Experiment 8

Thin Lenses

8.1 Objectives

- Measure the focal length of a converging lens.
- Measure the focal length of a diverging lens.
- Investigate the relationship between power and focal length.

8.2 Introduction

Lenses are devices that manipulate the path of light rays, allowing us to bend the rays toward or away from each other. Our eyes have evolved naturally to perform this task and enhance our vision, while humans have figured out how to fashion lenses out of glass and plastic, giving us viewing access to previously inaccessible phenomena from galaxies to bacteria. The physics of lenses is also used to detect the presence of "dark matter" surrounding galaxies (*gravitational lensing*), as well as to direct beams of atomic nuclei in particle accelerators like in the Cyclotron Building across the courtyard. However, the first step is to get some experience with thin lenses (thin enough so that we can ignore their thickness) where the math is somewhat more tractable than with more exotic lenses.

8.3 Key Concepts

As always, you can find a summary online at Hyperphysics[1]. Look for keywords: lenses, thin lenses, images, focal length, real images, virtual images and magnification.

8.4 Theory

A **converging lens** will cause light rays passing through it to be bent toward each other and the principal axis of the lens. If parallel light rays are incident on a converging lens, the light rays will converge at the focal point a distance f from the lens as shown in Fig. 8.1. The distance f is called the **focal length** of the lens. For converging lenses, f is positive. In general, converging lenses are thicker in the middle than they are at the outer edge of the lens.

A **diverging lens** will cause light rays passing through it to bend away from each other and the principal axis of the lens. If parallel light rays are incident on a diverging lens, the light rays will appear to diverge from the focal point a distance f from the lens as shown in Fig. 8.2. For diverging lenses, f is negative. In general, diverging lenses are thicker at the outer edge than they are in the middle.

[1] http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

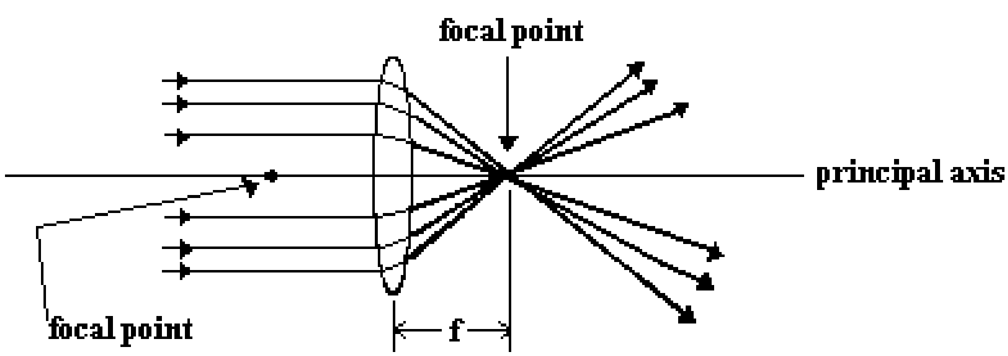

Figure 8.1: Parallel light rays passing through a converging lens converge at the focal point.

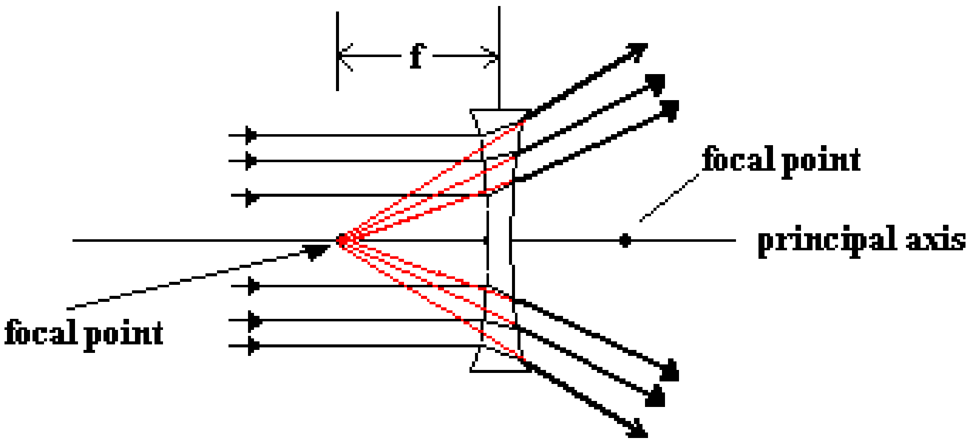

Figure 8.2: Parallel light rays passing through a diverging lens spread out. The light rays appear to diverge from the focal point on the front side of the lens.

Ray diagrams can be a useful tool for analyzing the behavior of light rays passing through thin lenses. Here are some rules for drawing light rays in ray diagrams.

1. A light ray approaching the lens parallel to the principle axis will pass through the focal point on the opposite side of the lens for a converging lens or appear to diverge from the focal point on the same side of the lens for a diverging lens (see Fig. 8.1 and Fig. 8.2).

2. A light ray passing through the center of the lens will not change direction as it passes through the lens.

3. A light ray passing through the focal point will be parallel to the principal axis when it leaves the lens.

Consider the ray diagrams shown in Figs. 8.3 and 8.4. For clarity, only rays 1 and 2 (explained above) are shown. Note in these figures that h is the height of the object, h' is the height of the image, p is the object distance (measured from the lens) and q is the image distance (measured from the lens). An image is formed where the rays intersect.

There are two types of images: real and virtual. **Real images** occur when the light rays actually converge on a point and form an image. These

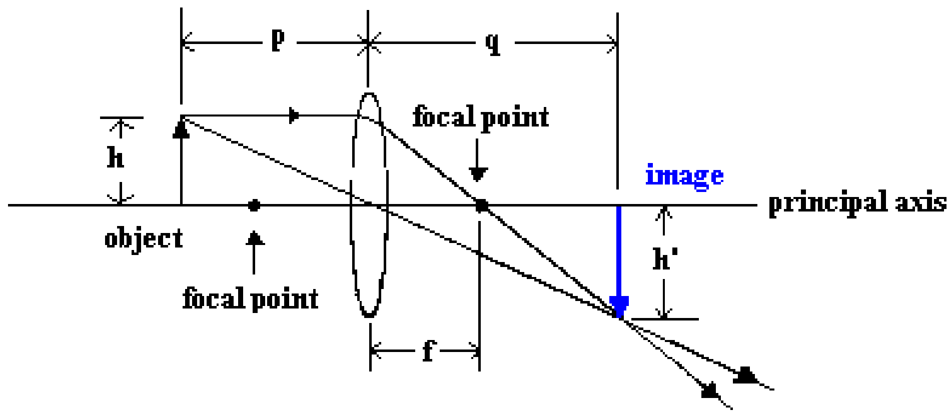

Figure 8.3: Real image formed by a converging lens.

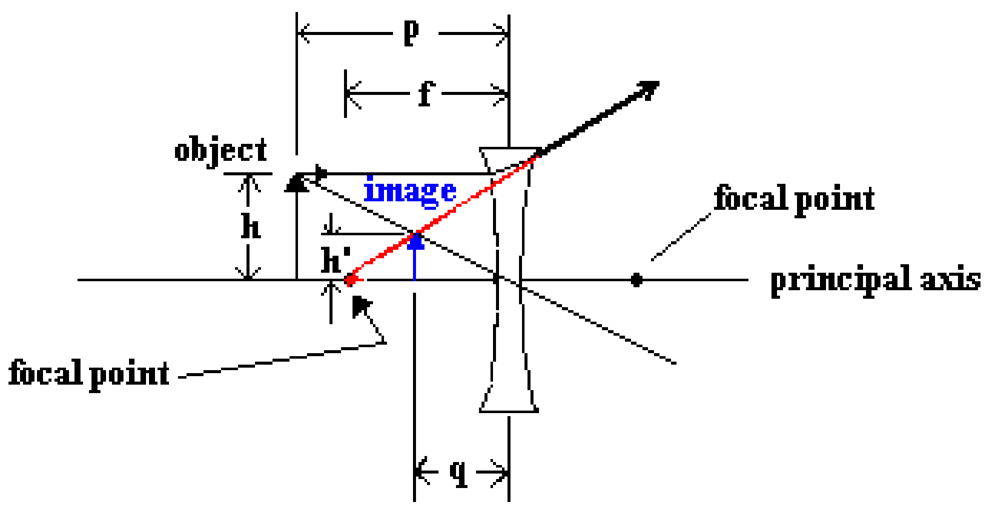

Figure 8.4: Virtual image formed by a diverging lens.

types of images can be displayed on a screen. A **virtual image** is formed when the light rays appear to diverge from a point. Because the rays do not actually diverge from this point (that is, pass through the point), they cannot be displayed on a screen. Converging lenses can form both real and virtual images. A diverging lens by itself can only form a virtual image. In the last part of this lab, you will use an image formed by a converging lens as the object for a diverging lens. The resulting image from this two-lens system can be real.

Sign conventions for lenses

When doing calculations for thin lenses you will need to follow the sign conventions listed below. Note that the front side of the lens is defined as the side of the lens from which the light is traveling (in the preceding diagrams it is the left-hand side of the lens). The back side of the lens is defined as the side to which the light is traveling (in the preceding diagrams it is the right-hand side of the lens).

- The object distance, p, is positive when the object is on the front side of the lens.
- The object distance, p, is negative when it is on the back side of the lens.
- The image distance, q, is positive when the image is on the back side of the lens.
- The image distance, q, is negative when it is on the front side of the lens.
- The object height, h, is always positive.
- Image height, h', is negative when the image is inverted (compared to the object).
- The image height, h', is positive when the image is upright (i.e. has the same orientation as the object).

For systems with a single optical element (i.e. just one lens), a real image will always have a negative image height (because the image is inverted) and a positive image distance. In contrast, a virtual image will always have a positive image height (because the image is upright) and a negative image distance.

Magnification

The intuitive definition of magnification is how much bigger (or smaller) an image is compared to the original object.

$$M = \frac{\text{image height}}{\text{object height}} = \frac{h'}{h} \tag{8.1}$$

Notice that for a real image formed by a single lens[2] the magnification is negative; the negative sign tells you that the image is inverted.

Referring back to Fig. 8.3, we can use the geometric notion of similar triangles to find an alternate expression for magnification in terms of the object and image distances, p and q.

$$\frac{h}{p} = -\frac{h'}{q}$$

Rearranging this expression we find:

$$-\frac{q}{p} = \frac{h'}{h}$$

The right-hand side is the same as in Eq. 8.1, therefore:

$$M = -\frac{q}{p} \tag{8.2}$$

Notice that you need the minus sign in Eq. 8.2 to get the same answer as Eq. 8.1. For example, both the image and object distances, p and q, for a real (inverted) image from a converging lens are positive, but because we defined magnification to be negative for inverted images you need a minus sign in Eq. 8.2.

[2]The single lens must be a converging lens, as a diverging lens cannot make a real image by itself.

Focal Length

The focal length f of a thin lens is related to the object distance p and image distance q by the following expression:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \tag{8.3}$$

If we rearrange Eq. 8.3 we obtain:

$$\frac{1}{q} = -\frac{1}{p} + \frac{1}{f} \tag{8.4}$$

Notice Eq. 8.4 has the same form as the equation of a straight line, $y = mx + b$. With $\frac{1}{q}$ playing the role of y, $\frac{1}{p}$ playing the role of x and $\frac{1}{f}$ as the intercept. What is the slope m in Eq. 8.4? (You will need this for question #2.)

If you wear corrective lenses you may have noticed that your eyeglass prescription has units of Diopters (D). This is because your prescription tells you the refractive **power** of your lens, which is a measure of its ability to bend light rays. The power P is related to the focal length f by

$$P = \frac{1}{f} \tag{8.5}$$

where the focal length is in meters; so a Diopter D is equivalent to meter^{-1}. (Be careful with units and make sure to convert centimeters to meters before calculating the power of a lens.) The power will be positive or negative depending on whether you are farsighted (hyperopic) or nearsighted (myopic). If you are farsighted (i.e. you can only see things that are far away) your prescription will have a positive power and focal length which tells the lens maker to use a converging lens. If you are nearsighted (i.e. you can only see things that are nearby) your prescription will have a negative power and focal length which tells the lens maker to use a diverging lens. For example, if your prescription is -4.0 D then you are nearsighted and need diverging lenses with a focal length of 0.25 m = 25 cm. Alternatively, if your prescription is +1.0 D then you are farsighted and need converging lenses with a focal length of 1 m.

8.5 In today's lab

First we'll become accustomed to forming images using a converging lens and use object and image distances to determine its focal length. Then we will form an image with a two lens system, using a converging and diverging lens, to determine the focal length of a diverging lens.

Measuring the focal length of a converging lens

In this experiment, you will measure a series of object distances and corresponding image distances for a converging lens and calculate its focal length. You will do this by constructing a graph of $\frac{1}{q}$ vs $\frac{1}{p}$ and using Eq. 8.4 to find the focal length.

Measuring the focal length of a diverging lens

To measure the focal length of a diverging lens, we will first use a converging lens to create a real image. This real image will then become the object for the diverging lens.[3] We will place the diverging lens such that the image formed by the converging lens is on the diverging lens's back side as shown in Fig. 8.5. Remember from the earlier section on "Sign conventions for lenses" the object distance p is defined to be negative if it is on the back side of the lens. This two-lens system, one converging and one diverging lens, allows us to make a real image that can be viewed on a screen. (Remember that a diverging lens alone can only make a virtual image which is not viewable on a screen.) Using the object and image distances from the diverging lens and Eq. 8.3, the focal length of the diverging lens can be calculated.

[3]For any multi-lens system (involving 2 or more lenses) understanding the entire system only involves following the simple rule that the image from the first lens acts as the object for the second lens. If there are > 2 lenses then the image from the second lens acts as the object for the third lens, etc.

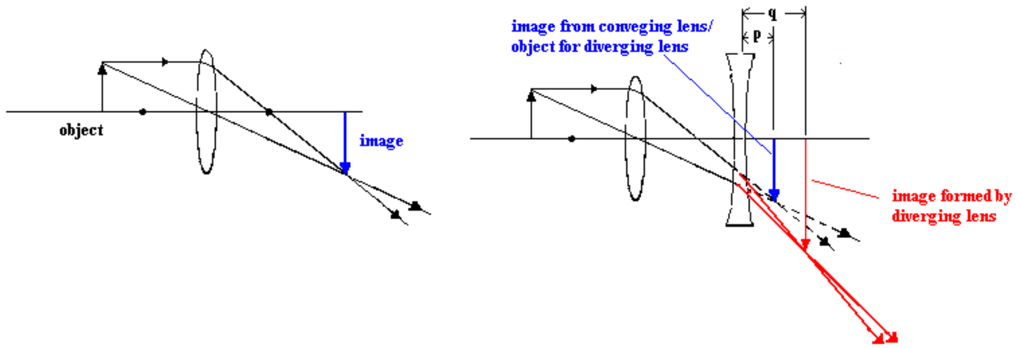

Figure 8.5: A converging lens forms a real image then a diverging lens is added to form a new real image which can be viewed on a screen.

8.6 Equipment

The experimental setup we will use to measure the focal length of a converging lens is shown in Fig. 8.6 and the experimental setup we will use to measure the focal length of a diverging lens is shown in Fig. 8.7.

- Optics bench
- LED object (shown in Fig. 8.8)
- Converging lens
- Diverging lens
- Screen

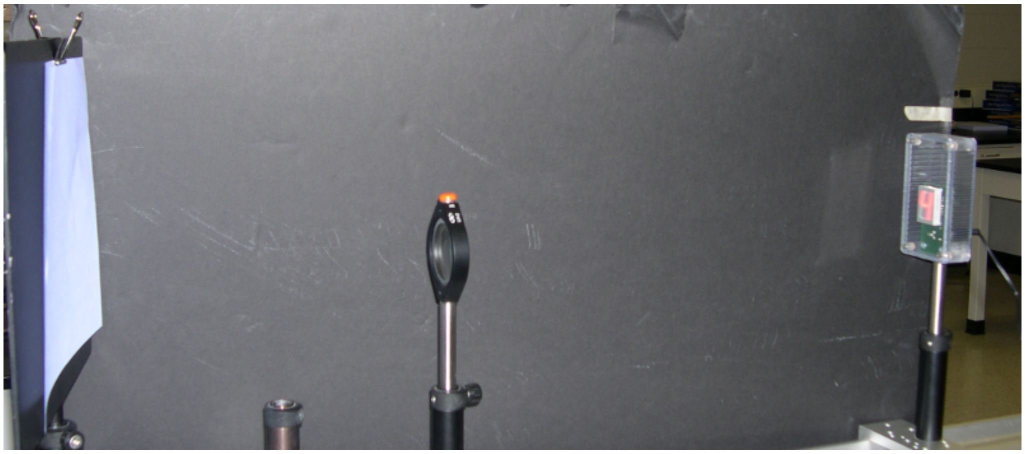

Figure 8.6: Optics bench with an object, a converging lens and a screen.

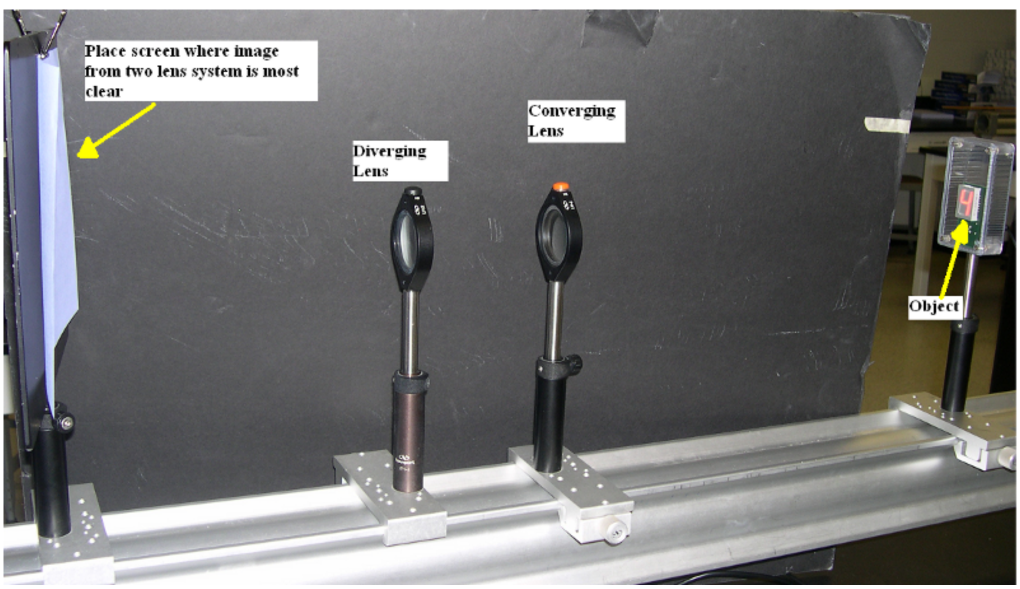

Figure 8.7: Optics bench with an object, a converging lens, a diverging lens and a screen.

8.7 Procedure

Focal length of a converging lens

1. Measure the height of the object as shown in Fig. 8.8 and record it in the thin lenses Excel spreadsheet.

2. Place a 25.0 cm focal length converging lens at a distance of $p = 60.0$ cm away from the object. Then adjust the screen until a clear image is formed. Assign a reasonable uncertainty to your object distance p and record it in your Excel spreadsheet. You should notice as you move the screen back and forth that there is a range over which the image appears clear. You should place the screen in the center of this range and use half of the range as your uncertainty in the image distance q. (Is this distance large compared to your uncertainty in reading the meter stick?) Measure the image distance q and record it and its uncertainty in your spreadsheet. Measure the height of your image as shown in Fig. 8.9 and record it plus a reasonable estimate of its uncertainty in your Excel spreadsheet.

3. Repeat step 2 for object distances of 55 cm, 50 cm, 45 cm and 40 cm.

4. Have Excel calculate the magnification (M_1) for each of your measurements using Eq. 8.1. Also, calculate the uncertainty[4] using:
$\delta M_1 = |M_1| \left(\left|\frac{\delta h}{h}\right| + \left|\frac{\delta h'}{h'}\right| \right)$

5. Have Excel calculate the magnification (M_2) for each of your measurements using Eq. 8.2. Also, calculate the uncertainty using
$\delta M_2 = |M_2| \left(\left|\frac{\delta p}{p}\right| + \left|\frac{\delta q}{q}\right| \right)$

6. Have Excel calculate $\frac{1}{p}$, $\frac{1}{q}$, and $\delta\left(\frac{1}{q}\right) = \frac{1}{q}\frac{\delta q}{q} = \frac{\delta q}{q^2}$. Import these data columns into Kaleidagraph and construct a graph of $\frac{1}{q}$ vs. $\frac{1}{p}$. Include vertical error bars and have Kaleidagraph fit it with a best-fit line with the relevant uncertainties included.

[4] The straight vertical lines "|" mean "absolute value of" and are needed because the size of an uncertainty is never negative. The Excel command for absolute value of a number is ABS(number).

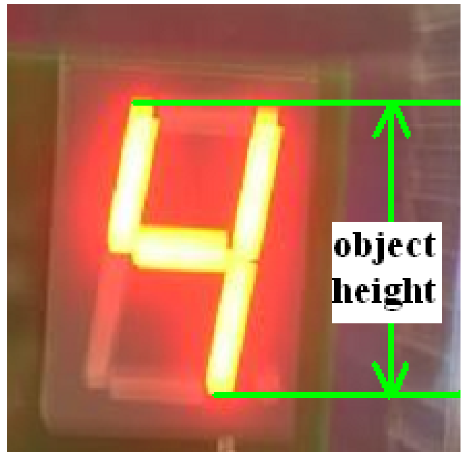

Figure 8.8: A picture of the LED number 4 that you will be using as your object showing how to measure its height.

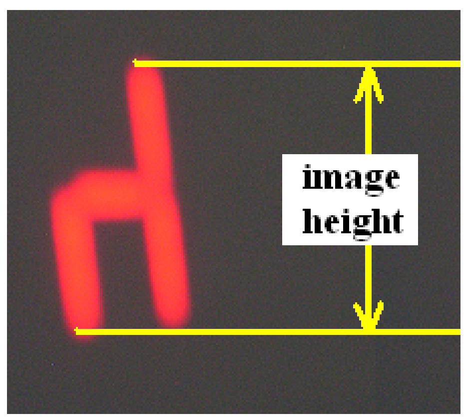

Figure 8.9: A picture of what your image should look like on the screen, notice that it is inverted (i.e. real), showing how to measure the image height.

Focal length of a diverging lens

1. Return the converging lens to a distance of 40.0 cm from the object and adjust the screen until a clear image is formed.

2. Place a diverging lens having a focal length of −30.0 cm at a distance of 10.0 cm from the screen (i.e. the diverging lens should be between the screen and the converging lens). By placing it in this location, your **object** distance for the diverging lens is −10.0 cm. Assign a reasonable uncertainty to this distance (remember you should include the uncertainty in finding the image formed by the converging lens).

3. Adjust the screen until a clear image is formed. Measure the image distance q. (Note this is the distance between the diverging lens and the screen.) Use Eq. 8.3 to calculate the focal length of the diverging lens. To calculate the uncertainty, use: $\delta f = f^2 \left(\left| \frac{\delta p}{p^2} \right| + \left| \frac{\delta q}{q^2} \right| \right)$ Remember that uncertainties are **always** positive. These calculations are part of Question 5.

8.8 Questions

1. Is M_1 consistent with M_2? Use your M_1 and M_2 values for an object distance of 50 cm. Explain.

2. What is the slope of your graph? Is it consistent with what you expect from Eq. 8.4? Explain.

3. From equation Eq. 8.4, the y-intercept of your graph should equal $\frac{1}{f}$. Use Eq. 8.4 and your graph to determine the focal length of the converging lens. Does it agree with the established value of $f = +25.0 \pm 2.0\,\text{cm}$? Explain.
   Hint: If the y-intercept and its uncertainty are $(\text{int}) \pm \delta(\text{int})$, then $\frac{\delta(\text{int})}{(\text{int})} = \frac{\delta f}{f}$.

4. Based on your measurement (not the established value), what is the power of the converging lens (in diopters)?

5. From your measurements, calculate the focal length of the diverging lens and its uncertainty. The formula for calculating the uncertainty in the focal length is given in the procedure.

6. Is your calculated focal length in Question 5 consistent with the established value of $f = -30.0 \pm 2.0\,\text{cm}$? Explain.

7. Based on your measurement (not the established value), what is the power of the diverging lens?