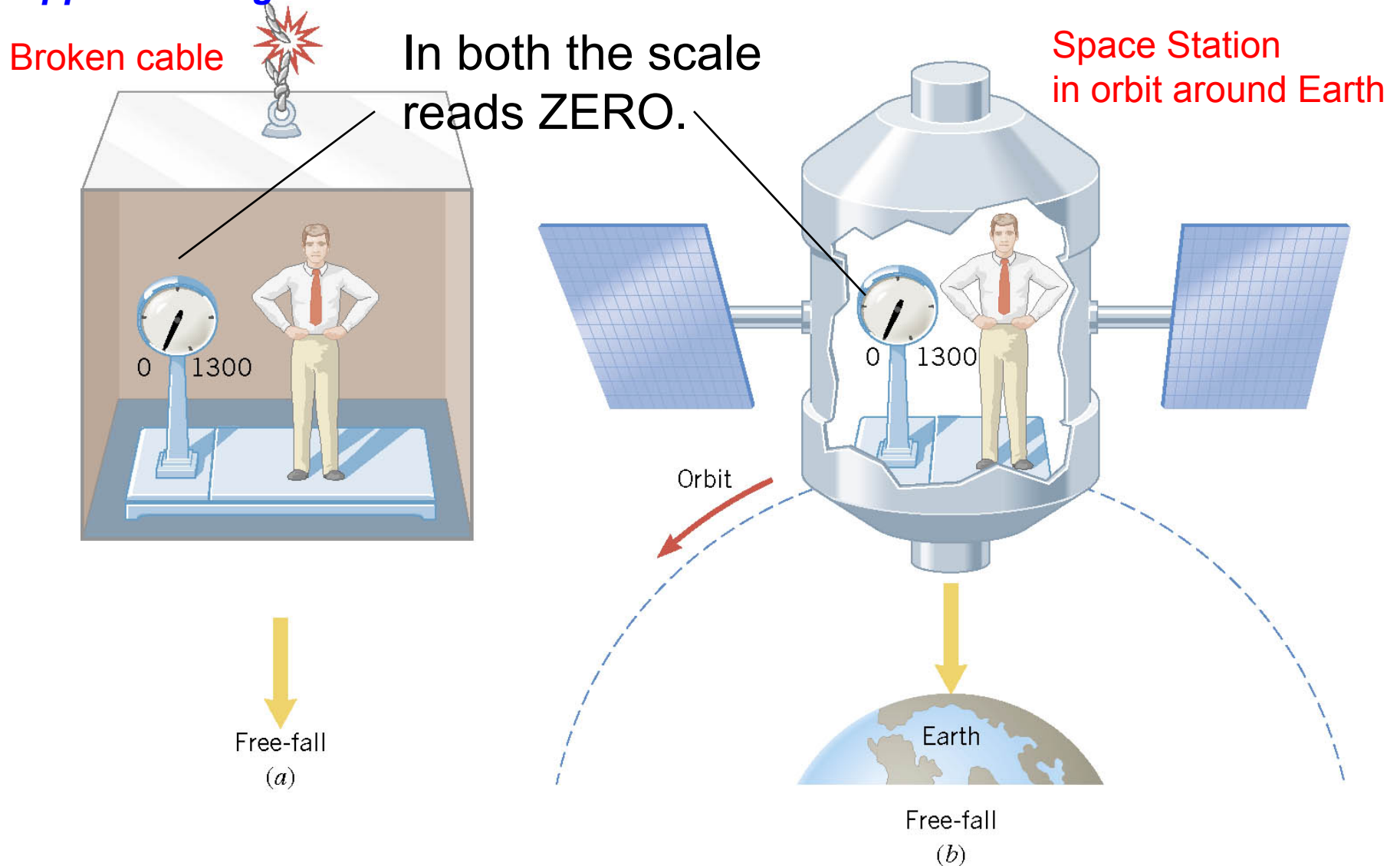
- 1) Hanging from a 100 m high diving board
– your arms feel stretched by the upward force of bent board.
- 2) Standing on a bed – your legs feel compressed by the compressed springs in the mattress.

The bent diving board or the compressed springs provide the force to balance the gravitational force on your whole body.

When you let go of the diving board and before you hit the ground the ONLY force on you is gravity. It makes you accelerate downward, but it does not stretch or compress your body.

In free fall one cannot feel the force of gravity!

9.5 Apparent Weightlessness



But it is the Gravitational Force (definition of weight) that makes both the elevator and the body free-fall with the same acceleration. **FEELING** weightless and **BEING** weightless are **VERY** different.

Chapter 8

Rotational Dynamics ***continued***

8.3 Angular Variables and Tangential Variables

ω = angular velocity - **same at all radii** (radians/s)

α = angular acceleration - **same at all radii** (radians/s²)

$\vec{v}_T$ = tangential velocity - **different at each radius**

$\vec{a}_T$ = tangential acceleration - **different at each radius**

Direction is tangent to circle at that θ

$$\vec{v}_T = \omega r$$

$$\vec{a}_T = \alpha r$$

$$\vec{v}_T \text{ (m/s)}$$

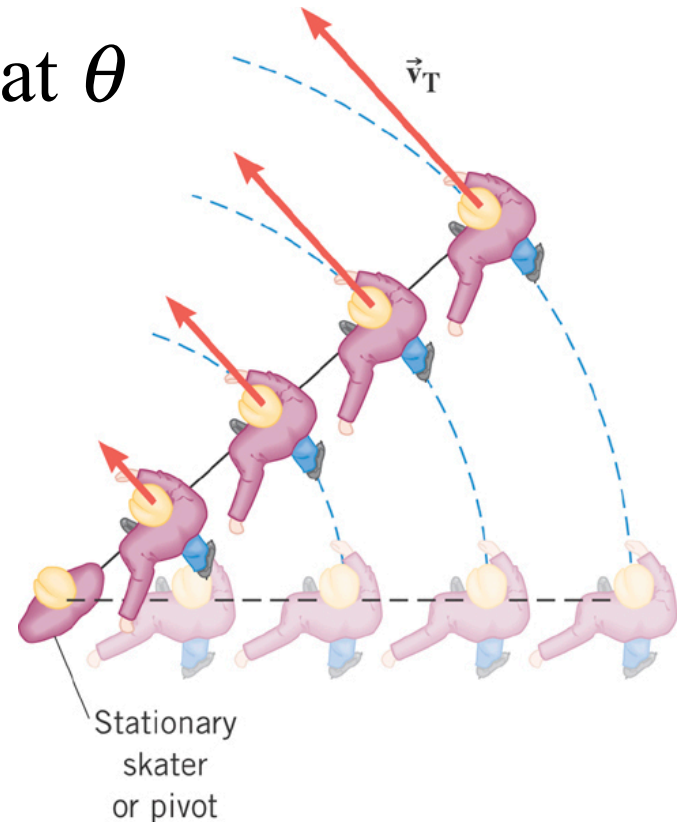
$$\vec{a}_T \text{ (m/s}^2\text{)}$$

$$\omega \text{ (rad/s)}$$

$$\alpha \text{ (rad/s}^2\text{)}$$

$$r \text{ (m)}$$

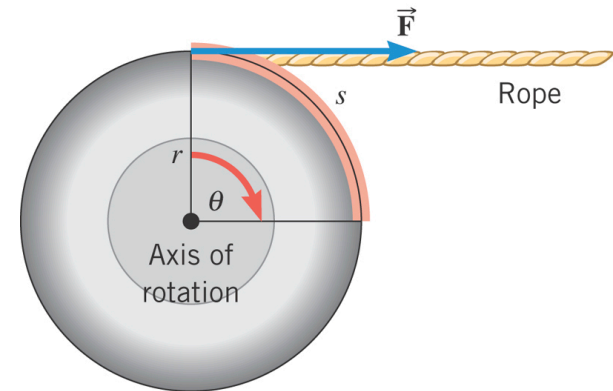
$$r \text{ (m)}$$



8.4 Rotational Work and Energy

Work
rotating
a mass:

$$\begin{aligned} W &= F(\cos\theta)x & x &= s = r\theta \\ &= Fr\theta & Fr &= \tau \text{ (torque)} \\ &= \tau\theta \end{aligned}$$



DEFINITION OF ROTATIONAL WORK

The rotational work done by a constant torque in turning an object through an angle is

$$W_R = \tau\theta$$

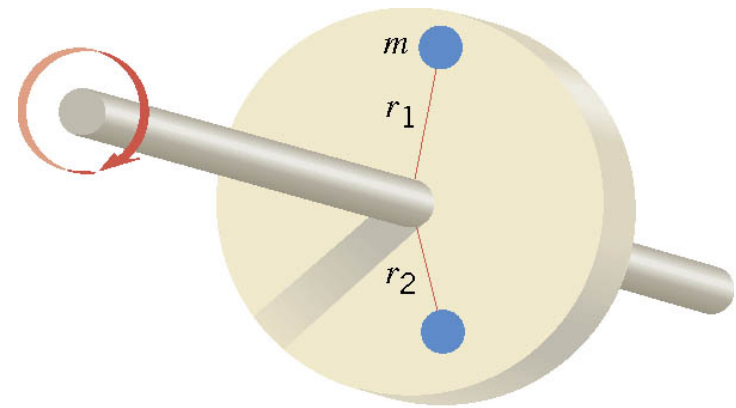
Requirement: The angle must be expressed in radians.

SI Unit of Rotational Work: joule (J)

8.4 Rotational Work and Energy

Kinetic Energy of a rotating **one** point mass

$$\begin{aligned} KE &= \frac{1}{2} m v_T^2 & v_T &= r\omega \\ &= \frac{1}{2} m r^2 \omega^2 \end{aligned}$$



Kinetic Energy of many rotating point masses

$$KE = \sum \left(\frac{1}{2} m r^2 \omega^2 \right) = \frac{1}{2} \left(\sum m r^2 \right) \omega^2 = \frac{1}{2} I \omega^2$$

DEFINITION OF ROTATIONAL KINETIC ENERGY

The rotational kinetic energy of a rigid rotating object is

$$KE_R = \frac{1}{2} I \omega^2$$

$$I = \sum_{i=1}^n (m r^2)_i$$

Requirement: The angular speed must be expressed in rad/s.

SI Unit of Rotational Kinetic Energy: joule (J)

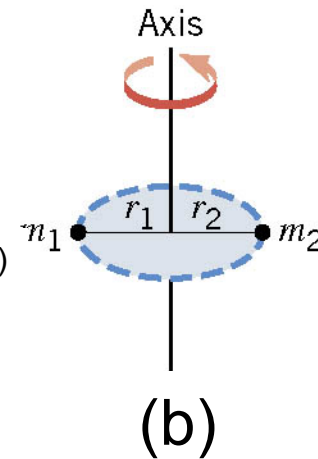
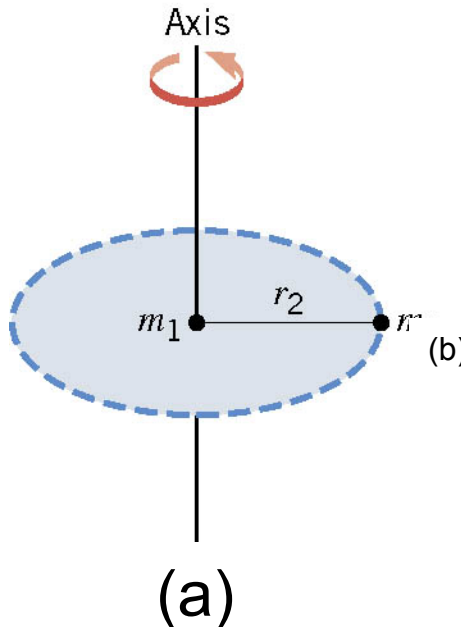
8.4 Rotational Work and Energy

Moment of Inertia depends on axis of rotation.

Two particles each with mass, m , and are fixed at the ends of a thin rigid rod. The length of the rod is L . Find the moment of inertia when this object rotates relative to an axis that is perpendicular to the rod at (a) one end and (b) the center.

$$m_1 = m_2 = m$$

$$r_1 = 0, \quad r_2 = L$$



$$r_1 = \frac{L}{2}, \quad r_2 = \frac{L}{2}$$

$$(a) \quad I = \sum_i (mr^2)_i = m_1 r_1^2 + m_2 r_2^2 = m(0)^2 + m(L)^2 = mL^2$$

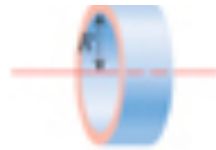
$$(b) \quad I = \sum_i (mr^2)_i = m_1 r_1^2 + m_2 r_2^2 = m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 = \frac{1}{2}mL^2$$

8.4 Rotational Work and Energy

Moments of Inertia of Rigid Objects Mass M

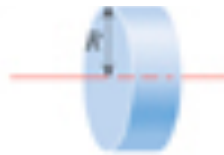
Thin walled hollow cylinder

$$I = MR^2$$



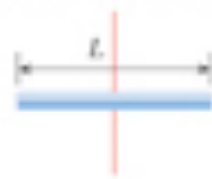
Solid cylinder or disk

$$I = \frac{1}{2} MR^2$$



Thin rod length ℓ through center

$$I = \frac{1}{12} M\ell^2$$

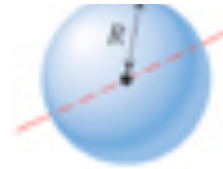


Thin rod length ℓ through end

$$I = \frac{1}{3} M\ell^2$$

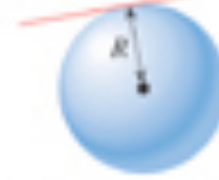


Solid sphere through center



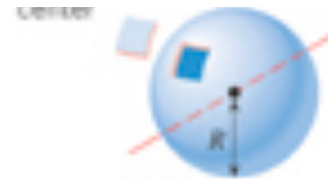
$$I = \frac{2}{5} MR^2$$

Solid sphere through surface tangent



$$I = \frac{7}{5} MR^2$$

Thin walled sphere through center



$$I = \frac{2}{3} MR^2$$

Thin plate width ℓ , through center



$$I = \frac{1}{12} M\ell^2$$

Thin plate width ℓ through edge

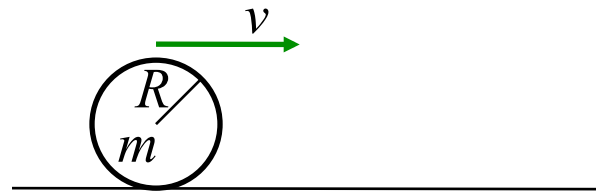


$$I = \frac{1}{3} M\ell^2$$

Clicker Question 8.3 Rotational Energy

A bowling ball is rolling without slipping at constant speed toward the pins on a lane. What percentage of the ball's total kinetic energy is translational kinetic energy?

- a) 50%
- b) 71%
- c) 46%
- d) 29%
- e) 33%



Hint: Express both KE types with $\omega^2 R^2$

$$KE = \frac{1}{2} m v^2, \text{ with } v = \omega R$$

$$KE_{rot} = \frac{1}{2} I \omega^2, \text{ with } I = \frac{2}{5} m R^2$$

$$\text{fraction} = \frac{KE}{KE + KE_{rot}}$$

Clicker Question 8.3 Rotational Energy

A bowling ball is rolling without slipping at constant speed toward the pins on a lane. What percentage of the ball's total kinetic energy is translational kinetic energy?

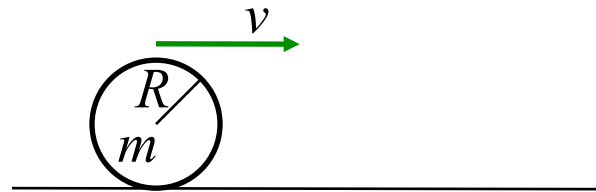
a) 50%

b) 71%

c) 46%

d) 29%

e) 33%



$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 R^2, \text{ because } v = \omega R$$

$$KE_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{5}m\omega^2 R^2, \text{ because } I = \frac{2}{5}mR^2$$

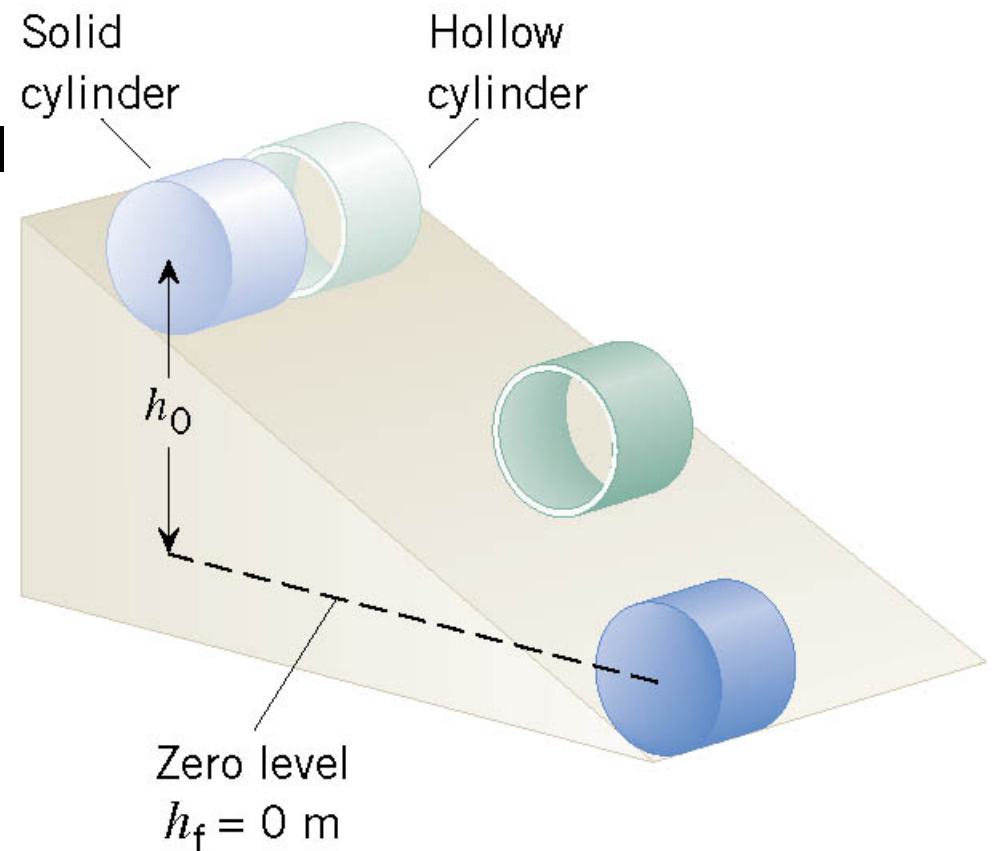
$$\frac{KE}{KE + KE_{rot}} = \frac{\frac{1}{2}}{\frac{1}{5} + \frac{1}{2}} = \frac{\frac{1}{2}}{\frac{2}{10} + \frac{5}{10}} = \frac{5}{7} = 71\%$$

8.5 Rolling Bodies

Example: Rolling Cylinders

A thin-walled hollow cylinder (mass = m_h , radius = r_h) and a solid cylinder (mass = m_s , radius = r_s) start from rest at the top of an incline.

Determine which cylinder has the greatest translational speed upon reaching the bottom.



8.5 Rolling Bodies

Total Energy = Kinetic Energy + Rotational Energy + Potential Energy

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgy$$

ENERGY CONSERVATION $E_f = E_0$

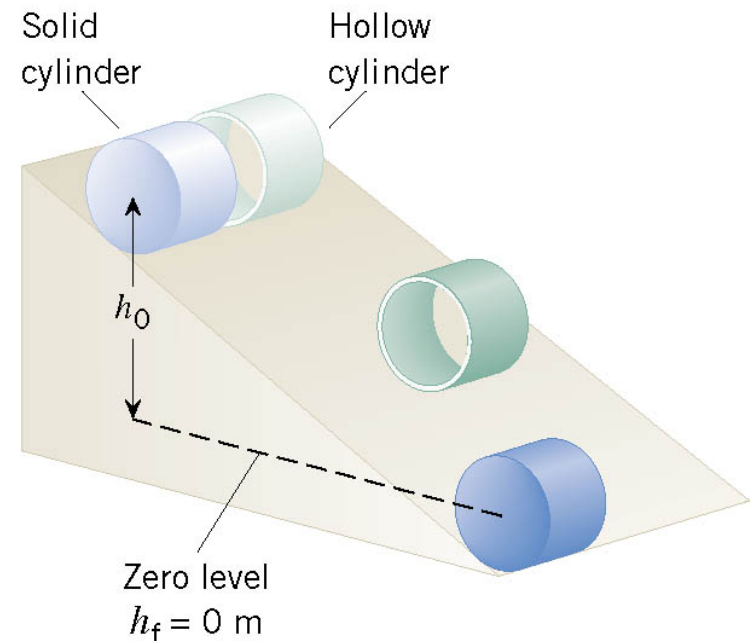
$$\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + mgh_f = \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2 + mgh_0$$

$$\boxed{\omega_f = v_f / r} \quad \frac{1}{2}mv_f^2 + \frac{1}{2}Iv_f^2 / r^2 = mgh_0$$
$$v_f^2(m + I/r^2) = 2mgh_0$$

$$\boxed{v_f = \sqrt{\frac{2mgh_0}{m + I/r^2}}}$$

The cylinder with the **smaller** moment of inertia will have a **greater** final translational speed.

Same mass for cylinder and hoop



8.6 *The Action of Forces and Torques on Rigid Objects*

Chapter 8 developed the concepts of angular motion.

θ : angles and radian measure for angular variables

ω : angular velocity of rotation (same for entire object)

α : angular acceleration (same for entire object)

$v_T = \omega r$: tangential velocity

$a_T = \alpha r$: tangential acceleration

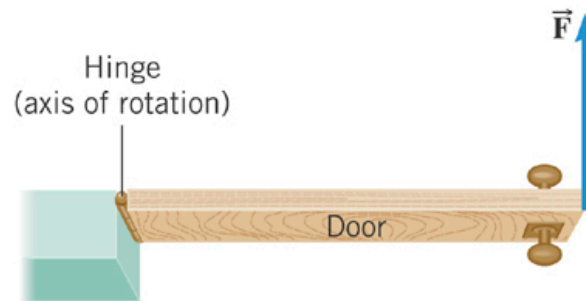
According to Newton's second law, a net force causes an object to have a ***linear acceleration***.

What causes an object to have an ***angular acceleration***?

TORQUE

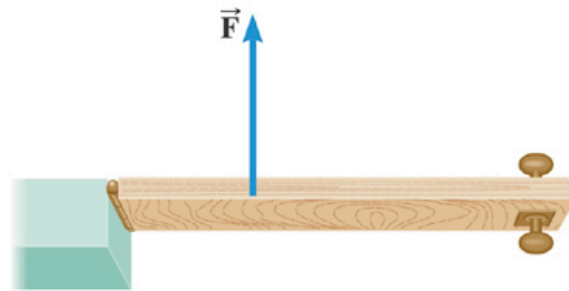
8.6 *The Action of Forces and Torques on Rigid Objects*

The amount of torque depends on where and in what direction the force is applied, as well as the location of the axis of rotation.



Maximum rotational effect of the force F .

(a)



Smaller rotational effect of the force F .

(b)



Rotational effect of the force F is minimal; it compresses more than rotates the bar

(c)

8.6 The Action of Forces and Torques on Rigid Objects

DEFINITION OF TORQUE

Magnitude of Torque = $r \times$ (Component of Force $\perp$ to $\vec{r}$)

$$\tau = rF_{\perp} = rF \sin \theta$$

Direction: The torque is **positive** when the force tends to **produce a counterclockwise rotation** about the axis.

SI Unit of Torque: newton x meter (N·m)

